Performance of BDFM as generator and motor

R.A. McMahon, P.C. Roberts, X. Wang and P.J. Tavner

Abstract: The performance of the brushless doubly-fed machine (BDFM) is analysed using a per-phase equivalent circuit. An expression for the rating of the machine as a function of magnetic and electric loadings is developed, and the rating is compared to those of the doubly-fed induction machine and cascaded induction machines. As the magnetic field in a BDFM is complex, the magnetic loading is considered in detail and a new generalised loading is derived. The BDFM suffers a reduction rating of about one-quarter in comparison to comparable conventional machines, arising from penalties in magnetic and electric loadings consequent on the presence of two stator to rotor couplings. The handling of reactive power has an important effect on the machine performance and this point is illustrated with experimental results from a frame size 180 BDFM. The tests were carried out at modest flux densities to avoid the effects of saturation, but the implications of saturation are considered.

List of symbols

- \( X_1, X_2, X_r, X_c \) indicating a stator 1, 2, rotor or core model quantity \( X \)
- \( \mathbb{R}\{X\}, \mathbb{I}\{X\}, |X| \) denotes real, imaginary part, magnitude of complex \( X \)
- \( \arg(X) \) argument of complex \( X \), i.e. \( \arctan(\mathbb{I}\{X\}/\mathbb{R}\{X\}) \)
- \( N, n \) winding turns, turns ratio (real)
- \( R, L \) resistance, self inductance (real)
- \( X' \) indicates an apparent (referred) quantity
- \( S, Q, P \) apparent power, reactive power (real), real power (real)
- \( V, I \) complex voltage, current
- \( p_1, p_2 \) BDFM winding pole pairs
- \( \omega_r \) BDFM rotational shaft speed
- \( s \) slip (real)
- \( j \) \( \sqrt{-1} \)
- \( \phi \) phase angle of current (real)
- \( \delta \) load angle in synchronous mode (real)

In general, symbols denote complex quantities.

1 Introduction

Doubly-fed machines are attractive as variable speed drives or generators because the associated inverter only needs a fractional rating compared to that of the machine. Potential applications include generation from wind power [1, 2] and integrated drives [3]. This paper is concerned with one type of doubly-fed machine, the brushless doubly-fed machine (BDFM). The BDFM has two stator windings, each producing an airgap field of a different pole number, chosen to avoid transformer coupling between the stator windings. The rotor is specially designed to couple to both airgap fields. A comparison of the BDFM with other types of doubly-fed systems has been given by Hopfensperger and Atkinson [4].

Recent work on the BDFM includes the generalised analysis published by Williamson et al. [5], studies of power flow through the machine [6, 7] and the equivalent circuit approach proposed by Roberts et al. [8]. However, there remains a need to consider the operation of the machine from the perspective of terminal quantities.

In this paper we show how the use of the equivalent circuit for the machine leads to an understanding of operating conditions and hence how the rating of a BDFM can be established. [5, 9].

2 Basic configuration

A BDFM can operate in several ways but the synchronous, doubly-fed, mode is used for controlled variable-speed operation [5, 9]. In this arrangement, one winding, the power winding, is connected directly to the grid. The other winding, the control winding, is supplied with variable voltage at variable frequency from a converter also connected to the grid.

Stator and rotor quantities are shown for the synchronous mode in Fig. 1. The shaft angular velocity is given by:

\[
\omega_r = \frac{\omega_1 + \omega_2}{p_1 + p_2}
\]

and slips for the two windings can be defined as:

\[
s_1 \triangleq \frac{\omega_1 - p_1 \omega_r}{\omega_1} = \frac{\omega_1}{\omega_1}
\]

\[
s_2 \triangleq \frac{\omega_2 - p_2 \omega_r}{\omega_2} = \frac{\omega_2}{\omega_2}
\]
The frequencies of the fields due to stator 1 and stator 2 in the rotor reference frame are:

\[
\omega_{r1} = \omega_1 - p_1\omega_r \quad (4)
\]

\[
\omega_{r2} = \omega_2 - p_2\omega_r \quad (5)
\]

A further relationship for the angular velocity at the so-called 'natural' speed, that is the synchronous speed when the control winding is fed with DC (\(\omega_2 = 0\)), is given by:

\[
\omega_n = \frac{\omega_1}{p_1 + p_2} \quad (6)
\]

In the synchronous mode, from (1), (4) and (5) \(\omega_{r1} = -\omega_{r2}\).

Hence from (2) and (3):

\[
s_2 = -\frac{s_1}{s_2} \quad (7)
\]

### 3 Equivalent circuits for BDFM

A per-phase equivalent circuit for the BDFM based on two induction machines with connected rotors is given in [8]. As it was not possible to determine all the parameters explicitly from measurements, an electrically equivalent circuit for which all parameters could be found was proposed. This circuit, shown in Fig. 2, is used in this paper; the machine is taken to be three-phase. Parameters are shown referred to the power winding and iron losses are neglected. The circuit is valid for all modes of operation, including the synchronous mode. The parameters, in the unreferred form, are defined in Table 1.

![Fig. 1 BDFM synchronous mode of operation](image)

![Fig. 2 Referred per-phase equivalent circuit](image)

### Table 1: Definition of parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Power winding</th>
<th>Control winding</th>
<th>Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance</td>
<td>(R_1)</td>
<td>(R_2)</td>
<td>(R_s)</td>
</tr>
<tr>
<td>Series inductance</td>
<td>–</td>
<td>–</td>
<td>(L_r)</td>
</tr>
<tr>
<td>Magnetising inductance</td>
<td>(L_m)</td>
<td>(L_m)</td>
<td>–</td>
</tr>
<tr>
<td>Turns ratio to rotor</td>
<td>(n_1:1)</td>
<td>(n_2:1)</td>
<td>–</td>
</tr>
</tbody>
</table>

### Table 2: Power flows in a lossless BDFM

<table>
<thead>
<tr>
<th>Electrical power flows in or out of BDFM stator windings at different modes and speeds</th>
<th>Speed Below (\omega_n)</th>
<th>(\omega_n)</th>
<th>Above (\omega_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generating</td>
<td>Power winding</td>
<td>Control winding</td>
<td>Power winding</td>
</tr>
<tr>
<td>Motoring</td>
<td>Power winding</td>
<td>Control winding</td>
<td>Power winding</td>
</tr>
<tr>
<td>Generating</td>
<td>OUT</td>
<td>IN</td>
<td>OUT</td>
</tr>
<tr>
<td>Motoring</td>
<td>IN</td>
<td>OUT</td>
<td>IN</td>
</tr>
</tbody>
</table>

In this form of the equivalent circuit, the stator leakage inductances are not shown explicitly and the values of the magnetising inductances and turns ratios for the two windings therefore have slightly modified values. The referred and actual stator 2 supply voltage are related by:

\[
V_{2n} = \frac{n_1}{n_2} V_2 \quad (8)
\]

To simplify further analysis, we can now consider an equivalent circuit for the inner core of a BDFM, obtained by omitting the magnetising reactances and the stator and rotor resistances. However, in rotor designs reported to date, the rotor leakage reactance is relatively large and cannot be neglected [8]. A significant portion of the leakage inductance in these rotors is due to the high spatial harmonic content of the rotor field. The equivalent circuit, again referred power winding, then reduces to that shown in Fig. 3.

![Fig. 3 Referred per-phase equivalent circuit for core of BDFM](image)

The circuit resembles that for a synchronous machine with negligible stator resistance and can be analysed as such. The major, and crucial, difference is that the 'excitation', \(\frac{n_1}{n_2} F^{n_1}_c\), is induced from the control winding, via the rotor.

### 4 Machine operation

The BDFM can be operated as a motor or generator. The relative power flows in the machine windings depend on whether the machine is motoring or generating, and whether it is running at, below or above natural speed. The power flows in a lossless BDFM are listed in Table 2. The power supplied to or from the control winding, \(P_c\), can be related to that from the power winding, \(P_s\), by the fractional deviation in speed from the natural speed. From Fig. 3:

\[
P_{c} = 3\Re\{V_{c1}I_{c1}^*\} = 3\Re\{V_{c2}I_{c2}^*\} \quad (9)
\]

\[
P_{s} = 3\Re\{V_{s1}I_{s1}^*\} = 3\Re\{V_{s2}I_{s2}^*\} \quad (10)
\]

But from Fig. 3, \(\frac{n_1}{n_2} F^{n_1}_c\) = \(V_{c1} - j\omega_1 L_r I_{c1}\):

\[
P_{c} = -\frac{s_1}{s_2} \Re\{V_{s1}I_{c1}^* - j\omega_1 L_r I_{c1}I_{s1}^*\} = -\frac{s_1}{s_2} \Re\{V_{c1}I_{c1}^*\} - \frac{s_1}{s_2} P_{c} \quad (11)
\]
\[ P_{c} = P_{c1} \left( \frac{\omega_{r}}{\omega_{n}} - 1 \right) \text{ from (1), (6)} \]  

Previous analysis [8] shows that in the synchronous mode there are both synchronous and induction torque components. These can be seen from the following:

\[ T = \frac{3s_{2}R_{c} \{ V_{c2}^{2} \} \omega_{n}}{s_{1}\omega_{n}} + 3|V_{c2}^{2}| \left( \frac{p_{n} - p_{s}}{\omega_{1}s_{1}} \right) \]  

The torques collapse as the machine approaches the synchronous speed of the power winding. Away from that speed, the slip \( s_{1} \) is not small and the second component of torque is therefore relatively small. This is consonant with ignoring the rotor resistance in the equivalent circuit of the core of the BDFM and so the expression for the torque can be written in a similar way to that for an ideal conventional synchronous machine as (for proof see the Appendix):

\[ T_{k} = \frac{3(p_{1} + p_{2})}{\omega_{1}} \left| V_{c1} V_{c2}^{*} \omega_{n} \right| \sin \delta \]  

### 5 Analysis of operation

The following analysis is for the generating mode, with the load angle \( \phi \) taking a positive value. For a BDFM operating with a lagging load the circuit of Fig. 3 leads to the phasor diagram shown in Fig. 4. The diagram shows how varying the referred control winding voltage, \( V_{c2}^{*} \), has the same effect as varying the excitation in a synchronous machine, namely providing the ability to control the flow of VAr.

\[ V_{c2}^{*} = \frac{V_{c1} s_{1}}{s_{2}} \left( 1 - \frac{s_{1}}{s_{2}} \right) \cos \phi \cos(\phi + \delta) \]  

This is a referred quantity; the actual control winding quantity \( |V_{c2}| \) is given by

\[ |V_{c2}| = \frac{n_{2}}{n_{1}} V_{c1} \frac{s_{1}}{s_{2}} \cos \phi \cos(\phi + \delta) \]  

Substituting (6) into (1) and using (7) we may relate the slips and shaft angular velocities:

\[ \omega_{r} = \omega_{n} \left( 1 - \frac{s_{1}}{s_{2}} \right) \]  

\[ \therefore \]  

\[ |V_{c2}| = \frac{n_{2}}{n_{1}} |V_{c1}| \left( 1 - \frac{\omega_{r}}{\omega_{n}} \right) \cos \phi \cos(\phi + \delta) \]  

Inspection of (19) shows how the control winding voltage \( |V_{c2}| \) varies in proportion to the change in speed from natural. From (7) and (18) the frequency of \( |V_{c2}| \) also varies in this way:

\[ f_{2} = f_{1} \left( \frac{\omega_{r}}{\omega_{n}} - 1 \right) \]  

Furthermore, (19) shows how \( |V_{c2}| \) needs to be varied to reflect changes in the load angle \( \delta \) (reflecting machine torque) and power factor of the power winding.

From Fig. 3 the referred current in the control winding, \( I_{c2} \), can be written in terms of \( t_{c1} \) (note \( I_{c1} = -I_{c2}^{*} \)):

\[ I_{c2} = -\frac{I_{c1} m_{1}}{n_{2}} \]  

The control winding power, VArS and VA can now be found. It is convenient to express them in terms of \( P_{c1} \), the power delivered by the power winding:

\[ P_{c} = P_{c1} + P_{c2} = P_{c1} \frac{\omega_{r}}{\omega_{n}} \]  

The VArS in the power winding, \( Q_{c1} \), are

\[ Q_{c1} = -P_{c1} \tan \phi \]  

From (13), and noting that \( I_{c2} \) leads \( V_{c2} \) by \( \pi + \delta + \phi \), the control winding VArS, \( Q_{c2} \), are given by

\[ Q_{c2} = -P_{c2} \left( \frac{\omega_{r}}{\omega_{n}} - 1 \right) \tan(\phi + \delta) \]  

Substituting (24) into (25), \( Q_{c2} \) is related to \( Q_{c1} \) by

\[ Q_{c2} = Q_{c1} \tan(\phi + \delta) \left( \frac{\omega_{r}}{\omega_{n}} - 1 \right) \]  

This shows that, for a particular value of \( Q_{c1} \), \( Q_{c2} \) increases in magnitude with the deviation from natural speed. Alternatively, the ratio \( Q_{c2} \) to \( Q_{c1} \) can be described as an amplification of VArS and is given by

\[ \frac{Q_{c2}}{Q_{c1}} = \tan(\phi + \delta) \left( \frac{\omega_{r}}{\omega_{n}} - 1 \right)^{-1} \]  

The effective VAr amplification falls with increasing deviation from natural speed and for \( \omega_{r} > \omega_{crit} \), given by (28), every VAr exported by the power winding requires more than one extra VAr to be supplied to the control winding:

\[ \omega_{crit} = \omega_{n} \left( 1 + \frac{\tan \phi}{\tan(\phi + \delta)} \right) \]  

Adding (24) and (25), the total VArS \( Q_{c} \) in the BDFM is given by

\[ Q_{c} = Q_{c1} + Q_{c2} = P_{c1} \left( \frac{\omega_{r}}{\omega_{n}} - 1 \right) \tan(\phi + \delta) + \tan \phi \]  

The VAr flow in the control winding is reflected in the expression for the VA, \( |S_{c2}| \):

\[ |S_{c2}| = P_{c1} \left( \frac{\omega_{r}}{\omega_{n}} - 1 \right) \sec(\phi + \delta) \]  

The total rating of the machine, \( |S_{c1}| \), is given by the sum of the rating of the two windings, as they are electrically
independent, and the total rating is
\[ |S_e| = |S_{e1}| + |S_{e2}| \]
\[ = P_{c1} \left( \left| \frac{\omega_r}{\omega_n} - 1 \right| \sec(\phi + \delta) \right) + |\sec \phi| \]  
(31)

6 Machine ratings and effects of turns ratios

6.1 Turns ratios

Before developing an expression for the rating of the machine, it is necessary to define the turns ratios in the machine. It is convenient to use a rotor turns ratio, \( n_r \), and a stator turns ratio, \( n_s \), rather than turns ratios for the two airgap pole number systems. The stator turns ratio, \( n_s \), is given by
\[ n_s = \frac{N_{1s}K_{W1s}}{N_{2s}K_{W2s}} \]  
(32)

where \( N_{1s} \) and \( N_{2s} \) are the numbers of turns per phase in the power and control windings, respectively; \( K_{W1s} \) and \( K_{W2s} \) are the winding factors of the two windings, given by for example [10, Appendix B]. The rotor turns ratio \( n_r \) is
\[ n_r = \frac{N_{1r}K_{W1r}}{N_{2r}K_{W2r}} \]  
(33)

Similarly, \( N_{1r} \) and \( N_{2r} \) are the numbers of turns per phase in the power and control windings; \( K_{W1r} \) and \( K_{W2r} \) are their winding factors. The special design of BDFM rotors means that the rotor winding factors are not generally simple to evaluate. The ratio of stator to rotor turns for the power winding, \( n_1 \), is given by
\[ n_1 = \frac{N_{1s}K_{W1s}}{N_{1r}K_{W1r}} \]  
(34)

and the ratio for the control winding, \( n_2 \), is given by
\[ n_2 = \frac{N_{2s}K_{W2s}}{N_{2r}K_{W2r}} \]  
(35)

From (34) and (35) it can be shown that
\[ \frac{n_2}{n_1} = \frac{n_r}{n_s} \]  
(36)

6.2 Magnetic fields

From [10, equation (B-2)], the two stator voltages are related to the airgap fields by:
\[ |V_{e1}| = \frac{|Id|\omega_1}{p_1} N_{1s}K_{W1s}B_1 \]  
(37)

\[ |V_{e2}| = \frac{|Id|\omega_2}{p_2} N_{2s}K_{W2s}B_2 \]  
(38)

where \( B_1 \) and \( B_2 \) are the rms values of the fundamental 2\( p_1 \) and 2\( p_2 \) pole airgap flux densities, \( l \) is the axial length of the BDFM and \( d \) is the mean airgap diameter.

Substituting (7) into (18) gives:
\[ \frac{\omega_r}{\omega_n} - 1 = \frac{\omega_2}{\omega_1} \]  
(39)

The ratio obtained from (37) and (38), substituting into (39), is:
\[ \frac{|V_{e2}|}{|V_{e1}|} = \frac{p_1B_2}{n_1p_2B_1} \left| \frac{\omega_r}{\omega_n} - 1 \right| \]  
(40)

The stator voltages are also related by (19). Substituting (40) into (19) gives:
\[ \frac{B_2}{B_1} = \frac{n_r \cos \phi}{\cos(\phi + \delta)} \frac{p_2}{p_1} \]  
(41)

If the factor \( \frac{\cos \phi}{\cos(\phi + \delta)} \) can be approximated to unity, this expression reduces to the form derived in [11] from consideration of rotor voltages. Equation (41) is fundamental to the operation of a BDFM as it shows how the rotor windings define the relative magnitudes of the \( p_1 \) and \( p_2 \) pole pair fields.

6.3 Specific magnetic loading

In conventional induction machines designers use a specific magnetic loading, dependent on the electrical steel chosen, to achieve a balance between the effective use of the iron and undue saturation [12]. The question of what is the 'rated' magnetic flux density for a BDFM is not straightforward as the magnetic flux circulating in the BDFM is complex, as shown by the flux plots in [5] for a power winding voltage of 90 V rms. Details of the construction of the machine can be found in [8] and both rotor and stator use 0.65 mm thick laminations of a grade of steel with a nominal loss of 5.3 W kg\(^{-1} \) at a flux density of 1.5 T. With a supply voltage of 90 V rms, the highest flux density in the machine, found in the rotor teeth, has a value of 1.23 T\(_{\text{peak}} \). So saturation is not occurring in any part of the iron circuit. If the supply voltage is increased to 150 V rms, the peak flux density in the rotor teeth rises to 1.9 T\(_{\text{peak}} \) and therefore there is a degree of saturation. The corresponding peak flux density in a stator tooth is 1.34 T\(_{\text{peak}} \). With this in mind, experimental work has been carried out at a power winding voltage of 120 V rms to avoid the effects of saturation, although this will result in a magnetic loading below that prevailing in commercial machines.

The traditional explanation for the operation of the BDFM considers two independent flux systems related to the power and control windings, respectively. However, in a BDFM the fluxes are coupled via the rotor and the flux circulating in the machine is the resultant of MMFs from the three windings acting on either side of the airgap:

- The explicit MMF waves of \( p_1 \) and \( p_2 \) pole pairs of the power and control stator windings. These MMF waves are substantially sinusoidal, with a corresponding low space harmonic content, as the windings are distributed.
- The implicit MMF wave from the rotor winding with components of \( p_1 \) and \( p_2 \) pole pairs. The space harmonic content will depend on the degree of distribution of the rotor winding.

The resulting airgap flux comprises components of \( p_1 \) and \( p_2 \) pole pairs, plus space harmonics. If the two components are similar in magnitude, the flux can also be considered to be predominantly the product of \((p_1 + p_2)/2\) and \((p_1 - p_2)/2\) pole pair fields, which explains the six-pole field pattern apparent in Fig. 5 for a four-pole/eight-pole BDFM. The design of the rotor winding is particularly important. Distribution of the windings reduces space harmonics, hence rotor leakage inductance, which also results in a higher pull-out torque. However, this needs to be balanced against turns ratio, rotor resistance and ease of manufacture [13].

The issue of magnetic loading for the BDFM has received little attention. It is believed that the only significant contribution in this area is due to Broadway [15]. However, Broadway only discussed the issue for a
specific example, rather than presenting any general conclusions. Approaching from a different angle, Ferreira and Williamson [16], studied iron loss and saturation in a BDFM with a nested loop rotor in an attempt to produce saturation factors that could be used in analytical models but this work was not pursued further. Two independent flux systems are present in the induction motor described by Munoz-Garcia et al. [17] and they report a mode of operation in which the fields have different angular velocities, as in a BDFM.

The specific magnetic loading is traditionally defined as the mean absolute flux per pole in the airgap of a machine. We propose the following generalisation of the definition for use with a BDFM:

$$\bar{B} = \lim_{T \to \infty} \frac{1}{T} \int_0^T \frac{1}{2\pi} \int_0^{2\pi} |B(\theta)| d\theta dt$$

(42)

where \(B(\theta)\) is the flux density in the airgap, assumed uniform along the axis of the machine. It is clear that this definition is consistent with the concept of 'mean flux per pole', and the well-known result of \(B = \frac{2\sqrt{2}B_{\text{rms}}}{\pi}\) may be verified.

Ignoring harmonic fields, the magnetic fields in the airgap of the BDFM may be written as [5]:

$$B(\theta) = \sqrt{2}B_1 \cos(\omega_1 t - \omega_1 \theta) + \sqrt{2}B_2 \cos(\omega_2 t - \omega_2 \theta + \gamma)$$

(43)

where \(\gamma\) is an arbitrary phase offset. These fields may be referred to the rotor reference frame as follows:

$$\theta = \omega_0 t + \phi$$

(44)

The frequency of the currents in the rotor reference frame under steady-state conditions can be denoted by \(\omega_0\):

$$\omega_0 = \omega_1 - \omega_2 = -p_2 \omega_2 + \omega_2$$

(45)

From (45) we may write:

$$B_i(\phi) = \sqrt{2}B_1 \cos(\omega_1 t + p_1 \phi) + \sqrt{2}B_2 \cos(\omega_2 t - p_2 \phi + \gamma)$$

(46)

Therefore, from (42) the specific magnetic loading for the BDFM is:

$$B = \lim_{T \to \infty} \frac{1}{T} \int_0^T \frac{1}{2\pi} \int_0^{2\pi} |B_1 \cos(\omega_1 t + p_1 \phi) + B_2 \cos(\omega_2 t - p_2 \phi + \gamma)| d\phi dt$$

(47)

$$= \frac{\omega_0}{2\pi} \int_0^{2\pi} \int_0^{2\pi} |B_1 \cos(\omega_1 t + p_1 \phi + \gamma) + B_2 \cos(\omega_2 t - p_2 \phi + \gamma)| d\phi dt$$

(48)

This integral may be evaluated either analytically or numerically. It may easily be shown that when \(B_2 = 0\) then \(B = \frac{2\sqrt{2}}{\pi}B_1\) and when \(B_2 = B_1\) then \(B = 2\sqrt{2} (\frac{p_1}{p_2})^2\), regardless of the value of \(p_1, p_2, \omega_0, \gamma\).

Figure 6 shows how magnetic loading (scaled by \(\frac{3}{2\sqrt{2}}\)) given by (48) varies with \(B_2\) for unity \(B_1\). Although a simple proof has not been forthcoming, the same relationship was found regardless of the pole number combinations or offset angle chosen. Seventy pole pair number combinations were tried under the standard restrictions for a BDFM, that is \(p_1 \neq p_2\) and \(|p_1 - p_2| > 1\). Figure 6 also includes a plot of
The specific electric loading for an electrical machine with limited magnetic and electric loadings, which is otherwise ideal, is given by [12]:

\[ |S_{lm}| = \frac{\pi^2}{\sqrt{2}} \left( \frac{d}{2} \right)^2 \frac{1}{p} \frac{|\omega_s|}{BJ} \left( \frac{1}{n_r} + \frac{1 + n_p p_1 \cos \phi}{n_r p_1 \cos(\phi + \delta)} \right)^2 \times \left( \frac{\omega_s}{\cos \phi} - 1 \right) \frac{\cos \phi + 1}{\cos(\phi + \delta)} \]  

(56)

The BDFM total output power, \( P_c \), may be written in terms of the electric and magnetic loadings. Substituting for \( P_c \) into (23) using (31) gives the total power in terms of the total rating. Then substituting for \( |S_{lm}| \) using (56) gives:

\[ P_c = \frac{\pi^2}{\sqrt{2}} \left( \frac{d}{2} \right)^2 l_{10r} BJ \frac{p_1 + p_2}{p_1 (1 + \frac{1}{n_r})} \left( \frac{n_p p_2 \cos \phi}{p_1 \cos(\phi + \delta)} \right)^2 \cos \phi \]  

(57)

For a given physical size, magnetic and electric loadings, pole numbers and rotational speed the only free parameters are the rotor turns ratio and the phase angles \( \phi \) and \( \delta \). The value of \( n_r \) giving the maximum value of (57) may be obtained by solving \( \partial P_c / \partial n_r = 0 \), which is equivalent to solving \( \partial f(n_r) / \partial n_r = 0 \), \( f(n_r) \) being given as:

\[ f(n_r) = \left( \frac{1}{n_r} + \frac{1}{n_r} \right) \left( \frac{n_p p_2 \cos \phi}{p_1 \cos(\phi + \delta)} \right)^2 \]  

(58)

This gives a value of

\[ n_{rem} = \left( \frac{p_1}{p_2} \right)^{2/3} \frac{\cos(\phi + \delta)}{\cos \phi} \]  

(59)

The value of \( n_{rem} \) depends on the operating conditions, but assuming the term \( \cos(\phi + \delta) / \cos \phi \) is close to unity will give a benchmark value of \( n_r \). For a four-pole/eight-pole BDFM, this gives \( n_{rem} = 0.63 \). The turns ratio, \( n_1 / n_2 \), of the nested loop rotor used in the authors’ BDFM, including an adjustment allowing for stator leakage inductances, has a measured value of \( n_1 / n_2 = 0.685 \), and in this case
\( n_s \approx P_1 / P_2 \) by design. Hence \( n_s \approx 0.730 \), which is close to the optimum value.

### 7.1 Comparison with conventional wound rotor induction machine as a generator

It is interesting to compare the total power outputs of the BDFM and an ideal conventional induction machine. The comparison is made here between a doubly-fed induction generator with 2\( p \) poles operating at \( \omega_s = \omega_n \) and a BDFM at the same speed. The physical sizes, magnetic and electric loadings of both machines are set equal and the power factor of the stator of the induction machine equals that of the power winding in the BDFM. Under these conditions both the rotor of the induction machine and the control winding of the BDFM will be fed with DC. The output power of an induction machine \( P_{IM} \), is:

\[
P_{IM} = \frac{\pi^2}{\sqrt{2}} \left( \frac{d}{2} \right)^2 \frac{\omega_s}{p} \tilde{B} J \cos \phi
\]  

(60)

The angular velocity of the induction machine corresponding to the synchronous speed, \( \omega_s / p \), is made equal to the angular velocity of the BDFM at natural speed, \( \omega_n \), given by (6) (hence \( p = p_1 + p_2 \), for the same supply frequency).

From (60) and (57), the ratio of the power output from the BDFM, \( P_s \), to that from a doubly-fed induction machine, \( P_{IM} \), is:

\[
P_s = \frac{P_{IM}}{P_{IM}} = \frac{p_1 + p_2}{p_1 (1 + \frac{1}{n_s})} \left[ 1 + \left( \frac{n_s p_2 \cos \phi}{p_1 \cos (\phi + \delta)} \right) \right]^2
\]

(61)

As a benchmark, if the term \( \cos \phi / \cos (\phi + \delta) \) is close to unity and the rotor has the consequent optimum turns ratio, the relative output of BDFMs with up to 12 pole windings varies between a minimum of 71% for an eight-pole/12-pole machine to a maximum of 78% for a two-pole/12-pole machine. The reduction arises from the need to accommodate two stator windings, mitigated to an extent by the lower pole numbers of these windings, and the penalty in magnetic loading from having two independent stator to rotor couplings in the machine.

### 7.2 Comparison with two wound rotor machines in cascade

The concept and steady-state operation of two slip-ring induction machines in cascade is well known [12] and a special form of this arrangement, the twin stator machine, is described in [18]. The comparison made here assumes that the BDFM and the cascaded machines have the same pole pair combinations, the same overall active volume and size, the same magnetic and electric loadings and the same power factors. If the second machine of a cumulatively cascaded pair is supplied with DC, the set will rotate at an angular velocity of \( \omega_s / (p_1 + p_2) \). The expression in (60) gives the output \( P_{IM} \) of an induction machine with \( p \) pole pairs so the output of the second machine, \( P_{CS2} \), is given by

\[
P_{CS2} = \frac{\pi^2}{\sqrt{2}} l_2 \left( \frac{d}{2} \right)^2 \frac{\omega_s}{p_1 + p_2} \tilde{B} J \cos \phi
\]  

(62)

From [12] the power outputs are in the ratio of the pole pairs so the output of the first machine, \( P_{CS1} \), is

\[
P_{CS1} = P_{CS2} \frac{p_1}{p_2}
\]

(63)

The total output power is then

\[
P_{CS} = \frac{\pi^2}{\sqrt{2}} l_2 \left( \frac{d}{2} \right)^2 \frac{\omega_s}{p_2} \tilde{B} J \cos \phi
\]

(64)

To make a fair comparison, the combined length of the cascaded machines is made equal to that of the BDFM, so \( l_2 \) is given by

\[
l_2 = l \left( \frac{P_2}{p_1 + p_2} \right)
\]

(65)

Substituting this relationship into (64) and using the expression for the output of the BDFM from (57), the ratio is

\[
P_s = \frac{p_1 + p_2}{p_1 (1 + \frac{1}{n_s})} \left[ 1 + \left( \frac{n_s p_2 \cos \phi}{p_1 \cos (\phi + \delta)} \right) \right]
\]

(66)

which is the same as that from the comparison between the BDFM and the standard induction machine given in (61).

### 8 Magnetising VArS and copper losses

So far the analysis has not included magnetising currents and copper losses. The magnetising VArS in the power winding are

\[
Q_m = \frac{3 \left\| V_c \right\|^2}{\omega_1 L_{m1}}
\]

(67)

The VArS in the control winding are

\[
Q_{m2} = \frac{3 \left\| V_c \right\|^2}{\omega_2 L_{m2}}
\]

(68)

From [10, equation (B-2)]

\[
L_{m1} = \frac{L_m p_1^2}{n_s p_2^2}
\]

(69)

The absolute values of the VArS depend on the airgap of the BDFM, but by substituting (19) into the ratio (68) over (67) and using (18), we can write the ratio of the magnetising VArS:

\[
\frac{Q_{m2}}{Q_{m1}} = \left( \frac{n_s^2 p_2^2}{p_1^2} \right)^{2/3} \left( \frac{\cos \phi}{\cos (\phi + \delta)} \right)^{2/3} \left( 1 - \frac{\omega_s}{\omega_n} \right)^{1/3}
\]

(70)

Substituting for the optimum turns ratio from (59), the ratio of VArS becomes

\[
\frac{Q_{m2}}{Q_{m1}} = \left( \frac{P_2}{p_1} \right)^{2/3} \left( \frac{\cos \phi}{\cos (\phi + \delta)} \right)^{2/3} \left( 1 - \frac{\omega_s}{\omega_n} \right)^{1/3}
\]

(71)

Equation (71) shows that the magnetising VArS for the control winding, relative to the power winding, depend on the ratio of the pole pair numbers raised to the power of two thirds and on the operating conditions through the term \( \left( \frac{\cos \phi}{\cos (\phi + \delta)} \right)^{2/3} \). The control winding VArS are also proportional to the deviation from natural speed, as expected. For a four-pole/eight-pole BDFM with the turns ratio from (59) and the term \( \left( \frac{\cos \phi}{\cos (\phi + \delta)} \right)^{2/3} \) equal to unity, the control winding VArS are almost 60% greater than the power winding VArS for the same excitation voltage and frequency and this has implications for the ratings of the control winding and the inverter which supplies it. The magnetising VArS obviously depend on the machine airgap and can be reduced by making the airgap as small as practical.

There are copper losses in the power, control and rotor winding resistances, which are calculated by the usual method.
9 Practical operation in generation mode

9.1 Consideration of reactive power flow
The flow of reactive power is important in the design and operation of a BDFM as a generator and directly affects the potential power output and losses. Although varying the voltage on the control winding alters the power factor on the power side of the BDFM, akin to varying the excitation in a conventional synchronous machine, the need to consider the supply of VArS to the control winding, other than at natural speed, is a major difference. In fact the flow of VArS in the BDFM is reversible and is subject to the ‘VAr amplification effect’ identified in Section 5. In addition, there are magnetising VArS to be considered.

The effects of VAr flow have been confirmed with experimental results from the authors’ frame size 180 machine with a nested loop type rotor. The power winding was connected to a 50 Hz supply of adjustable voltage and the control winding was fed with a variable voltage, variable frequency supply from an inverter. The BDFM was driven at a range of speeds and torques by a DC machine. Parameters for the equivalent circuit model of Fig. 2 are given in Table 3. Further details of the machine and the experimental method of determining the parameters are given in [8]. It should be noted that the fill factor in the slots is lower than would be achieved in normal manufacturing so the machine carries a resistance penalty of about 30%.

Table 3: Measured parameter values (referred to four-pole winding)

<table>
<thead>
<tr>
<th>$R_1$ (Ω)</th>
<th>$R_1^*$ (Ω)</th>
<th>$L_1^*$ (mH)</th>
<th>$L_m$ (mH)</th>
<th>$R_2^*$ (Ω)</th>
<th>$L_m^*$ (mH)</th>
<th>$\frac{n}{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.63</td>
<td>1.26</td>
<td>35.1</td>
<td>277</td>
<td>2.46</td>
<td>101</td>
<td>0.685</td>
</tr>
</tbody>
</table>

9.2 Generation and transfer of VArS
Calculated values of control winding VArS as a function of power winding VArS using the core model of Fig. 3 are shown in Fig. 7. The speed was 750 rpm, that is with 25 Hz supplied to the control winding, with a driving torque of 25 Nm. As there are no losses, the power outputs from the power and control windings are given by (73) and (13) and are 0.97 and 0.48 kW, respectively, from which power factors can be calculated. The load angle delta was 11.8°, calculated using (78). The power winding voltage was 120 V rms, chosen to avoid any effects of saturation.

The machine can be described as over-excited if (\(s_2/s_1\)) $V_{c2}$ is greater than $V_c$, and under-excited if smaller than $V_c$, using the notation of Fig. 3. When the two voltages are equal, half the rotor VArS are supplied from each winding.

As the excitation via the control winding is increased, the VArS drawn by the power winding decrease, falling to zero when all the rotor VArS are supplied by the control winding. Further increases in excitation cause the export of VArS from the power winding. In contrast, under-excitation causes the VArS drawn by the control winding to reduce, falling to zero when all the rotor VArS are supplied from the power winding. Further reductions in excitation result in the transfer of VArS to the control winding. The ratio of the VArS transferred is two to one as expected from (26).

Figure 7 also shows how the control winding VArS relate to the power winding VArS using the equivalent circuit of Fig. 2. Predictions are shown, along with experimental results, which are in close agreement. It is clear that the control winding magnetising VArS, given by (68), are significant, being much larger than the VArS transferred through the machine. Inspection of Fig. 7 shows that over the operating range of interest, generating a VAr on the power winding side requires approximately one VAr on the control winding side, a much less favourable situation than predicted from the core model.

The control winding voltage, as a function of the flow of reactive power in the power winding, is shown in Fig. 8. Once again, the predicted results agree well with the experimental data. Also shown is the voltage calculated from the core model; the difference between the two models reflects voltage drops in the additional series terms.

![Fig. 7 Control winding VArS against power winding VArS, using machine parameters shown in Table 3](image)

Power winding voltage is fixed at 120 V rms. Negative sign of VArS stands for absorbing VArS from grid; positive sign of VArS stands for providing VArS to grid.

9.3 Effect of reactive power flow on operating conditions
From the perspective of a grid connected power winding, over-exciting the control winding to enable the machine to supply its own rotor and power winding magnetising VArS, and even to export VArS to a load, appears attractive,
especially at low deviations from natural speed where (27) shows a substantial VAR magnification effect. However, the flow of VARs affects both the magnetic and electric loadings of the machine and hence the maximum potential output power. This is illustrated in Fig. 9 where the power output divided by the product of the electric and magnetic loadings, shaft speed and machine volume is shown against control winding voltage. Plots are shown for both the core model and the equivalent circuit model of Fig. 2, and the magnetic and electric loadings for each operating point are obtained from (49), (52) and (53). The power winding voltage was fixed at 120 V rms, the shaft speed was 750 rpm and the prime mover torque was 25 Nm.

From the core model, the optimum value of control winding voltage is 86 V rms, corresponding to a slight under-excitation of the control winding (at 750 rpm $\frac{2}{3}V_{c0}$ equals $V_c$, when the control winding voltage is 87.6 V rms). Including losses and magnetising VARs reduces the optimum voltage to 76 V rms. The difference can be largely explained by the fact that under-excitation the control winding reduces the reactive power flows within the machine. Optimum operating conditions will depend on speed and torque and a full exploration of these is beyond the scope of this paper.

10 Discussion

The analysis of the BDFM as a variable-speed generator, or drive, using a per-phase model provides a straightforward approach to predicting the performance of the machine, which in turn gives insight into how to design the machine. The analysis highlights several key issues in the use, performance and ultimately the economic viability of the BDFM.

10.1 Management of reactive power

The operation of the BDFM is similar to that of a synchronous machine in that VARs can be generated in the power winding, although this must be taken into account in the machine rating. However, in a BDFM the generation of VARs has consequences for the control winding. If the desired deviation from natural speed is relatively small, the VAR amplification effect will limit the VARs transferred from the control winding and the low excitation frequency will limit the magnetising VARs. Therefore the total VARs supplied to the control winding, and hence the VARs required from the inverter feeding the control winding, will be modest.

In contrast, where large variations in speed are needed, the reduction in VAR amplification and the rising magnetising VARs mean that the VARs in the control winding are large, increasing the required ratings of the control winding and the supplying inverter. It then becomes attractive to reduce the control winding voltage, even to the extent of reversing the flow of VARs through the BDFM, to limit the total VARs in the control winding. If this is not done the principal attraction of the BDFM, namely the use of a fractionally rated converter, is at least partially negated. In addition, minimising the magnetising VARs by keeping the machine airgap as small as practical is beneficial.

The reduction in the demand for VARs by reducing the control winding voltage has been shown in Section 9. However, this strategy will increase the VARs drawn by the power winding; in addition the load may well require VARs. When the control winding is fed from a bidirectional converter, the power winding VARs, and any load VARs, can be supplied by the line-side converter and the control winding VARs by the machine-side converter. This leads to the possibility of minimising the overall inverter and machine ratings.

10.2 Effect of rotor reactance

The term $\frac{\cos(\phi+d)}{\cos\phi}$ occurs in the expressions for VAR transfer, magnetic loading and the optimum turns ratio. In the nested loop type rotor the leakage reactance is larger than for a cage rotor, with approximately half the rotor resistance can be conceived.

10.3 Performance relative to other machines

A comparison with a doubly-fed induction machine with $p_1+p_2$ pole pairs shows that, with the same volume of active material, the output of the BDFM is about 25% less at the same speed if the definition of the magnetic loading given by (49) is used. This comparison is based on the core model assuming that the term $\frac{\cos(\phi+d)}{\cos\phi}$ is unity. The reduction arises from penalties in the electric and magnetic loadings consequent on having two independent stator to rotor couplings in the machine, mitigated to a degree by lower pole numbers of the two couplings. However, once the elimination of the slip rings and brushgear is taken into account, the size of the machines will be closer.

Analysis also shows that the same expression applies for the performance of the BDFM relative to two induction machines in cascade, including the special case of the twin stator machine of Smith [18]. The BDFM was in fact originally described as a self-cascaded machine, and both systems use two magnetic couplings. With two separate
machines in cascade, the couplings have independent iron circuits so standard magnetic loadings can be applied. There are two sets of magnetising VArS and the management of reactive power is subject to the same constraints as in the BDFM. However, the BDFM has the advantage of simpler construction and lower rotor circuit losses. In addition, the BDFM should have a lower rotor impedance leading to a higher torque capability.

In conclusion, the performance and the cost of the BDFM relative to its competitors depend directly on the achievable magnetic loading, and this is discussed in the next Subsection.

10.4 Magnetic loading
From experimental work, the effects of saturation become apparent when the peak flux density in the iron circuit, given by the sum of the amplitudes of the $p_1$ and $p_2$ pole pair fields, reaches the saturation value at some point in the magnetic circuit. If the machine is operated at higher voltages (flux densities), discrepancies between measured values and values calculated from the equivalent circuit become increasingly apparent. However, simulations performed using a finite-element analysis package (MEGA) incorporating the effects of saturation give correct predictions, confirming the view that the discrepancies arise from saturation effects; this is also consistent with observations made by Williamson and Ferreira [9].

In the present machine, saturation, that is a local flux density exceeding, say, 1.8 $T_{\text{peak}}$, commences in the rotor teeth at a power winding voltage of around 1.50 V rms. The corresponding specific magnetic loading ($B$) is only 0.3 T, so the available torque is restricted. Torques in excess of 100 Nm have been obtained from the machine by using a power winding voltage of 220 V rms but at the expense of increased magnetising current [8]. However, if the rotor were redesigned so that the rotor teeth and stator teeth saturated at the same magnetic loading, about 0.5 T, it should be possible to obtain, without undue saturation, the torque (140Nm) normally expected from a frame size 180 machine.

Nevertheless, the ratio of peak to rms values of the airgap flux density distribution is greater in a BDFM than in an electrical machine with only one MMF field on the stator, as is apparent in Fig. 6, which shows the difference between the summation value of $B_1$ and $B_2$ and the rms value. Therefore there will be a balance between achieving a high magnetic loading and the acceptance of a degree of local saturation, with the consequent redistribution of flux to lower reluctance paths. A more precise determination of the acceptable magnetic loading is an important future step in the study of the BDFM.

11 Conclusions
The equivalent circuit approach has been shown to be successful in predicting the performance of the BDFM. The equivalent circuit is valid and has constant parameters provided that saturation of the iron circuit does not exceed the localised saturation found in normal commercial induction motors. The rating of the BDFM has been established from the equivalent circuit. The relative magnitude of the two principal airgap fields, in turn related to the rotor turns ratio, and the operating conditions both have a marked effect on the rating. Given optimum operating conditions, the output of a BDFM is about 25% less than that from a conventional induction machine with the same volume of active material, assuming that the machines are designed for the same rotational speed.

12 Acknowledgments
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13 References

14 Appendix
14.1 Torque equation derivation
Substituting [8, equations (11) and (12)] into [8, equations (9) and (10)] gives:

$$P_{r_1} = \frac{s_2}{s_1} P_{r_2} = 3 \left( I_{r_1}^2 \right) \frac{R'}{s_1}$$

(72)

Substituting (72) into [8, equation (14)] gives:

$$T = \frac{P_{r_1}}{\Omega_1} - \frac{P_{r_2} s_2}{\Omega_1 s_1}$$

(73)
From (9), (10) and Fig. 2:

\[ T = \frac{3 p_1}{\omega_1} \left( \Re \left( V_{c} l'' \right) \right) + \frac{3 p_2 s_2}{s_1 \omega_1} \left( \Re \left( V_{1} l'' \right) \right) \]  

(74)

Writing \( l'' \) in terms of \( V_{c} \) and \( V_{c} \) = \( \frac{p_2}{p_1} \) \( l'' \):

\[
\begin{align*}
&= \frac{3 p_1}{\omega_1} \Re \left\{ V_{c} \left( \frac{V_{e1} - V_{e2}^*}{Z_r} \right) \right\} + \frac{3 p_2}{\omega_1} \Re \left\{ V_{c2} \left( \frac{V_{e1}^* - V_{e2}^*}{Z_r} \right) \right\} \\
&= \frac{3 p_1}{\omega_1} \Re \left\{ \frac{V_{e1} V_{e2}^*}{Z_r} + \frac{|V_{e1}|^2}{Z_r} \right\} + \frac{3 p_2}{\omega_1} \Re \left\{ - \frac{|V_{e2}|^2 V_{c1} V_{c2}^*}{Z_r} \right\} \\
&= \frac{3 p_1}{\omega_1} \left( - \frac{|V_{e2}|^2 \cos(\psi + \delta) + \frac{|V_{e1}|^2}{Z_r} \cos(\psi) \right) \\
&\quad + \frac{3 p_2}{\omega_1} \left( - \frac{|V_{e2}|^2}{|Z_r|} \cos(\psi) + \frac{|V_{e1} V_{c2}|}{|Z_r|} \cos(\psi - \delta) \right) \\
&= \frac{3}{\omega_1} \left| V_{c1} V_{c2} \right| \sqrt{p_1^2 + p_2^2 - 2 p_1 p_2 \cos 2\psi} \\
&\quad \times \sin \left( \delta - \arctan \left( \frac{(p_1 - p_2) \cos \psi}{(p_1 + p_2) \sin \psi} \right) \right) \\
&\quad + \frac{3}{\omega_1} \left( \frac{p_1 |V_{c1}|^2}{|Z_r|} - \frac{p_2 |V_{c2}|^2}{|Z_r|} \right) \cos(\psi) \\
\end{align*}
\]

(75)

where \( Z_r = R_{r}^{'}/s_1 + j \omega_1 L_{r}^{'} \), \( \psi = \angle Z_r \) and \( \delta = \angle V_{e1} - \angle V_{e2} \), the phase angle between \( V_{e1} \) and \( V_{e2} \). Hence if \( R_{r}^{'} = 0 \) then:

\[
T_{R_{r}^{'} = 0} = \frac{3(p_1 + p_2)}{\omega_1} \frac{|V_{c1} V_{c2}^*|}{|s_1 L_{r}^{'} L_{s1}^{'}|} \sin(\delta) \\
\]

(76)

(77)

(78)