

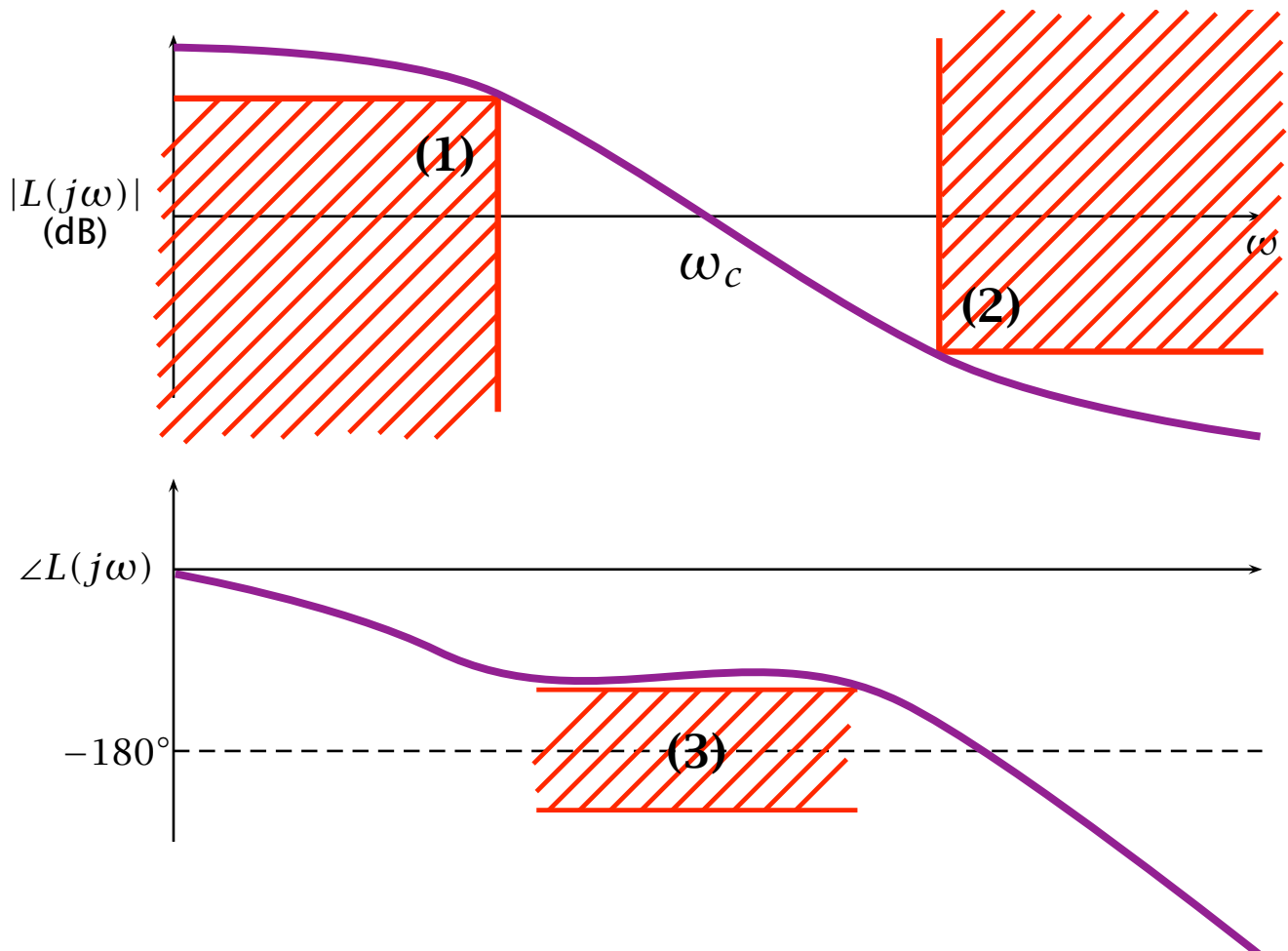
Part IB Paper 6: Information Engineering

LINEAR SYSTEMS AND CONTROL

Glenn Vinnicombe

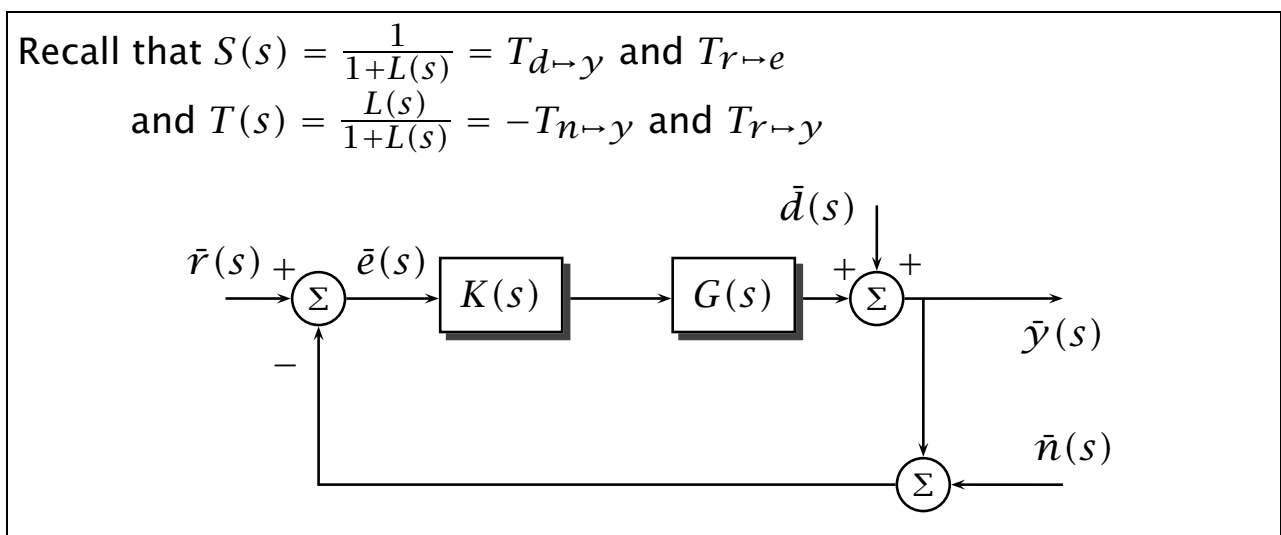
HANDOUT 7

“The design of feedback systems – an introduction”



7.1 Feedback system design, a loop-shaping approach

- This consists of choosing $K(s)$ to shape $L(s) = K(s)G(s)$ such that
 1. $|K(j\omega)G(j\omega)| \gg 1$ for frequency ranges where the benefits of feedback are sought (typically $\omega < \omega_c$)
(in order to ensure that the sensitivity function $|S(j\omega)| \ll 1$ at those frequencies.)
 2. $|K(j\omega)G(j\omega)| \ll 1$ at other frequencies (typically high frequencies $\omega \gg \omega_c$)
(ensuring that the complementary sensitivity function $|T(j\omega)| \ll 1$ at those frequencies.)
 3. $K(j\omega)G(j\omega)$ satisfies the Nyquist stability criterion, with adequate gain and phase margins. (ensuring that neither $S(j\omega)$ or $T(j\omega)$ have a large peak in the *crossover* region in between)

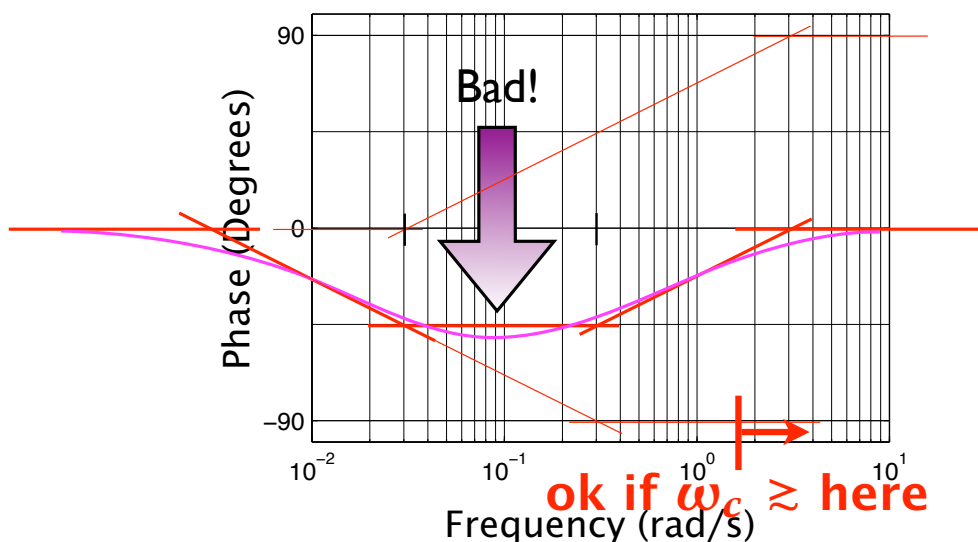
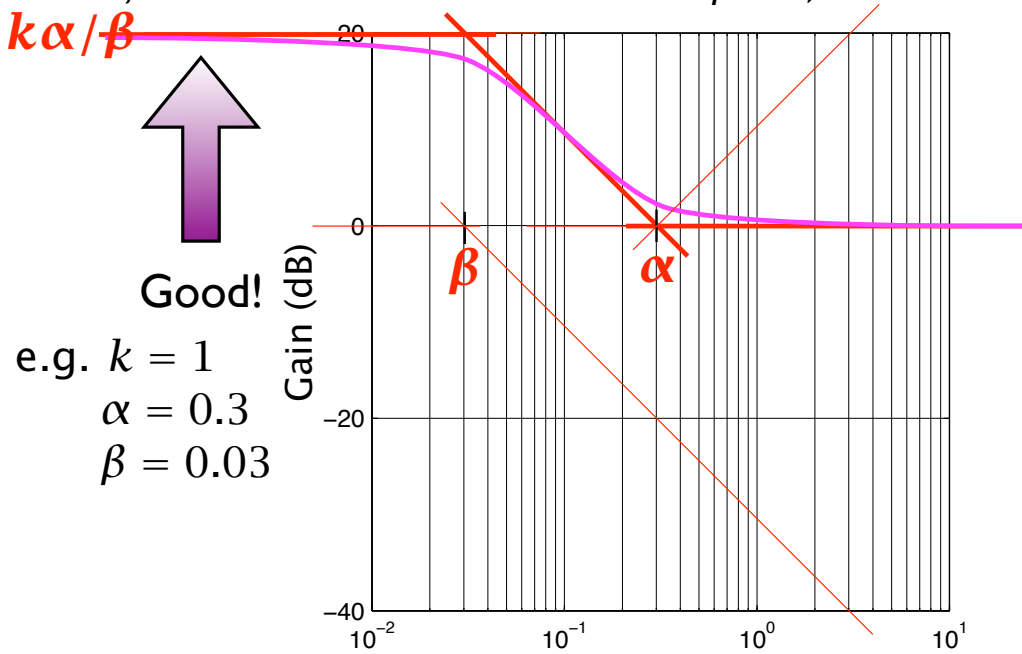


To achieve this, we can use combinations of phase lag and phase lead compensators. Phase lag compensators are a generalized form of P+I action and phase lead compensators are a generalized form of P+D action.

Note: Example 2 of Handout 4 is a phase lead compensator and Example 3 is a PI controller (and so a special case of a phase lag compensator).

- $$K(s) = k \frac{s + \alpha}{s + \beta} \text{ for } \beta < \alpha (< \omega_c \text{ typically}) = \frac{k\alpha}{\beta} \frac{1 + s/\alpha}{1 + s/\beta}$$

- **Phase lag compensator** (a generalized form of proportional+integral action, this becomes a PI controller for $\beta = 0$).



- improves low frequency gain (and so reduces steady-state errors) at the expense of introducing phase lag at frequencies between $\omega \approx \beta$ and $\omega \approx \alpha$ (although this is not an issue if $\alpha \ll \omega_c$).

- $K(s) = k \frac{s + \alpha}{s + \beta}$ for $\alpha < \beta$ (and $\alpha < \omega_c < \beta$ typically)

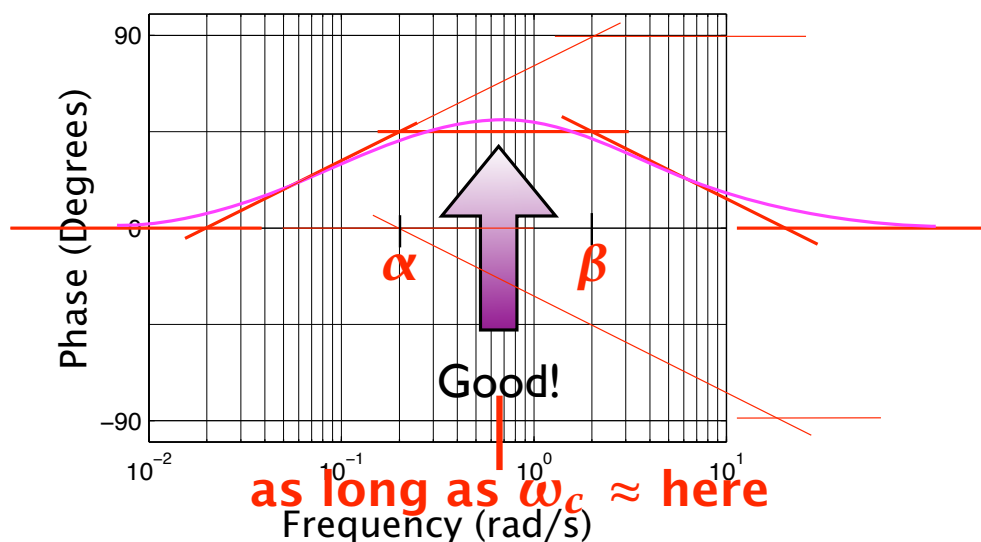
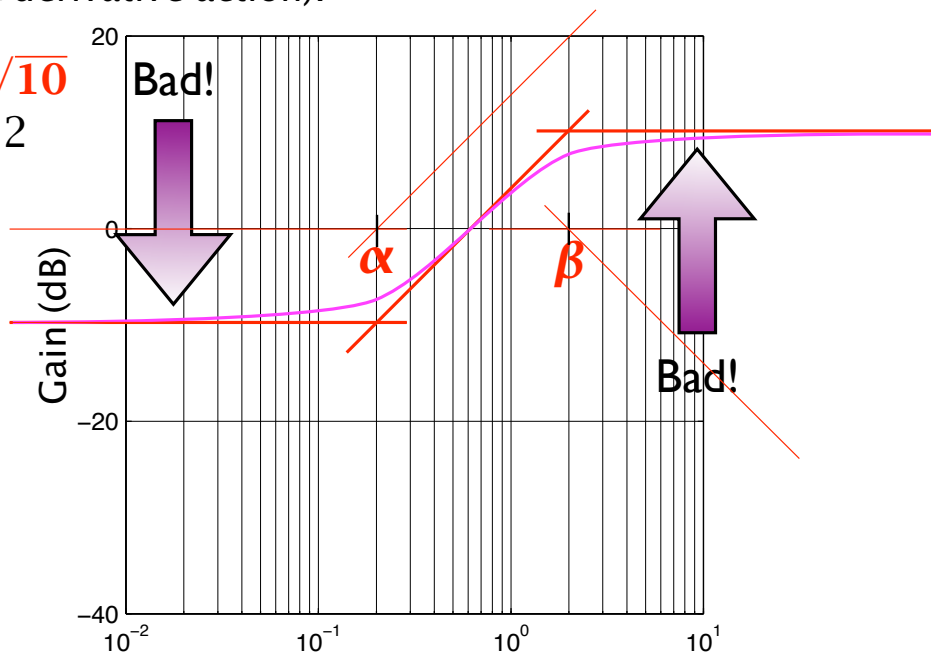
- **Phase lead Compensator** (a generalized form of proportional+derivative action).

e.g. $k = \sqrt{10}$

$\alpha = 0.2$

$\beta = 2$

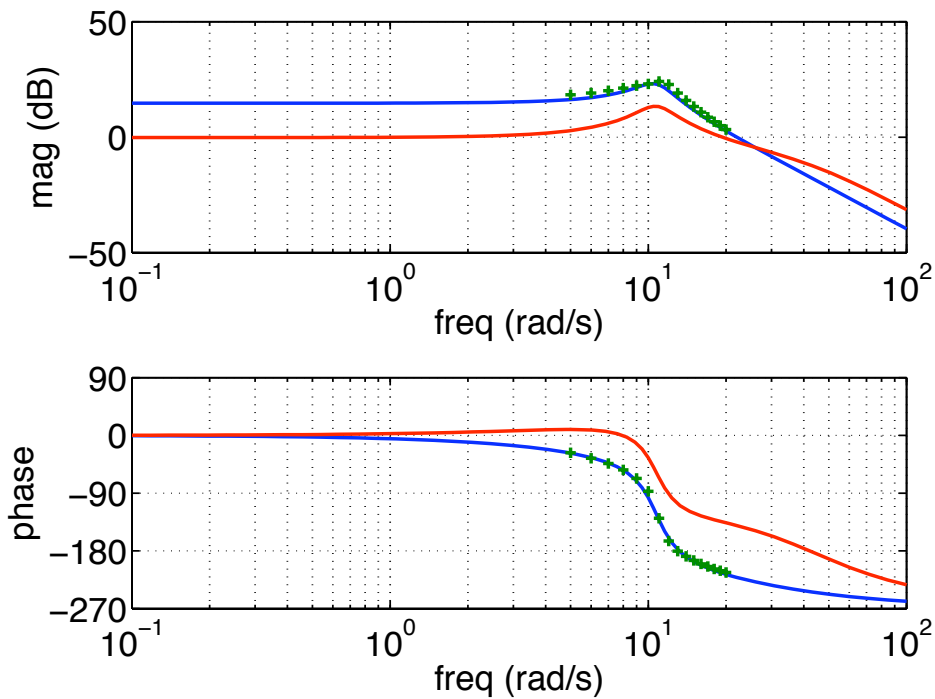
$k\alpha/\beta$



- can improve gain and phase margins (improving robustness and damping) at the expense of decreasing the low frequency gain (increasing steady-state errors) and increasing the high frequency gain (increasing sensitivity to noise).

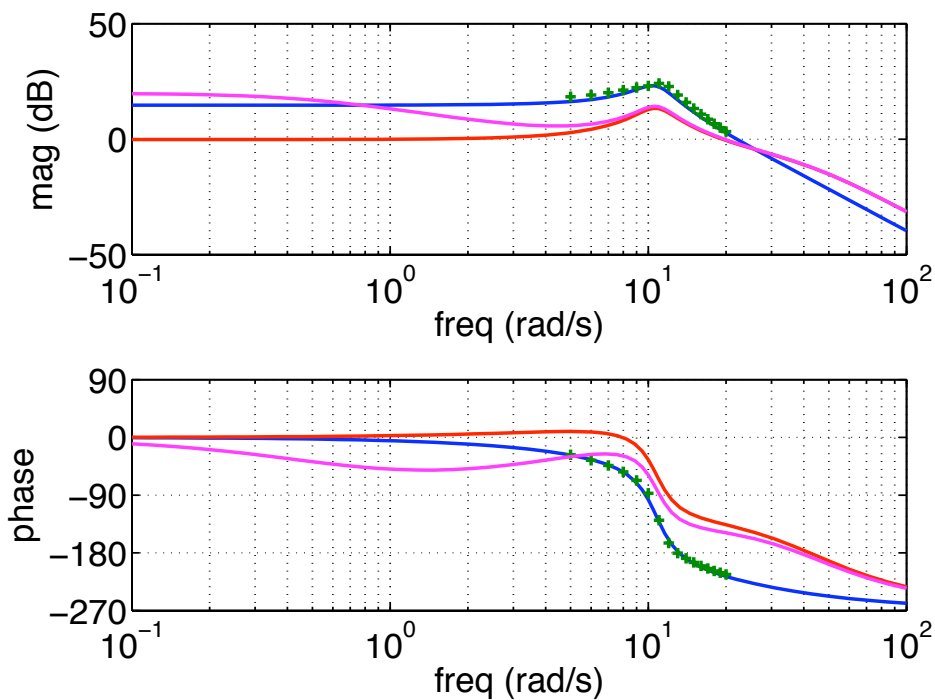
NOTE: WE CAN USE A COMBINATION OF PHASE LEAD AND PHASE LAG.

Lead/lag controller design for the Lego “Hubble”



$$G(s) = \frac{5.431}{\left(\frac{s^2}{10.745^2} + 2 \times 0.1613 \frac{s}{10.745} + 1\right)(s/16.54 + 1)}$$

Lead/lag controller design for the Lego “Hubble”



($L(s)$ is plotted in each case)

What the course was about:

The keys to understanding the course are the following *two* relationships:

- The relationship between the time and frequency (Laplace) domains:
 - Steady state responses (to both constant and sinusoidal inputs).
 - Pole locations.
- The relationship between open and closed loop properties (i.e. predicting properties of the feedback system from its return ratio).
 - Nyquist stability criterion (predicting stability of the feedback system).
 - Gain and phase margins (predicting the robustness of that stability and, indirectly, closed loop pole locations).
 - Understanding the map $L(j\omega) \mapsto L(j\omega)/(1 + L(j\omega))$ and $L(j\omega)/(1 + L(j\omega))$ (predicting the *performance* of the feedback system – which involves reading off $L(j\omega)$ and $1 + L(j\omega)$ from the Nyquist diagram).

That is, it is all about understanding the relationship between various diagrams – if you can understand the relationship between all the figures in the vc1 demo (which you can download from www-control.eng.cam.ac.uk/gv/p6) then you've got it!

