Progress in Conflict Detection Research

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Presentation

- Wind Correlated Conflict Probe
  - Wind Correlation Model
  - Monte Carlo Implementation
  - Results
- Sensitivity of Conflict Probes to Geometry
  - Sensitivity Problem
  - Proposed Solution
  - Validation
- Further Work

Recap - WG Model

Highly complex; one essential component is correlated wind:

$$\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_k \end{bmatrix} = \begin{bmatrix} B_{1,1} & 0 & \dots & \dots & 0 \\ B_{2,1} & B_{2,2} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ B_{k,1} & B_{k,2} & \dots & \dots & B_{k,k} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_k \end{bmatrix}$$

$$w_{k_1} = B_{k_1,k_1} n_{k_1} + B_{k_1,k_1-1} n_{k_1-1} + \dots + B_{k_1,1} n_1$$

B matrices determined from a/c positions and correlation structure:

$$\rho(t, t^*, P, P^*) = \sigma(z)^2 \exp -\lambda |t - t^*| \exp\left(-\beta \left\| P - P^* \right\|\right).$$

Wind Correlation Model

We wish to perform Monte Carlo simulation, which involves simulating a ‘large’ number of trajectories (particles). Efficient MC simulation requires a simple model.

Assumption 0.1 *Assumptions simplifying WG model for MC:*

1. *Use the same* B *matrix for all particles*
2. *Approximate flight path by straight line between observations.*

We repeat the MC simulation each time a new observation is received.

MC Implementation

Recall the additive model of wind disturbances:

$$\dot{\psi} = \frac{1}{mV}(L\sin(\phi) + T\sin(\phi)),$$
$$\dot{X} = V\cos(\psi) + w_1,$$
$$\dot{Y} = V\sin(\psi) + w_2.$$

Hence we may infer the wind acting on the aircraft, and the innovations n_i:

$$n_{k_1} = \left(w_{k_1} - \sum_{i=1}^{k_1-1} B_{k_1,i} n_i \right) B_{k_1,k_1}^{-1}$$

MC implementation

- The wind at time steps after k_1 has a deterministic and an uncertain component:

$$w_{k_2} = \tilde{w}_{k_2} + \breve{w}_{k_2},$$

$$\tilde{w}_{k_2} = \sum_{i=1}^{k_1} B_{k,i}\mathbf{n}_i,$$

$$\breve{w}_{k_2} = \sum_{i=k_1+1}^{k_2} B_{k,i}n_i.$$

- The terms for low i dominate the wind, so *uncertainty is greatly reduced*.

Results

- Reduced uncertainty improves trajectory prediction greatly.
- Conflict detection is much improved compared to 'uncorrelated' MC.

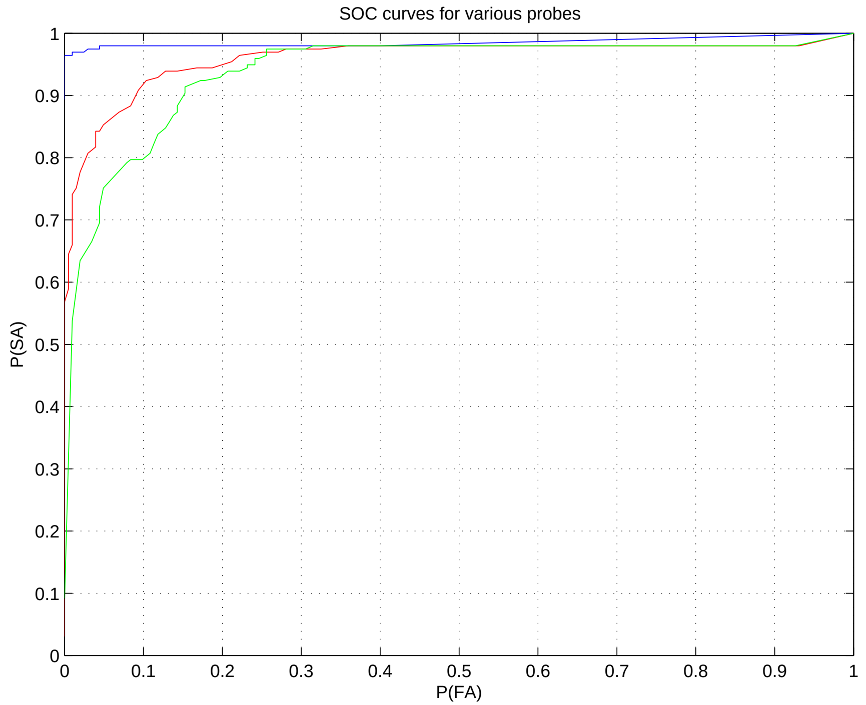

Figure 1: SOC curves for the wind correlated MC probe (blue), an uncorrelated MC probe (green) and the randomized probe of Lygeros and Prandini (Red).

Sensitivity Problem

- In the current and near term ATC architectures conflict alerting is a 1/0 decision.
- When the measure of conflict probability exceeds some threshold $\bar{C}$ an alert is given.
- The optimal threshold is given by

$$\bar{C}_{opt} = \arg\min_{\bar{C}} \sqrt{(1 - P(SA))^2 + P(FA)^2} \qquad (1)$$

And is *known to vary with conflict geometry*.

Solutions

Two solutions present themselves:

1. Apply one threshold over all geometries.
2. Attempt to approximate optimal threshold as a function of encounter geometry.

Option 1 is very simple and straightforward, however, *any success* in solving option 2 will outperform option 1.

Outline Method

Perform MC simulation to build up learning databases over ‘geometries of interest’ $\mathcal{S}$:

$$D_c = \{(s^{(i)}, I_v^{(i)}(s)) : s^{(i)} \in \mathcal{S}, I_v^{(i)} \in \{0,1\}\},$$
$$D_{FA} = \{(s^{(i)}, I_{FA}^{(i)}(s)) : s^{(i)} \in \mathcal{S}, I_{FA}^{(i)} \in \{0,1\}\},$$
$$D_{SA} = \{(s^{(i)}, I_{SA}^{(i)}(s)) : s^{(i)} \in \mathcal{S}, I_{SA}^{(i)} \in \{0,1\}\}.$$

Use a probabilistic classifier to approximate P_c, P_{FA} and P_{SA} at alert thresholds in $[0,1]$. We use a “Probit Classifier”, which returns:

$$P\{I^{(i)} = 1 | s^{(i)}\} \tag{2}$$

Outline Method

Now for any $s \in \mathcal{S}$, we can find the optimal alert threshold:

$$\bar{C}_{opt}(s) = \arg\min_{\bar{C}} \sqrt{\frac{P_{FA}(s)}{P_c(s)}^2 + \left(1 - \frac{P_{SA}(s)}{P_c(s)}\right)^2},$$

Tested with surfaces:

$$P_c(x_1, x_2) = x_1 x_2,$$
$$P_{FA}(x_1, x_2, \bar{C}) = x_1^2 x_2^2 (1 - \bar{C}),$$
$$P_{SA}(x_1, x_2, \bar{C}) = (1 - x_1 x_2)(1 - \bar{C}).$$

Chosen as they are analytically tractable, and have $\bar{C}_{opt} \in [0, 1]$.

Results

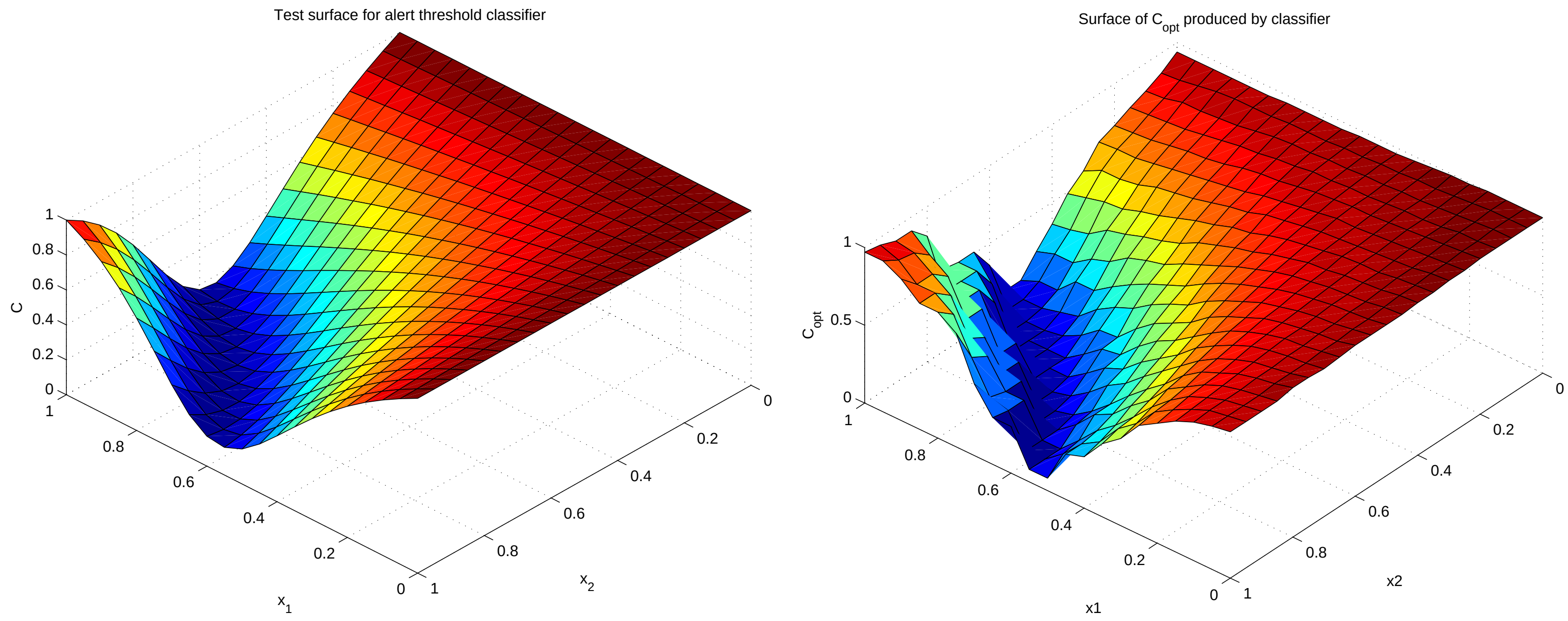

Figure 2: Values of $\bar{C}_{opt}$ produced using the classifier, and the error between these values and the true values. Average error is 0.04.

Conclusions

1. A method for wind correlated MC has been developed.
2. Although complex, the results appear to be very good.
3. An approach for reducing geometrical sensitivity has been developed.
4. Performance on test surfaces is good.
5. Simulations are currently being performed in a comparative study of conflict probes.
6. Results Imminent!