Part IB Paper 6: Information Engineering

LINEAR SYSTEMS AND CONTROL

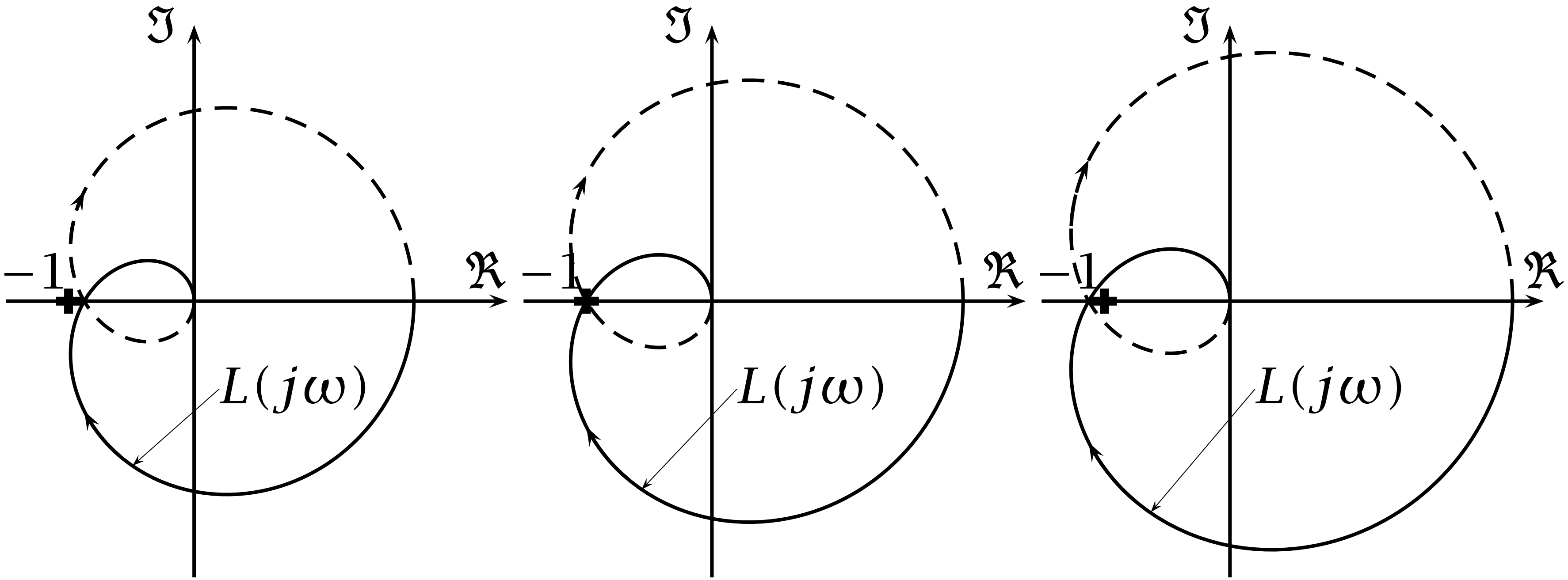
Glenn Vinnicombe

HANDOUT 6

“Feedback stability and the Nyquist diagram”

If $L(s)$ is stable , then:

(either marginally or asymptotically)



$\Longrightarrow \dfrac{L(s)}{1+L(s)}$ is asymptotically stable

$\Longrightarrow \dfrac{L(s)}{1+L(s)}$ is marginally stable

$\Longrightarrow \dfrac{L(s)}{1+L(s)}$ is unstable

That is, the *closed-loop* system is stable if the Nyquist diagram of the *return ratio* doesn't enclose the point "-1".

Summary

- The Nyquist diagram of a feedback system is a plot of the frequency response of the return ratio, with the imaginary part $\Im\big(L(j\omega)\big)$ plotted against the real part $\Re\big(L(j\omega)\big)$ on an Argand diagram (that is, like the Bode diagram, it is a plot of an *open-loop* frequency response).

- The Nyquist stability criterion states that, if
  - the open-loop system is asymptotically stable (i.e. the return ratio $L(s)$ has all its poles in the LHP) and
  - the Nyquist diagram of $L(j\omega)$ does not enclose the point "-1",

  then the closed-loop system will be asymptotically stable (i.e. the closed-loop transfer function $L(s)/(1+L(s))$ will have all its poles in the LHP)

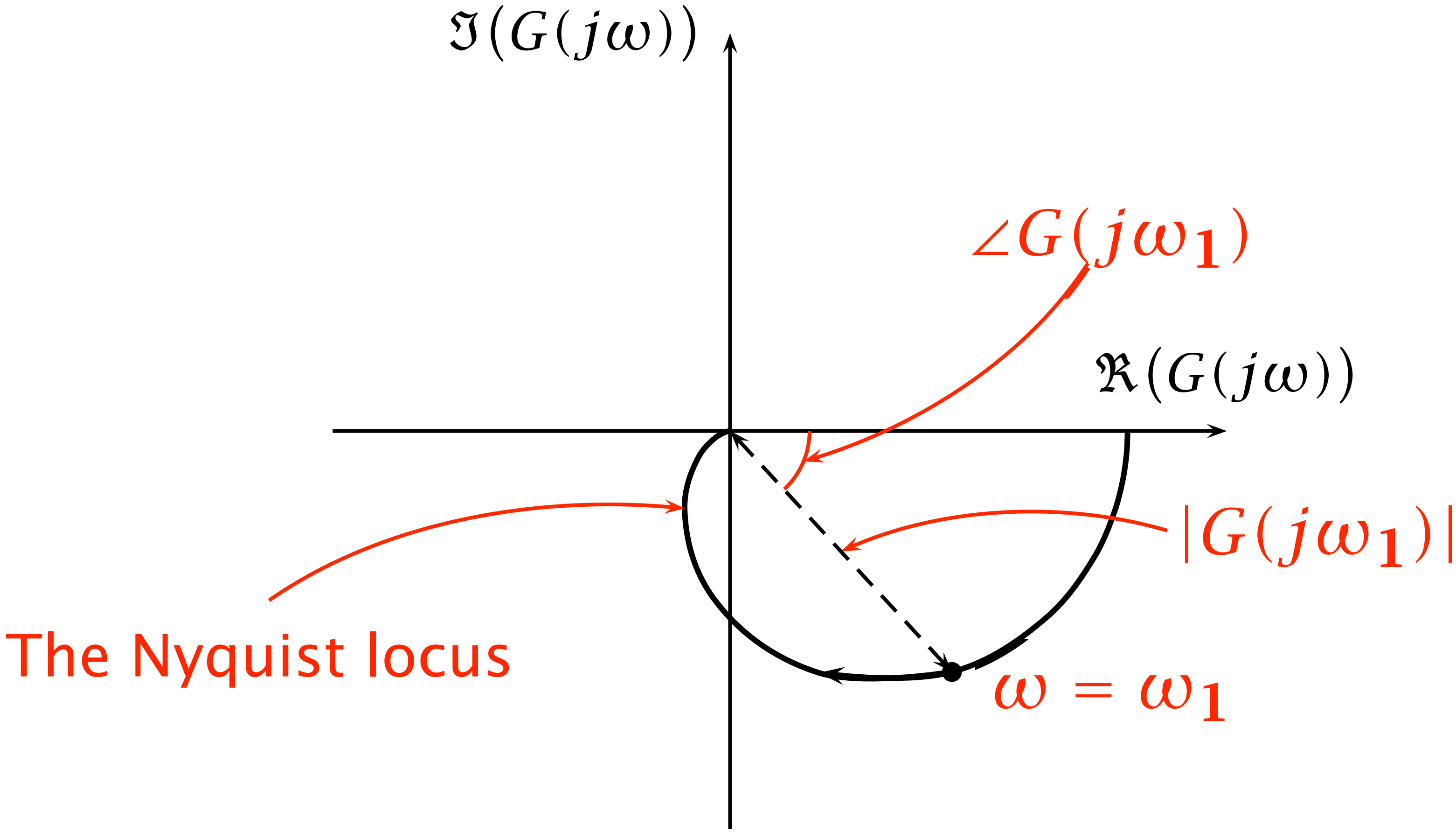
- The real power of the Nyquist stability criterion is that it allows you to determine of the stability of the *closed-loop* system from the behaviour of the *open-loop* Nyquist diagram. This is important from a design point of view, as it relatively easy to see how changing $K(s)$ affects $L(s) = H(s)G(s)K(s)$, but difficult to see how changing $K(s)$ affects $L(s)/(1 + L(s))$ directly, for example.
- In addition, the Nyquist diagram also allows more detailed information about the behaviour of the closed-loop system to be inferred. For example
  - Gain and phase margins measure how close the Nyquist locus gets to -1 (and hence how close the closed loop system is to instability).

Contents

6.1 The Nyquist Diagram

The Nyquist diagram of a system $G(s)$ is a plot of the frequency response $G(j\omega)$ on an Argand diagram.

That is: it is a plot of $\Im(G(j\omega))$ vs $\Re(G(j\omega))$.



Examples:

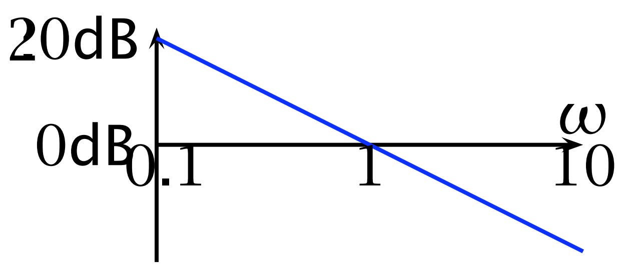
Integrator:

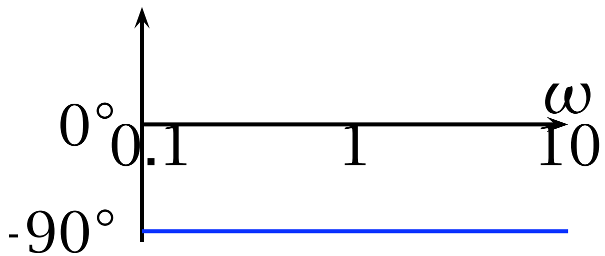
$$G(s) = \frac{1}{s}$$

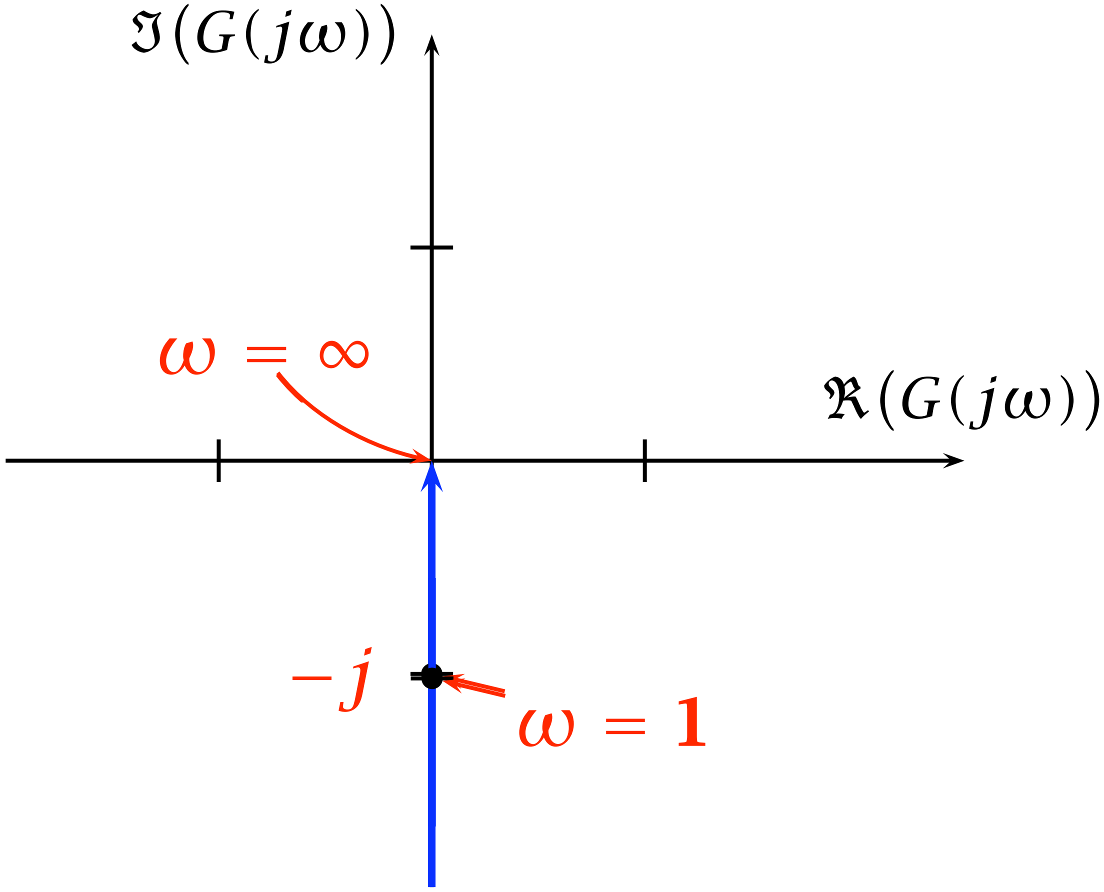
$$G(j\omega) = \frac{1}{j\omega} = \frac{-j}{\omega}$$

$$|G(j\omega)| = \mathbf{1/\omega}$$

$$\angle G(j\omega) = \mathbf{-90^\circ}$$







Time Delay:

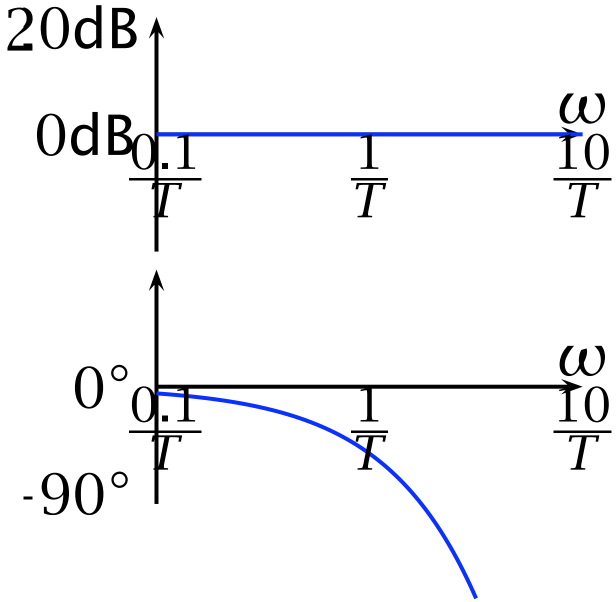
$$G(s) = e^{-sT}$$

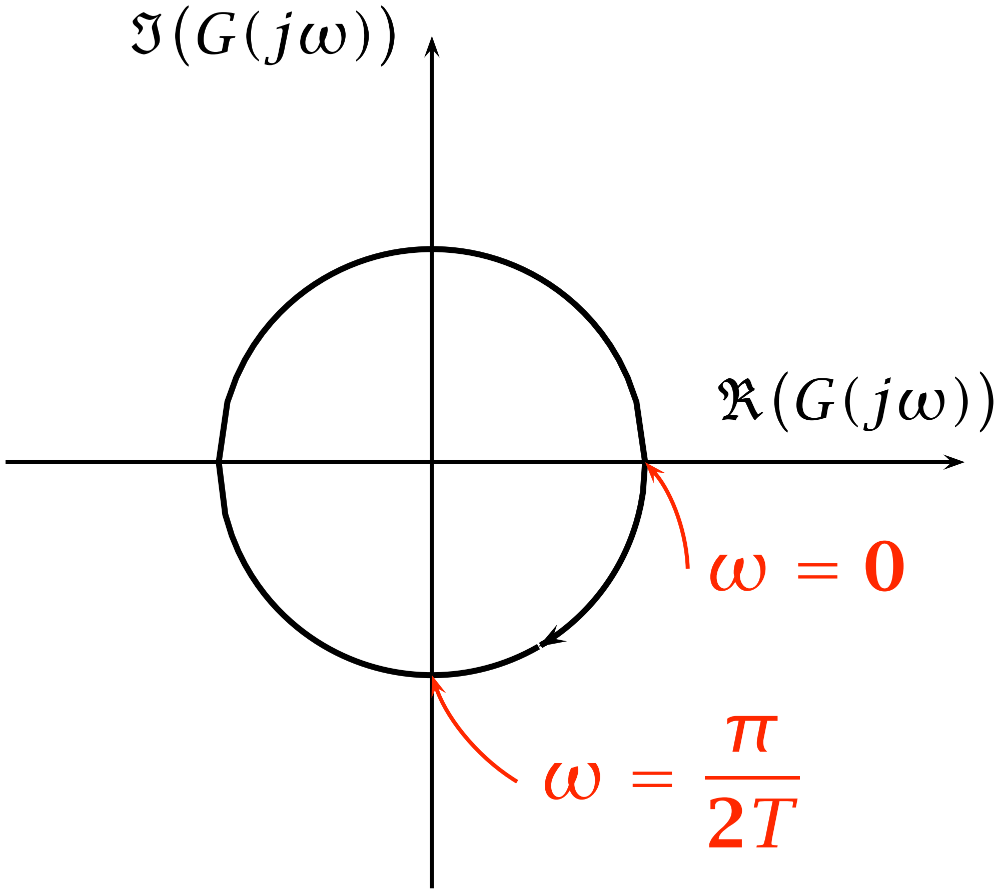
$$G(j\omega) = e^{-j\omega T}$$

$$|G(j\omega)| = \mathbf{1}$$

$$\arg G(j\omega) = -\omega T$$

(radians)





First-order lag:

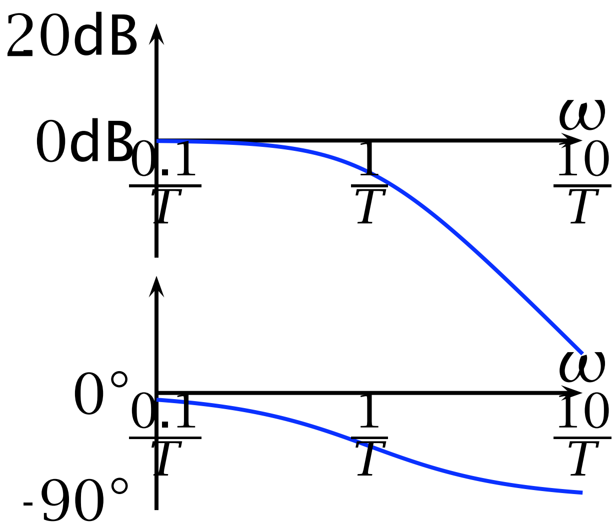
$$G(s) = \frac{1}{1 + sT}$$

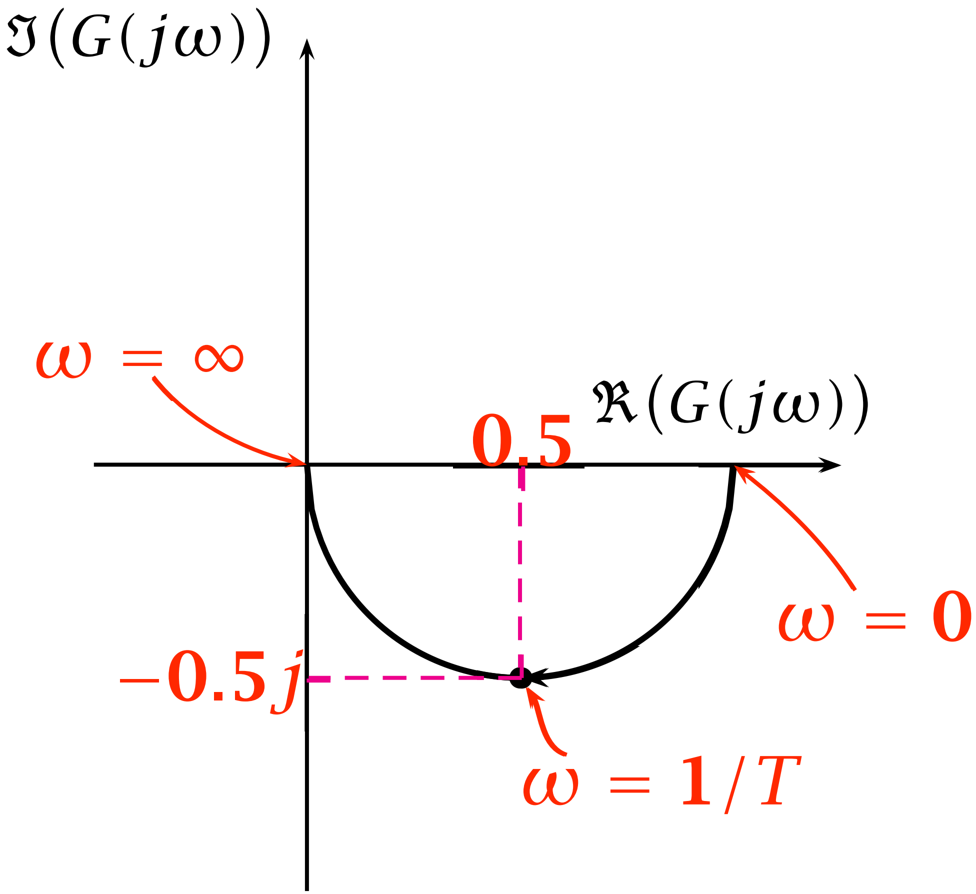
$$G(j\omega) = \frac{1}{1 + j\omega T}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1 + \omega^2 T^2}}$$

$$\angle G(j\omega) = -\arctan(\omega T)$$

	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
$\omega = 0$	**1**	**0**
$\omega = \frac{1}{T}$	$\mathbf{\frac{1}{\sqrt{2}}}$	-45°
$\omega = \to \infty$	**0**	-90°





Time Delay with Lag and Integrator:

$$G(s) = \frac{e^{-sT_1}}{s(1+sT_2)}$$

$$G(j\omega) = \frac{e^{-j\omega T_1}}{j\omega(1+j\omega T_2)}$$

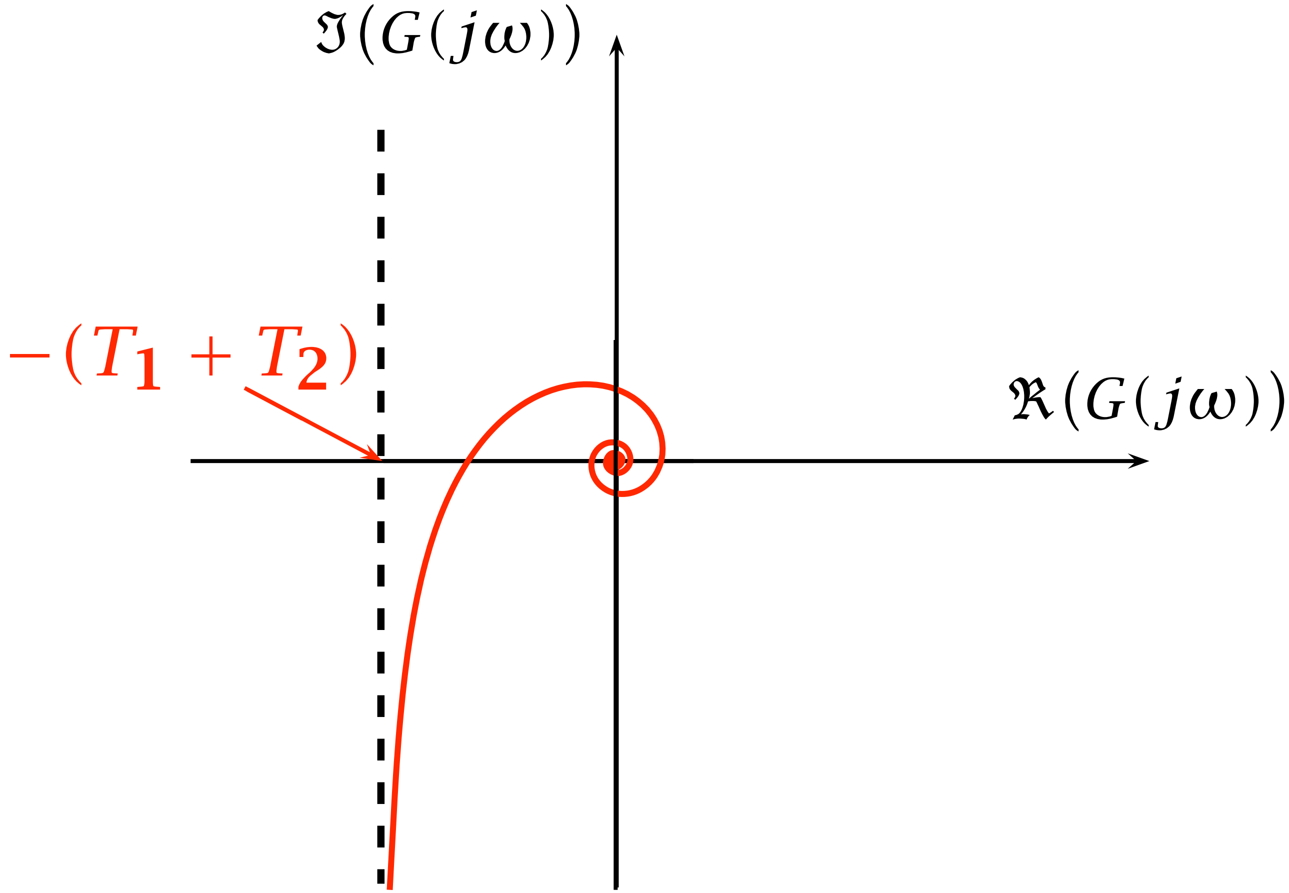
$$|G(j\omega)| = \underbrace{|e^{-j\omega T_1}|}_{\mathbf{1}} \times \frac{1}{|j\omega|} \times \frac{1}{|1+j\omega T_2|}$$

$$\angle G(j\omega) = \underbrace{\angle e^{-j\omega T_1}}_{-\omega T_1} - \underbrace{\angle(j\omega)}_{\mathbf{90^\circ}} - \angle(1+j\omega T_2)$$

Clearly, as $\omega \to 0$ then $|G(j\omega)| \to \infty$. But this is not enough information to sketch the Nyquist diagram. Precisely how does $|G(j\omega)| \to \infty$? To answer this, we use a Taylor series expansion around $\omega = 0$.

$$e^{-j\omega T_1} \rightarrow \quad \mathbf{1 - j\omega T_1} \quad \text{and}$$
$$1/(j\omega T_2 + 1) \rightarrow \quad \mathbf{1 - j\omega T_2},$$

$$\Longrightarrow \qquad G(j\omega) \rightarrow \frac{(1 - j\omega T_1)(1 - j\omega T_2)}{j\omega} = \frac{1}{j\omega} - (T_1 + T_2) + \cancelto{0}{j\omega T_1 T_2}.$$



Second-order lag:

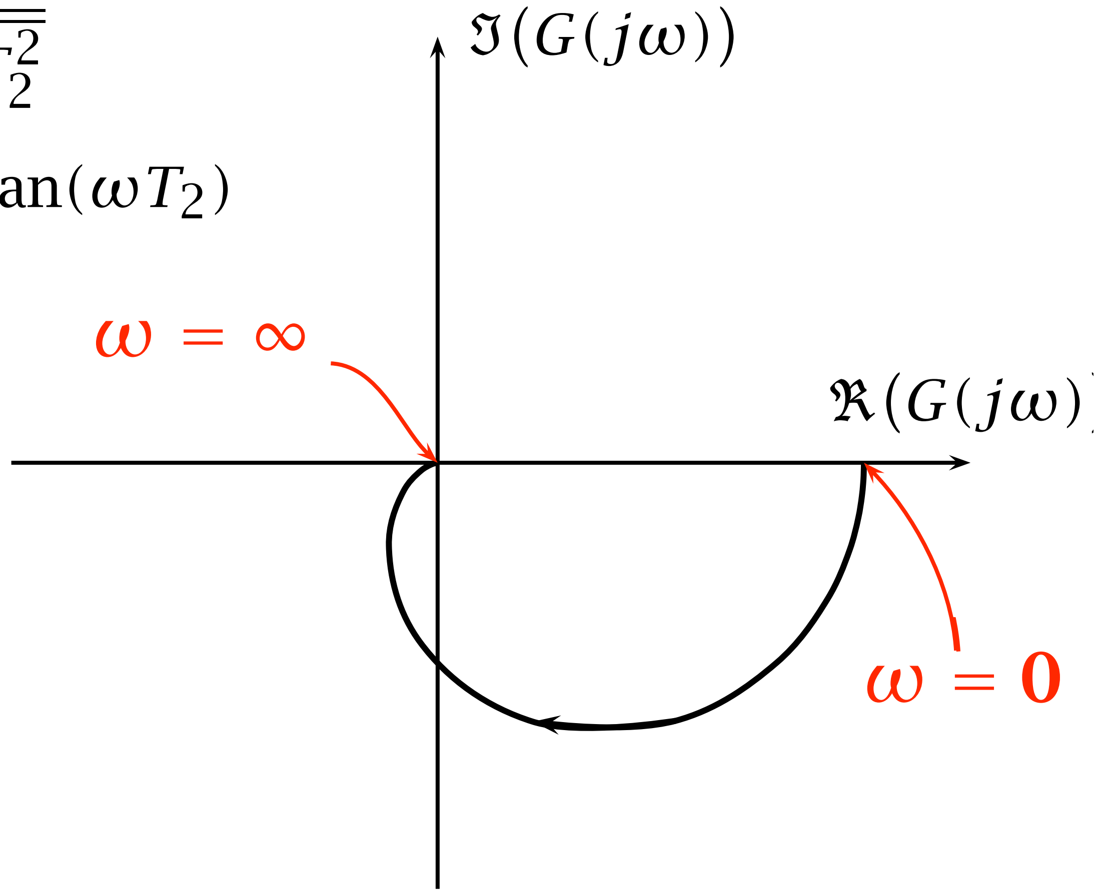
$$G(s) = \frac{1}{(1+sT_1)(1+sT_2)}$$

$$G(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2 T_1^2}\sqrt{1+\omega^2 T_2^2}}$$

$$\angle G(j\omega) = -\arctan(\omega T_1) - \arctan(\omega T_2)$$

	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
$\omega = 0$	**1**	**0**
$\omega = \rightarrow \infty$	**0**	**-180°**



6.1.1 Sketching Nyquist diagrams

Unlike the Bode diagram, there are no detailed rules for sketching Nyquist diagrams. It suffices to determine the asymptotic behaviour as $\omega \to 0$ and $\omega \to \infty$ (using the techniques we have seen in the examples) and then calculate a few points in between. Note that if $G(0)$ is a finite and non-zero, then the Nyquist locus will always start off by leaving the real axis at right angles to it. [1] If $G(0)$ is infinite, due to the presence of integrators, then we must explicity find the first two terms of the Taylor series expansion of $G(j\omega)$ about $\omega = 0$, as in the example with a time delay, a lag and an integrator.

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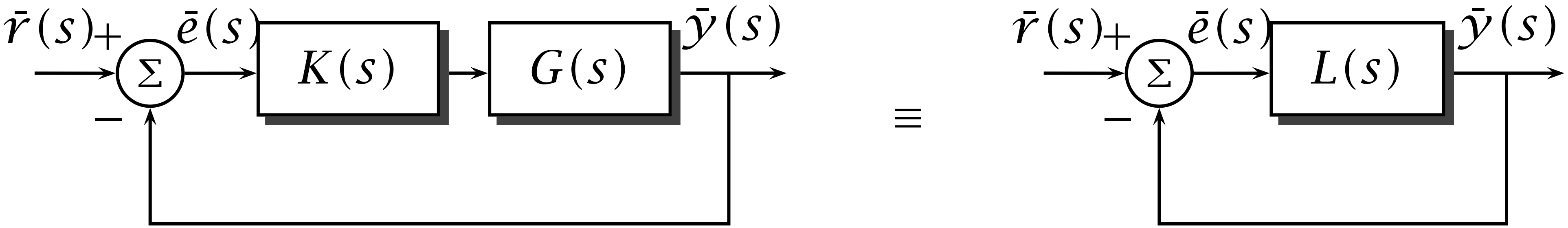
[1] This is since $G(j\epsilon) = G(0) + j\epsilon G'(0) - \epsilon^2 G''(0) - \cdots \approx G(0) + j\epsilon G'(0)$

6.2 Feedback stability



$$\begin{aligned} \text{Closed-loop poles} \quad &\equiv \text{poles of } \frac{G(s)K(s)}{1 + G(s)K(s)} \\ &\equiv \text{roots of } 1 + G(s)K(s) = 0 \end{aligned}$$

It is difficult to see how $K(s)$ should be chosen to ensure that all the closed-loop poles are all in the LHP. But . . .

Nyquist's Stability Theorem allows us to deduce closed-loop properties:

the location of the poles of $\dfrac{G(s)K(s)}{1+G(s)K(s)}$,

from open-loop properties

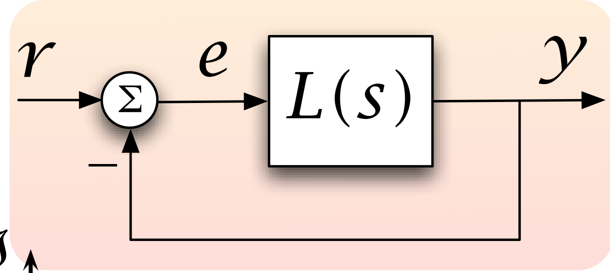
frequency response of the return ratio

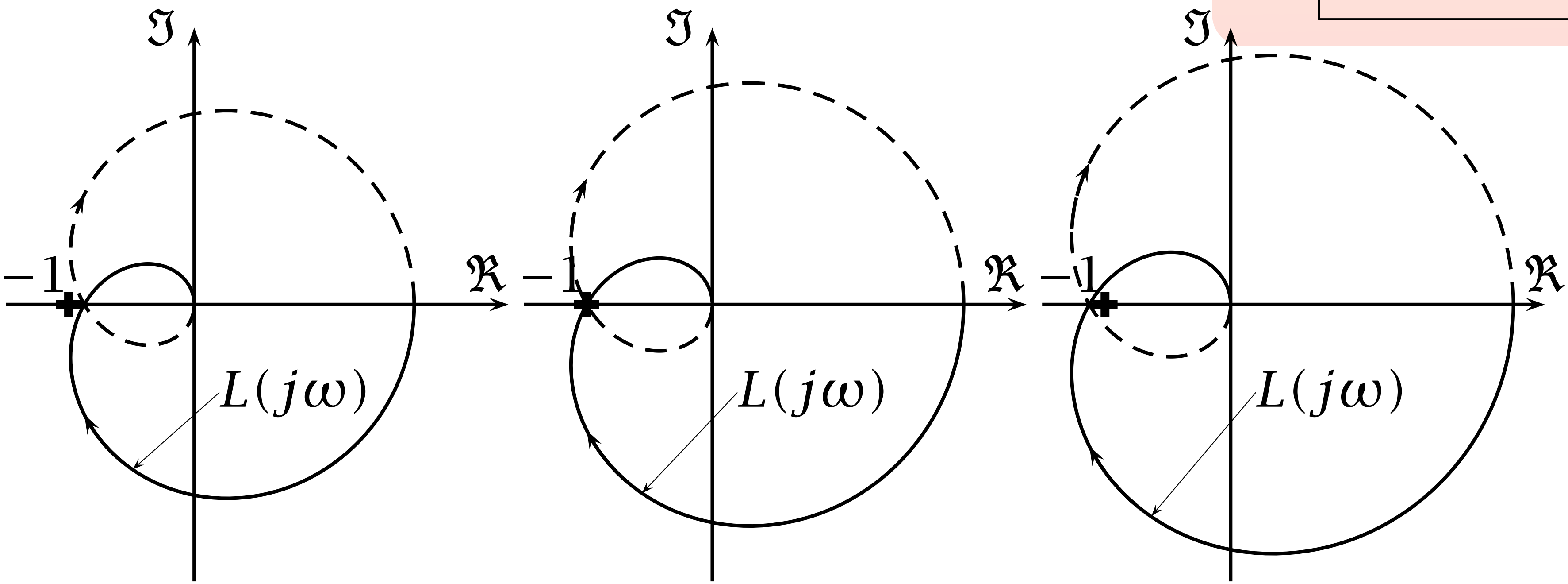
$$L(j\omega) = G(j\omega)K(j\omega).$$

The basic idea is as follows: Negative feedback is used to *reduce* the size of the error $e(t)$ in the above figures. If $y(t)$ is too large (i.e greater than $r(t)$) then $e(t)$ is negative, which will tend to reduce $y(t)$ (provided the signs of $K(s)$ and $G(s)$ have been chosen appropriately). However, for any real system the phase lag from the input to the output ($-\angle L(j\omega)$) will tend to increase with frequency, eventually reaching $180°$. When this happens, the negative feedback is turned into positive feedback. If the gain $|L(j\omega)|$ has not decreased to less than 1 by this frequency then instability of the closed-loop system will result.

If $L(s)$ is stable , then:

(either marginally or asymptotically)





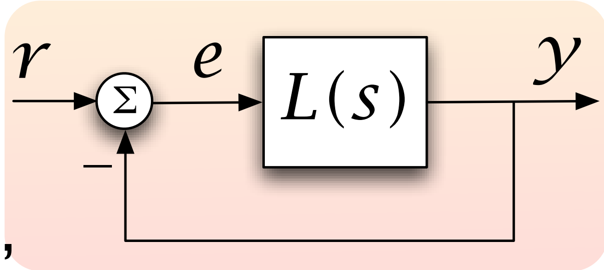
$\Longrightarrow \dfrac{L(s)}{1+L(s)}$ is asymptotically stable

$\Longrightarrow \dfrac{L(s)}{1+L(s)}$ is marginally stable

$\Longrightarrow \dfrac{L(s)}{1+L(s)}$ is unstable

That is, the *closed-loop* system is stable if the Nyquist diagram of the *return ratio* doesn't enclose the point "-1".

6.2.1 Significance of the point "-1"



If the Nyquist locus passes through the point "-1",

i.e. $L(j\omega_1) = -1$ for some ω_1

then the closed-loop frequency response $L(j\omega)/(1 + L(j\omega))$ becomes infinite at that frequency, ie

$$L(j\omega_1)/(1 + L(j\omega_1)) \to \infty$$

This is not a good thing!

In this case, if $e(t) = \cos(\omega_1 t)$ then in steady-state we have

$$y(t) = |L(j\omega_1)| \cos\left(\omega_1 t + \angle L(j\omega_1)\right)$$

$$= \cos(\omega_1 t + \pi) = -\cos(\omega_1 t)$$

However $e(t) = r(t) - y(t)$, which means that

$$r(t) = e(t) + y(t) = \cos(\omega_1 t) - \cos(\omega_1 t) \;\; = 0$$

That is, there is a sustained oscillation of the feedback system even when there is no external input!

6.2.2 Example:

Let

$$G(s) = \frac{1}{s^3 + s^2 + 2s + 1}, \quad K(s) = k,$$

$$\implies L(s) = \frac{k}{s^3 + s^2 + 2s + 1}.$$

The closed-loop poles are the roots of

$$1 + \frac{k}{s^3 + s^2 + 2s + 1} = 0 \iff \underbrace{\mathbf{s^3 + s^2 + 2s + 1 + k = 0}}_{CLCE}$$

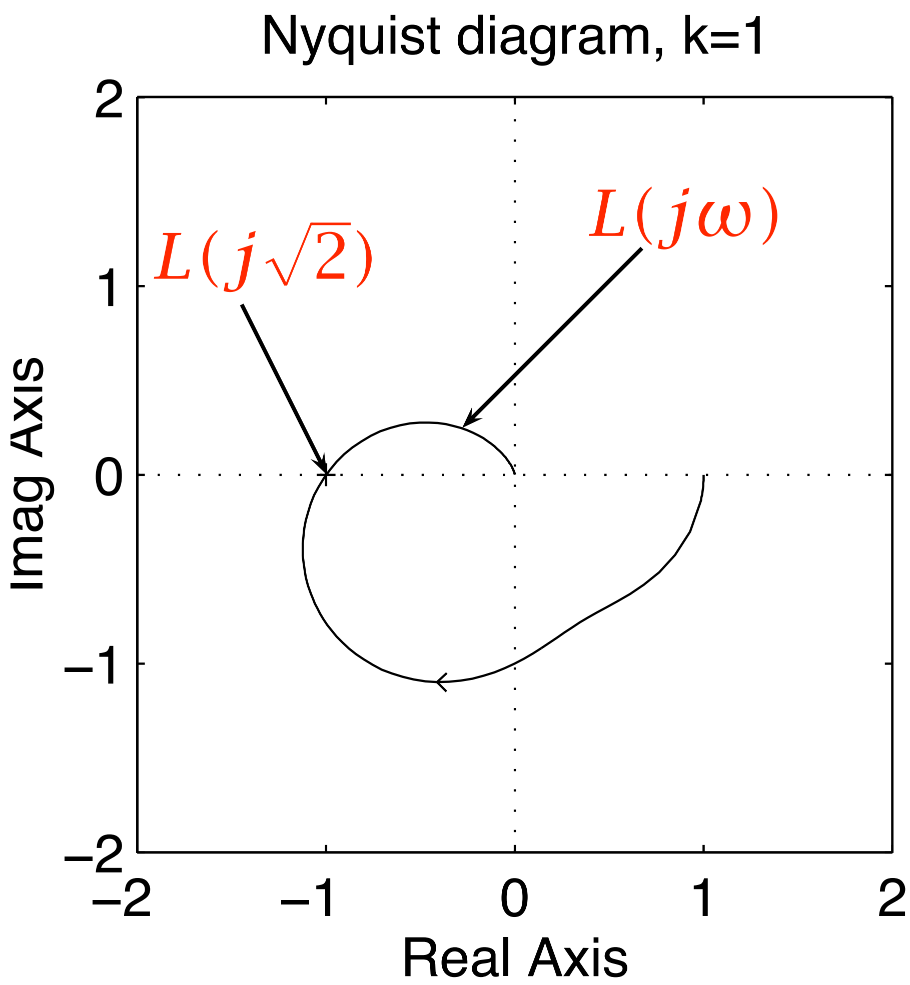
and the frequency response of the loop is:

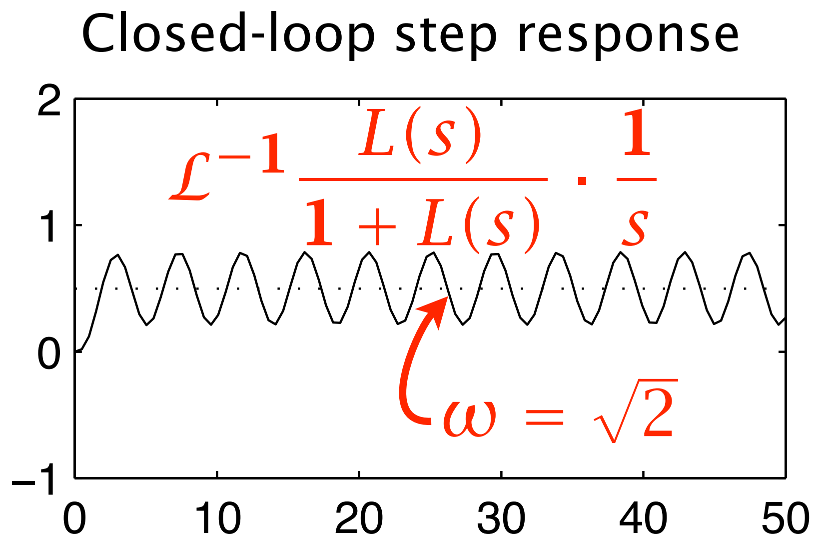
$$L(j\omega) = \frac{k}{j(-\omega^3 + 2\omega) + (-\omega^2 + 1)}$$

At $\omega = \sqrt{2}$, $L(jw)$ is purely real. That is

$$L(\sqrt{2}\,j) = \frac{k}{j(-2\sqrt{2} + 2\sqrt{2}) - 2 + 1} = -k$$

$\underline{k = 1}$



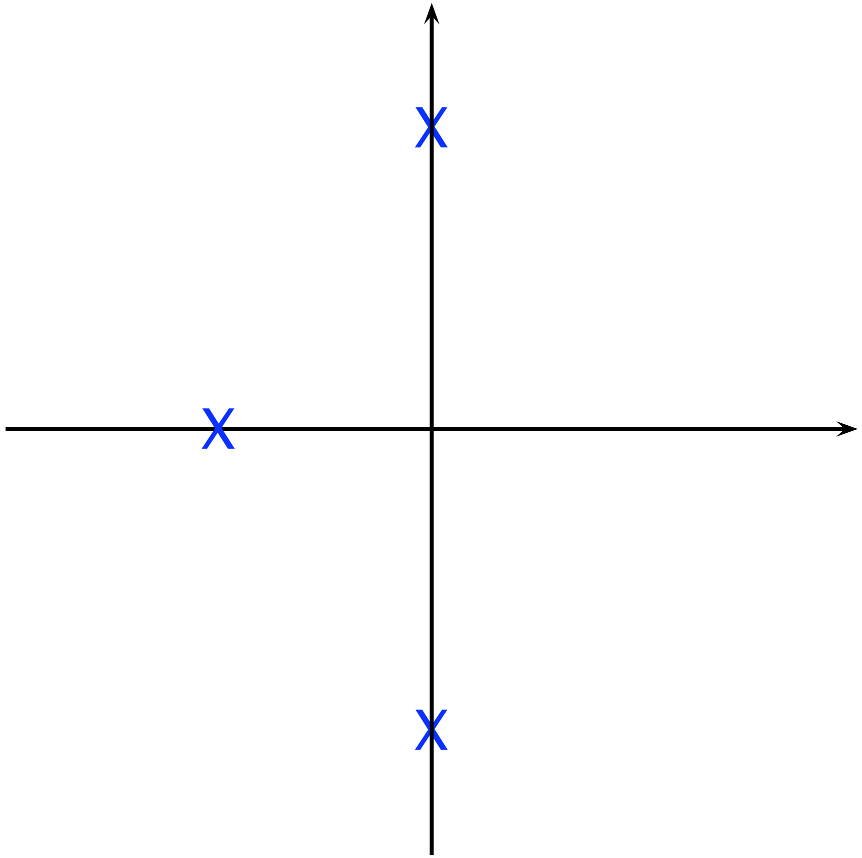


Closed-loop poles are at the roots of $s^3 + s^2 + 2s + 2 = 0$, i.e.

$$s = \boxed{-0.0000 + 1.4142j,}$$
$$-0.0000 - 1.4142j,$$
$$-1.0000$$

(because $L(j\sqrt{2}) = -1$,

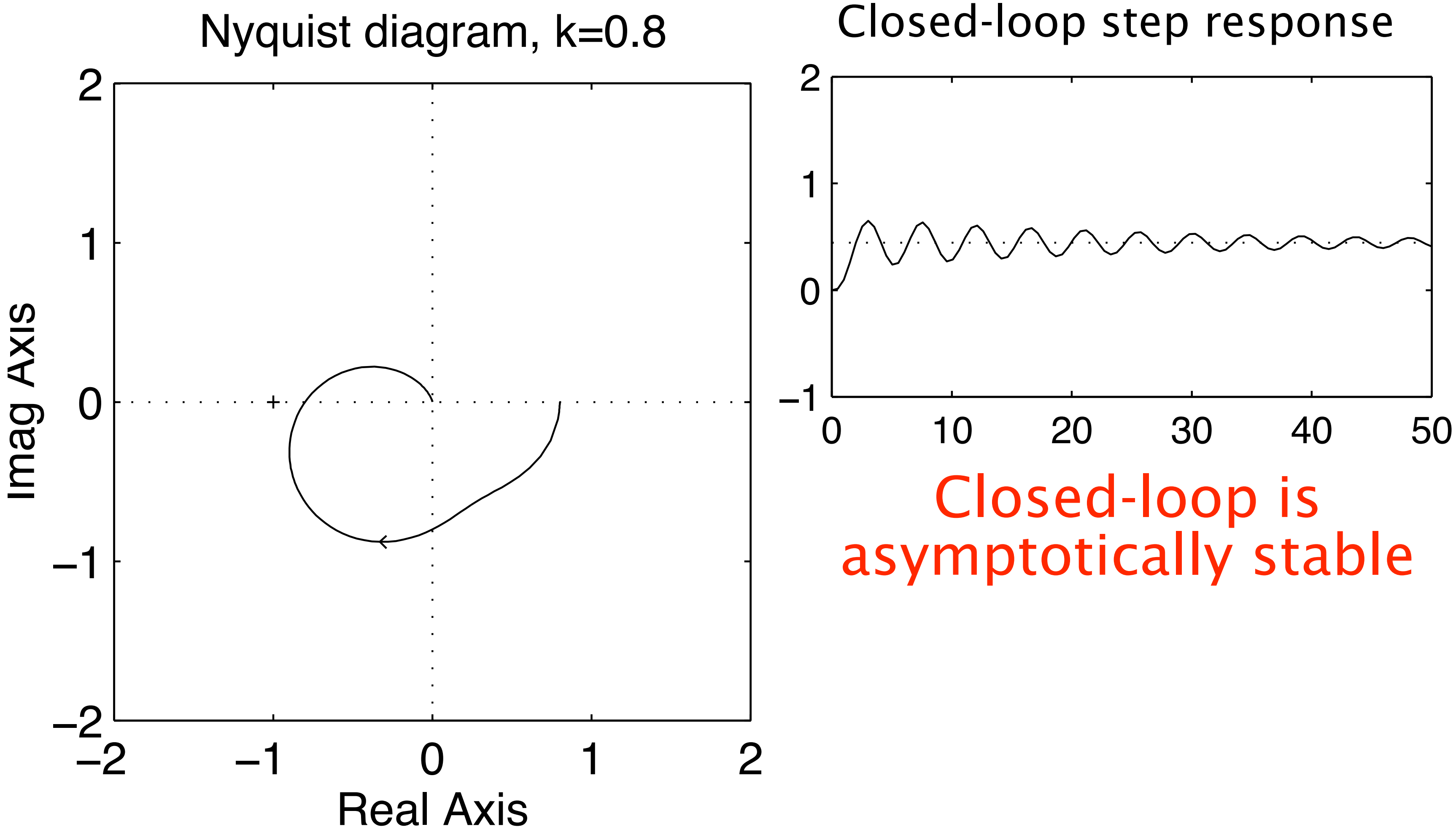
and so $1 + L(s) = 0$ at $s = j\sqrt{2}$)



Closed-loop poles

$\Longrightarrow$ **closed-loop system is marginally stable**

$\underline{k = 0.8}$

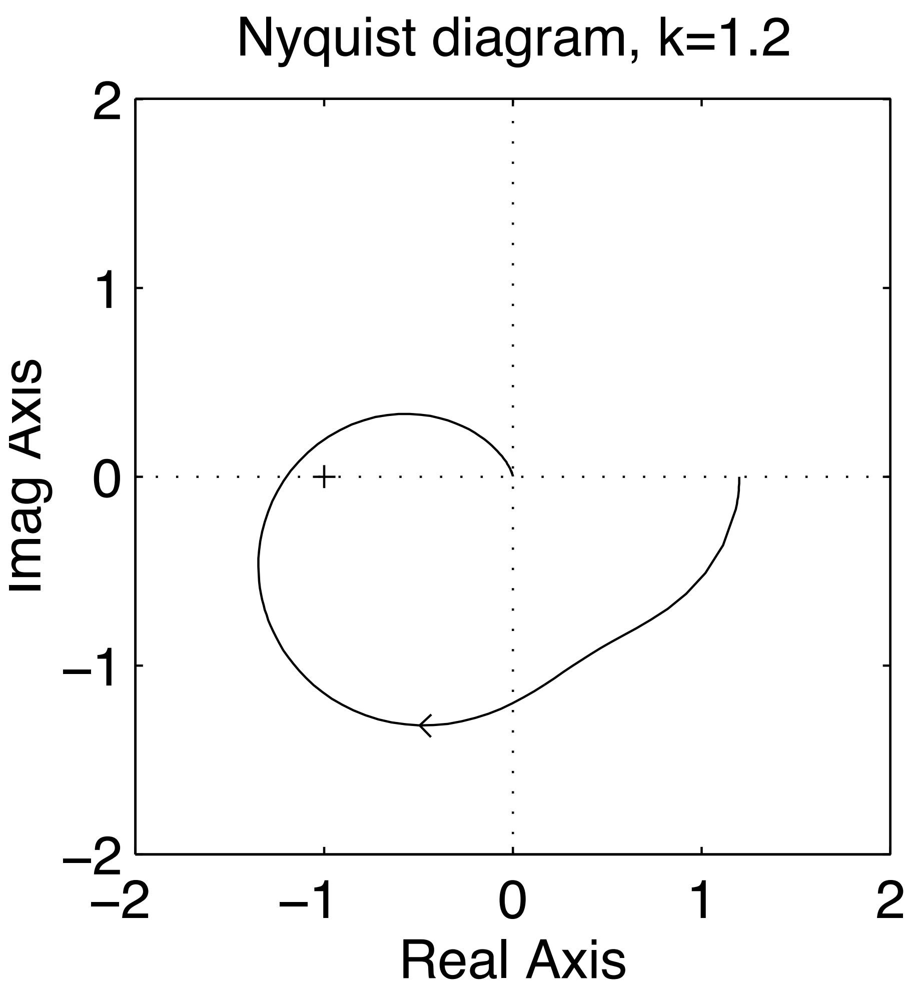


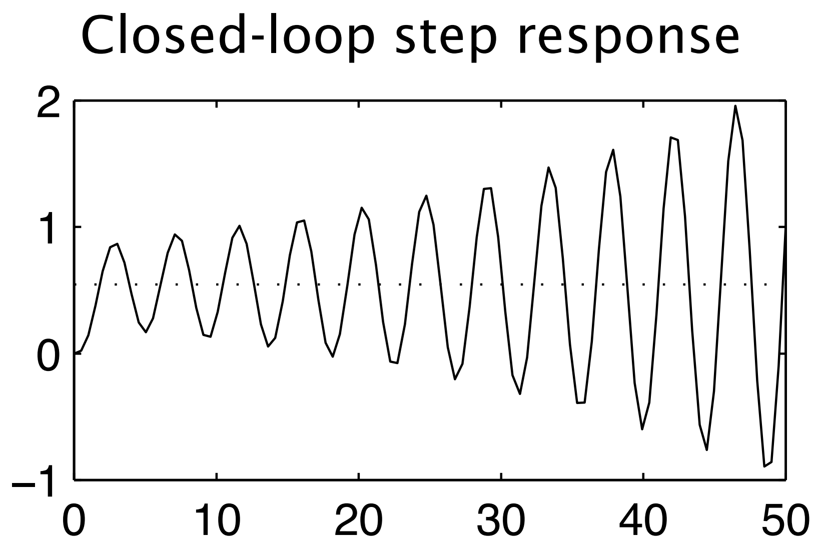
Closed-loop is asymptotically stable

Closed-loop poles are at the roots of $s^3 + s^2 + 2s + 1.8 = 0$, i.e.

$s = \boxed{-0.0349} + 1.3906j, \quad \boxed{-0.0349} - 1.3906j, \quad -0.9302$

$\underline{k = 1.2}$



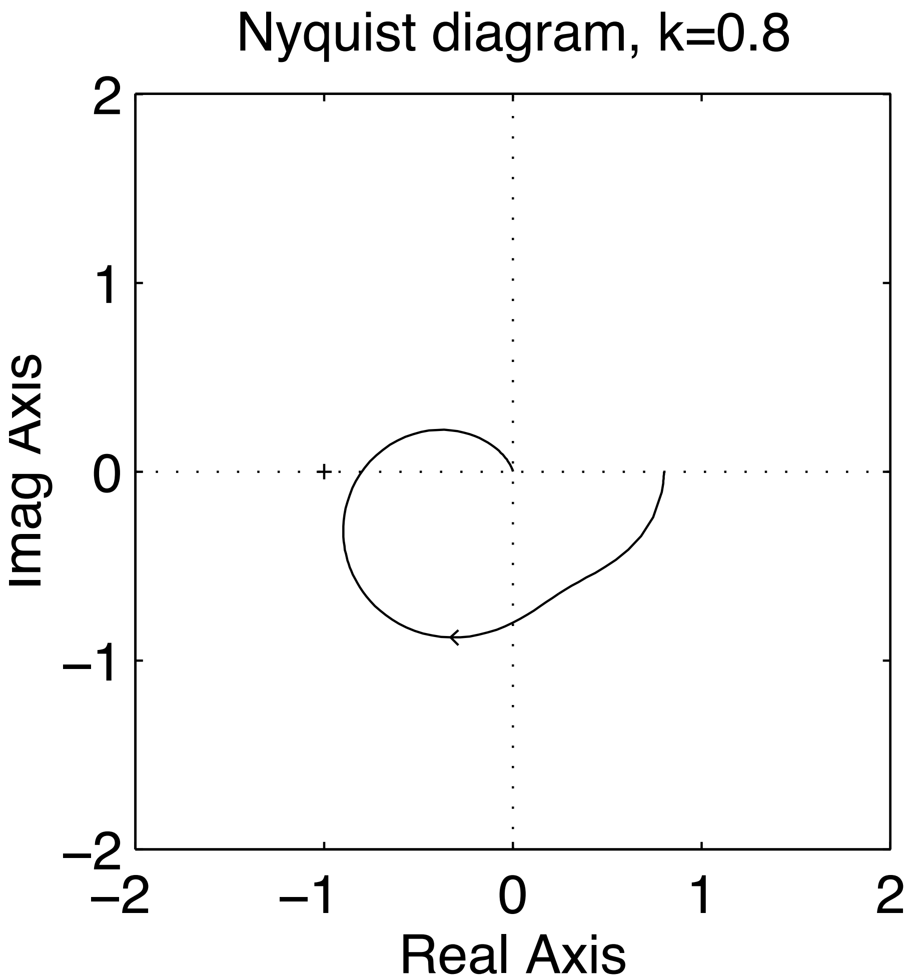


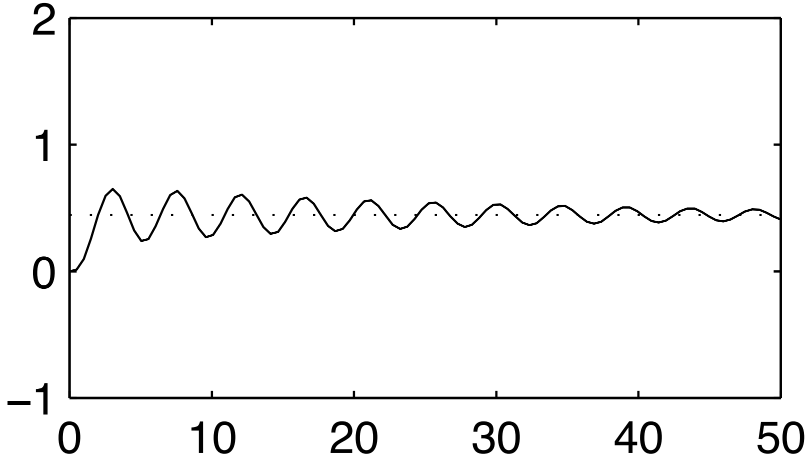
Closed-loop is unstable

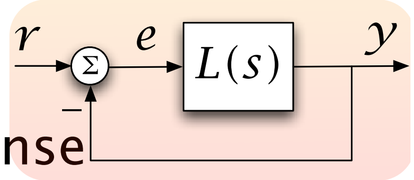
Closed-loop poles are at the roots of $s^3 + s^2 + 2s + 2.2 = 0$, i.e.

$s = \; 0.0319 + 1.4377j, \;\; 0.0319 - 1.4377j, \;\; -1.0639$

$\underline{k = 0.8}$





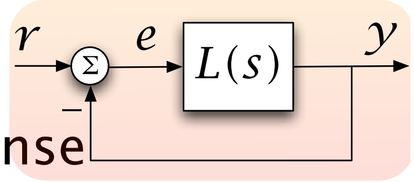


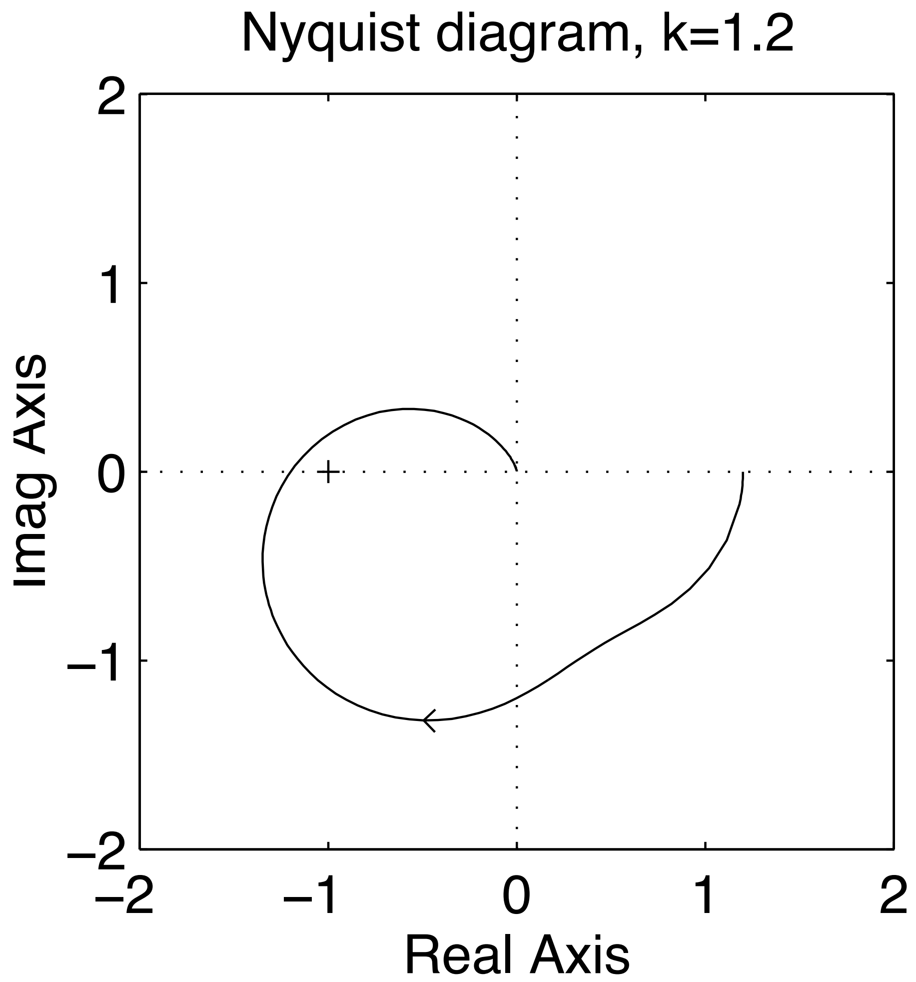
Closed-loop is asymptotically stable

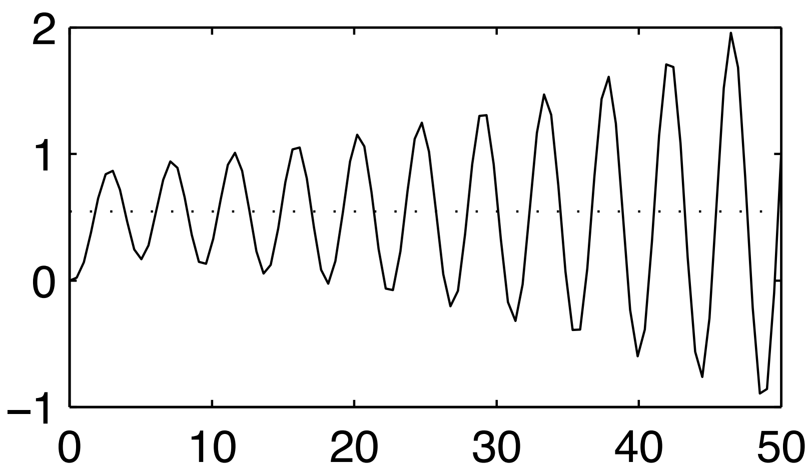
Closed-loop poles are at the roots of $s^3 + s^2 + 2s + 1.8 = 0$, i.e.

$s = \;\; -0.0349 + 1.3906j, \;\; -0.0349 - 1.3906j, \;\; -0.9302$

$\underline{k = 1.2}$







Closed-loop is unstable

Closed-loop poles are at the roots of $s^3 + s^2 + 2s + 2.2 = 0$, i.e.

$$s = \quad 0.0319 + 1.4377j, \quad 0.0319 - 1.4377j, \quad -1.0639$$

6.3 Nyquist Stability Theorem (informal version)

We can now give an informal statement of Nyquist's stability theorem:

"If a feedback system has an asymptotically stable return ratio $L(s)$, then the feedback system is also asymptotically stable if the Nyquist diagram of $L(j\omega)$ leaves the point $-1 + j0$ on its left".

This is unambiguous in most cases, and usually still works if $L(s)$ has poles at the origin or is unstable.

For completeness, a full statement of this theorem will be given later.

Definition: We say that the feedback system (or closed-loop system) is asymptotically stable if the closed-loop transfer function $\frac{L(s)}{1+L(s)}$ is asymptotically stable, that is if all the poles of $\frac{L(s)}{1+L(s)}$ (i.e. the roots of $1 + L(s) = 0$) lie in the LHP.

6.4 Gain and Phase Margins

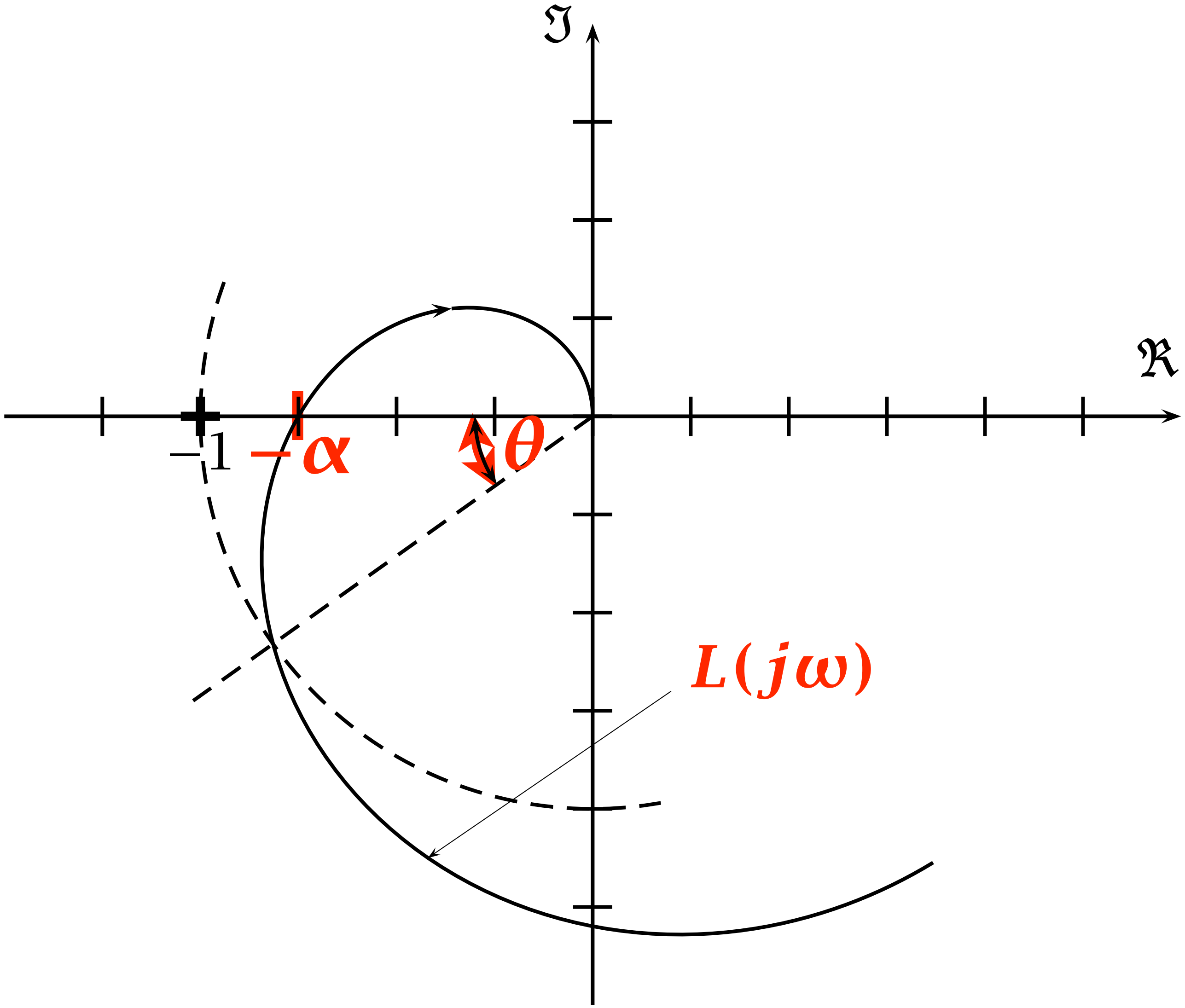
$L(j\omega)$ encirling or going through the -1 point is clearly bad, leading to the closed-loop not being asymptotically stable. However, $L(j\omega)$ coming close to -1 without encircling it is also undesirable, for two reasons:

- It implies that a closed-loop pole will be close to the imaginary axis and that the closed-loop system will be oscillatory.
- If $G(s)$ is the transfer function of an inaccurate model, then the "true" Nyquist diagram might actually encircle -1.

Gain and phase margins are widely used measures of how close the return ration $L(j\omega)$ gets to -1.

The **gain margin** measures how much the gain of the return ratio can be increased before the closed-loop system becomes unstable.

The **phase margin** measures how much phase lag can be added to the return ratio before the closed-loop system becomes unstable.

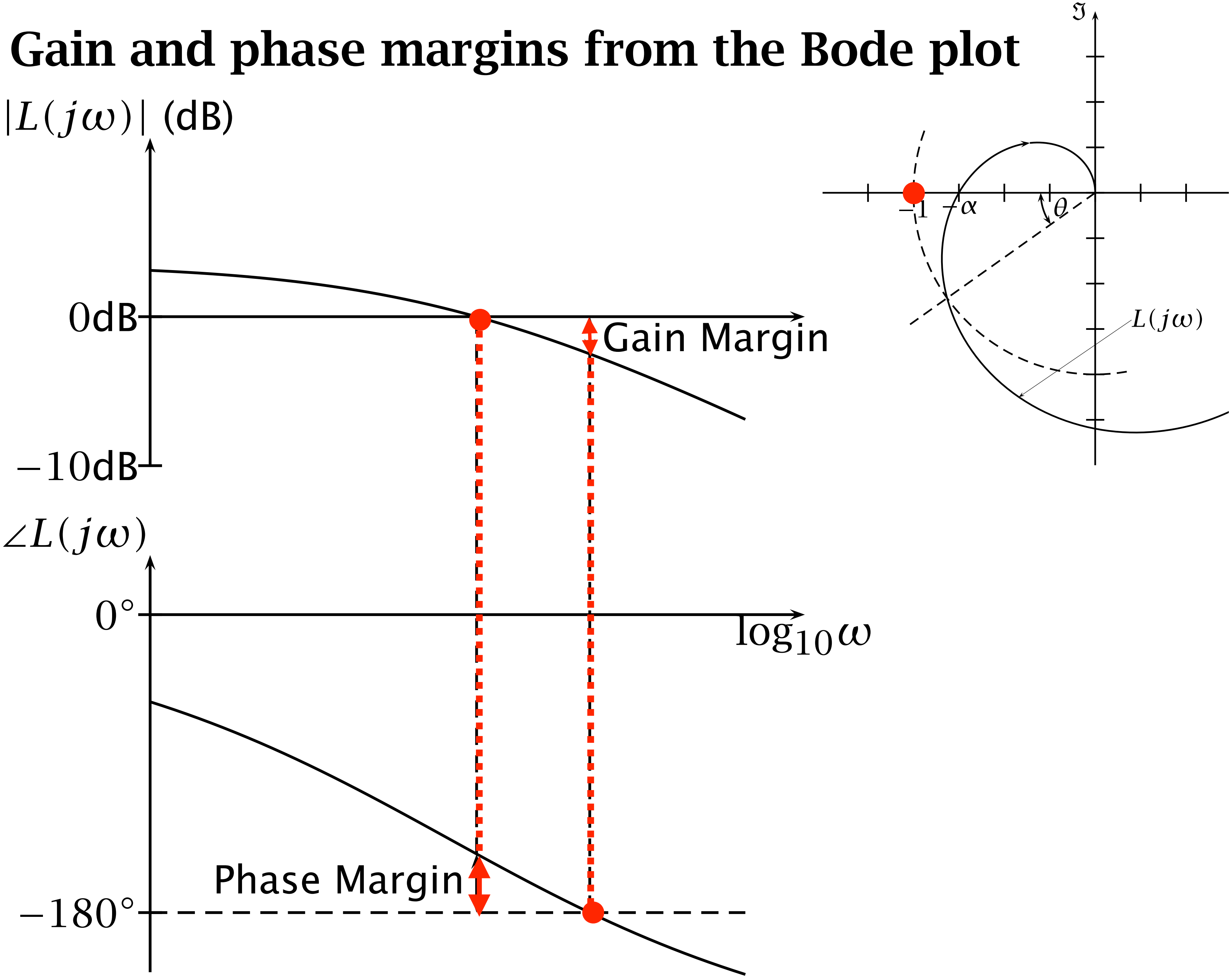


$$\text{Gain Margin} = \frac{1}{\alpha} \quad \text{Phase Margin} = \theta$$

In this example we have $\theta = 35^\circ$ and $-\alpha = -0.75$. Hence

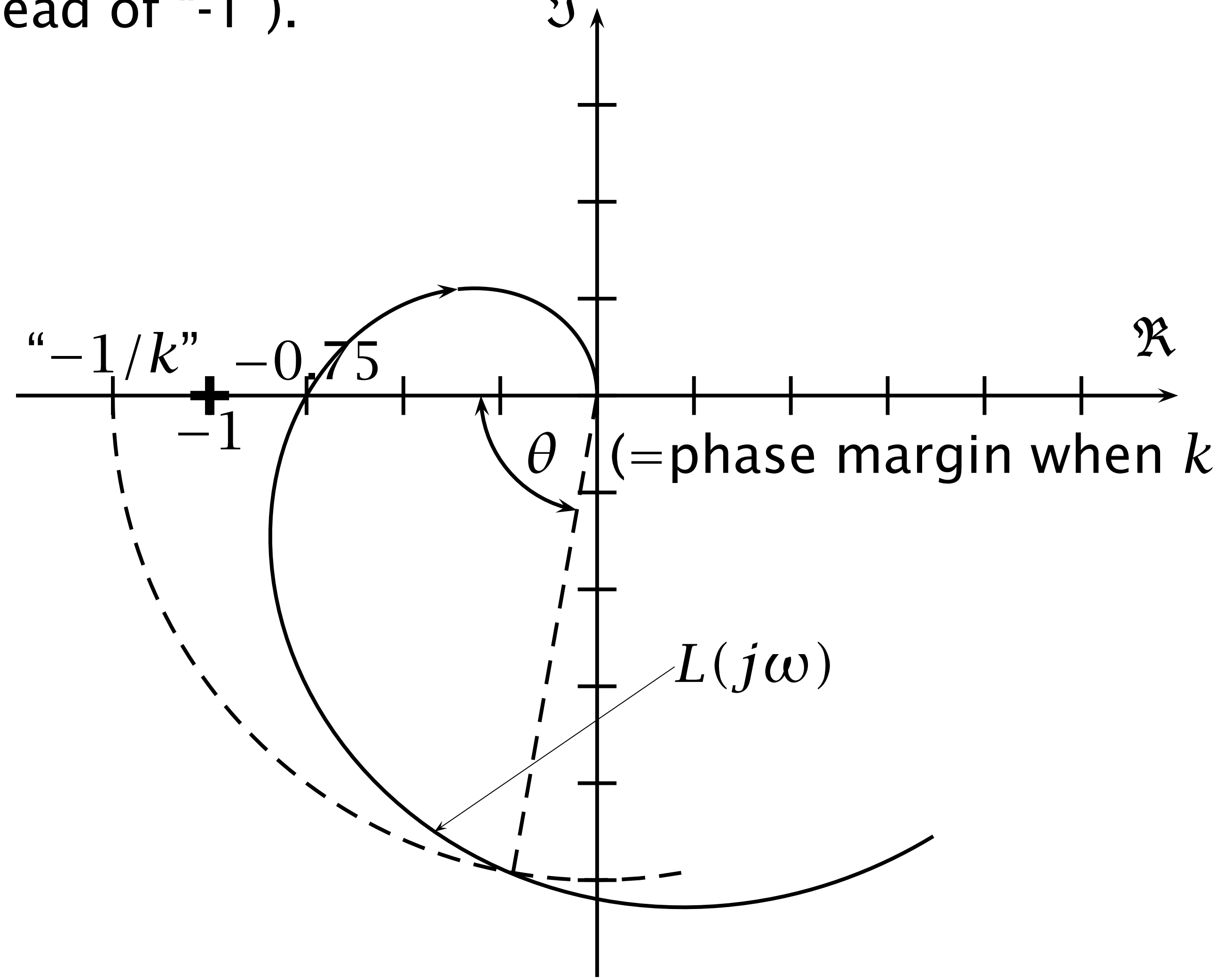
Phase Margin $= 35^\circ$ and Gain Margin $= 1/0.75 = 4/3$.

6.4.1 Gain and phase margins from the Bode plot



Gain Margin $= 20\log_{10}4/3 = 2.5$dB. Phase Margin $= 35°$ (as before)

Hint: Given a Nyquist diagram of $L(s) = kG(s)$ for $k = 1$, it is easy to find gain and phase margins for $k \neq 1$ (just look at the "$-1/k$" point instead of "-1").

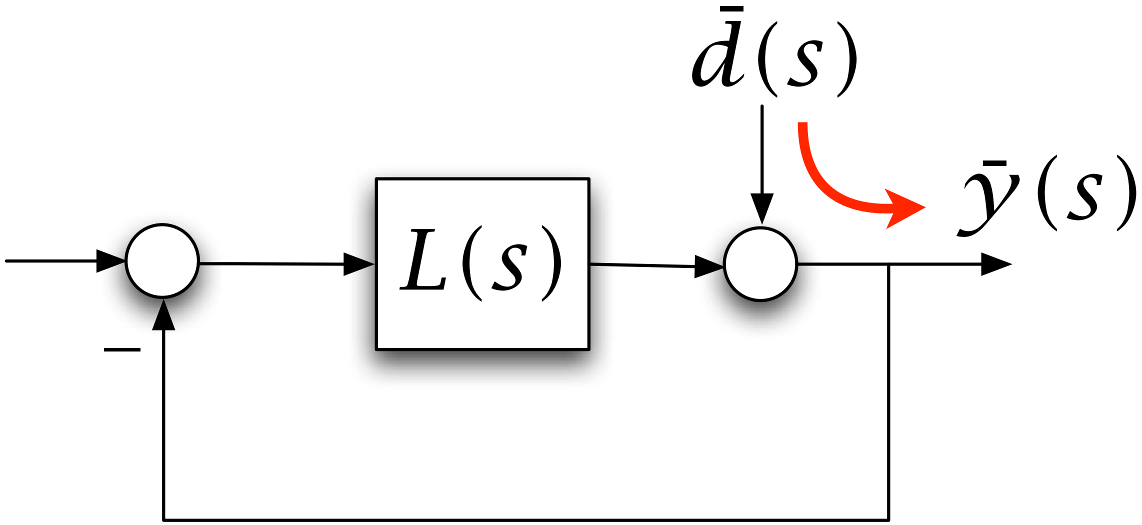


If $k = 0.8$, as here, then Gain Margin=$\frac{-1.25}{-0.75} = 5/3$ $(= 4.4dB)$, and Phase Margin=80°.

6.5 Performance of feedback systems

Good feedback properties $\iff$ "Small" sensitivity $\left| \dfrac{1}{1 + L(j\omega)} \right|$

For 1) rejection of disturbances.



$$\text{Transfer function with f/b} = \frac{1}{1+L(s)} \times \text{Transfer function without f/b}$$

Plus, 2) reducing the effects of uncertainty.

– if $L(s)$ depends on an *uncertain parameter* λ (eg $L(s) = \dfrac{1}{s^2 + 2\lambda s + 1}$) then

$$\underbrace{\frac{\dfrac{d}{d\lambda}\dfrac{L}{1+L}}{\dfrac{L}{1+L}}}_{\text{relative change in closed-loop}} = \frac{(1+L)\times\frac{dL}{d\lambda} - (L)\times\frac{dL}{d\lambda}}{(1+L)^2}\Big/\frac{L}{1+L}$$

$$= \underbrace{\frac{1}{1+L}}_{S} \quad \underbrace{\frac{\dfrac{d}{d\lambda}L}{L}}_{\text{relative change in open-loop}}$$

- Good design aims for sensitivity reduction over an appropriate range of frequencies

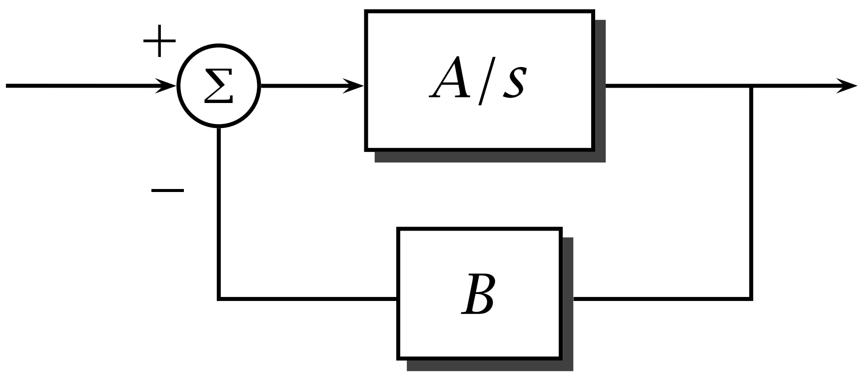
  Typically, by requiring that $\left|\dfrac{1}{1+L(j\omega)}\right| \ll 1$ for $\omega < \omega_1$ where ω_1 here denotes the desired control bandwidth.

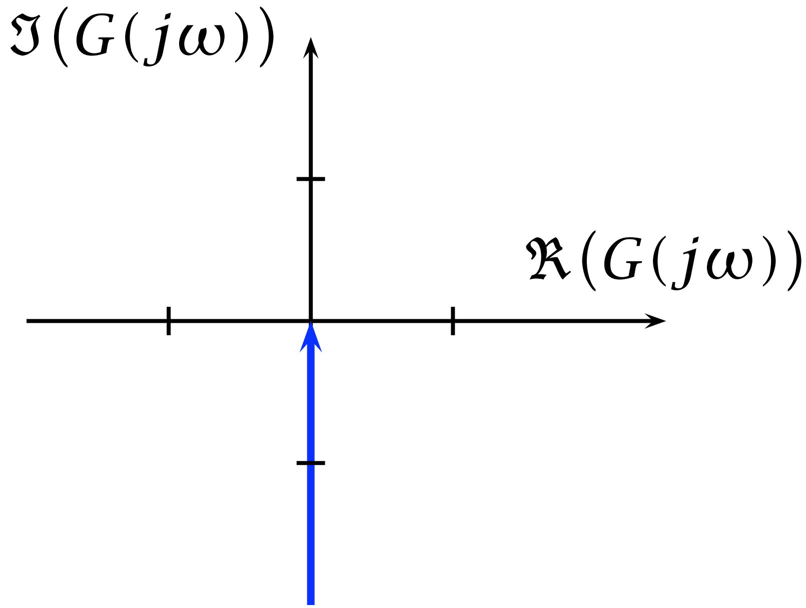
Fundamental limits on performance*

As described in Paper 5 (Linear Circuits) operational amplifiers are typically compensated so that their frequency response is similar to that of a pure integrator. Ideally they would have a transfer function

$$G(s) = A/s \quad \text{or} \quad G(j\omega) = A/j\omega.$$

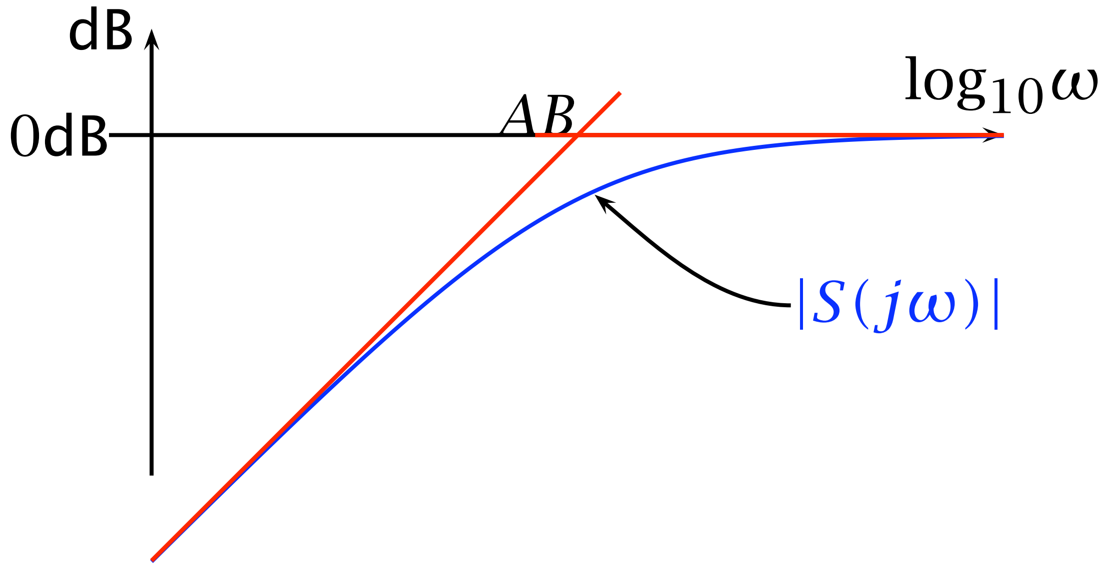
With a feedback gain of B, this would mean that the feedback system has a phase margin of 90°, for any A and B (see page 4).





In this case, the sensitivity function would be given by

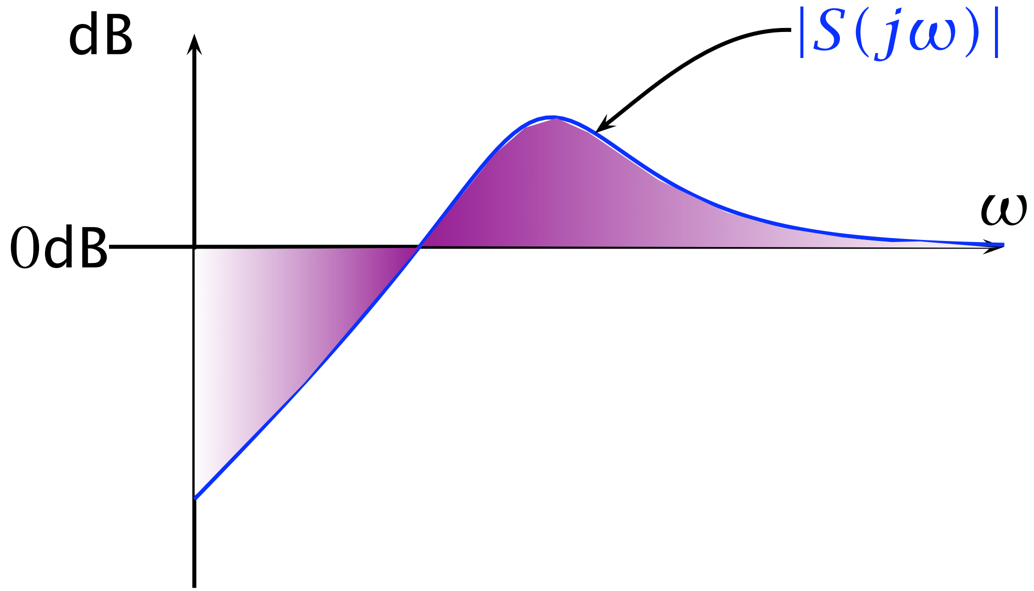
$$\frac{1}{1 + AB/s} = \frac{s}{AB} \times \frac{1}{(1 + s/AB)}.$$



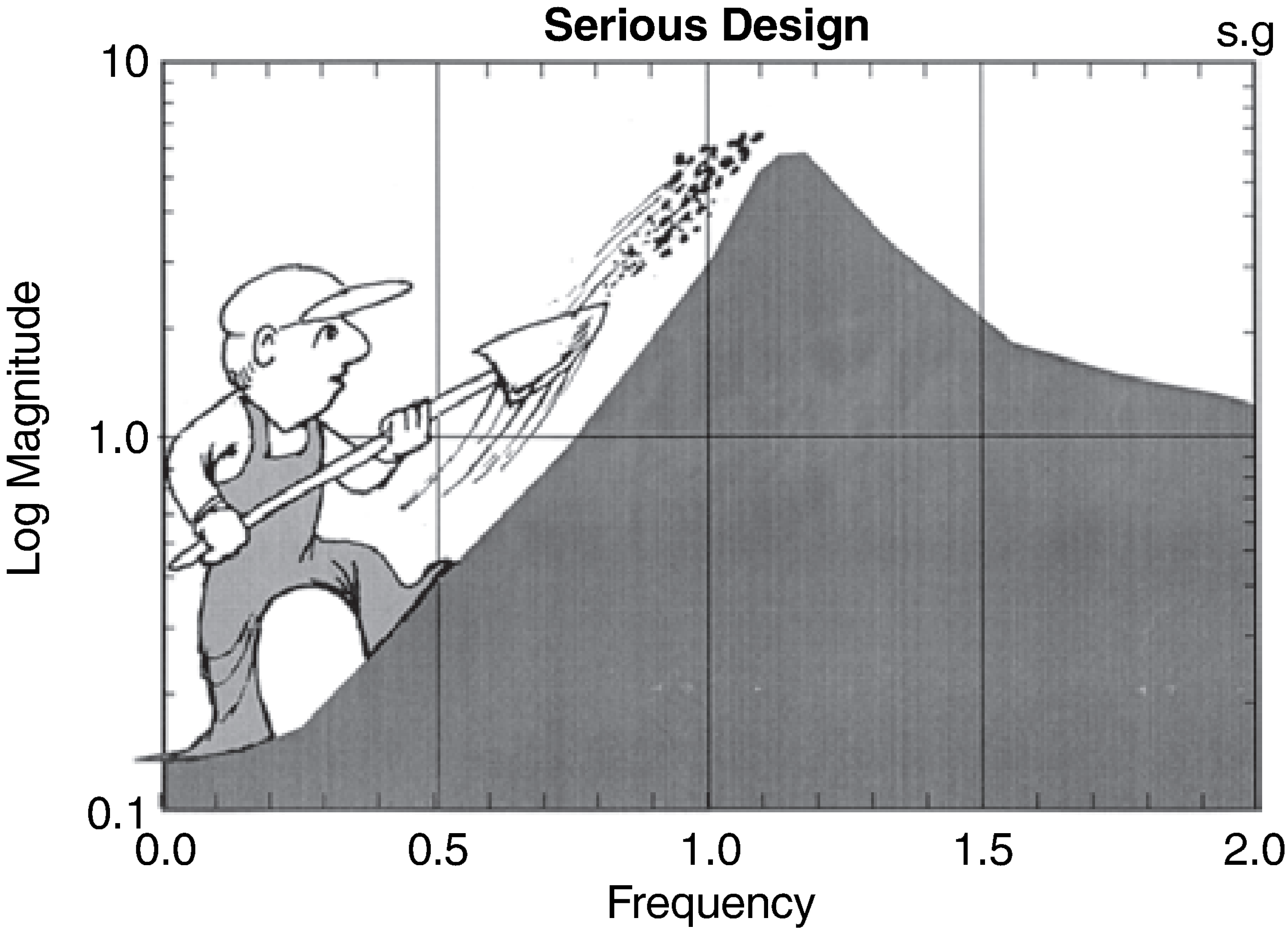
However, any real op-amp (and, indeed, any real system) will inevitably have an attenuation rate of greater $20\ \mathrm{dB/decade}$ (and a phase lag of greater than $90°$) at high frequencies. In this case, the Bode sensitivity integral applies: *This theorem will not be examined.*

Theorem: If both $L(s)$ and $1/(1+L(s))$ are asymptotically stable, and $L(j\omega)$ rolls off at a rate greater than $20\ \mathrm{dB/decade}$, then

$$\int_0^\infty 20\log_{10}\left|\frac{1}{1+L(j\omega)}\right| d\omega = 0$$



this is sometimes called the “waterbed” effect.



(from Gunter Stein’s Bode Lecture, CDC 1989)

6.5.1 The relationship between open and closed-loop frequency responses

Ultimately what we are always interested in are properties of the *closed-loop* system, such as its frequency response and pole locations. The following plots are representative of a typical feedback system, and correspond to a feedback system with a Return Ratio of

$$L(s) = \frac{2}{s(1+s)}$$

As is typical, the feedback reduces the effect of disturbances at low frequencies, up to ω_1, as evident from the plot of Sensitivity $S(j\omega)$. ω_1 is defined here as the lowest frequency at which $|S(j\omega)| = 1$. The closed-loop system will respond to reference inputs at frequencies up to around ω_2, as evident from the plot of the Complementary Sensitivity $T(j\omega)$. ω_2 is defined here as the highest frequency at which $|T(j\omega)| = 1$. Between these frequencies both disturbances and reference signals are amplified (because of the “waterbed” effect).

The actual value of the frequencies ω_1 and ω_2, and the size of these peaks, can be determined directly from the open-loop frequency response.

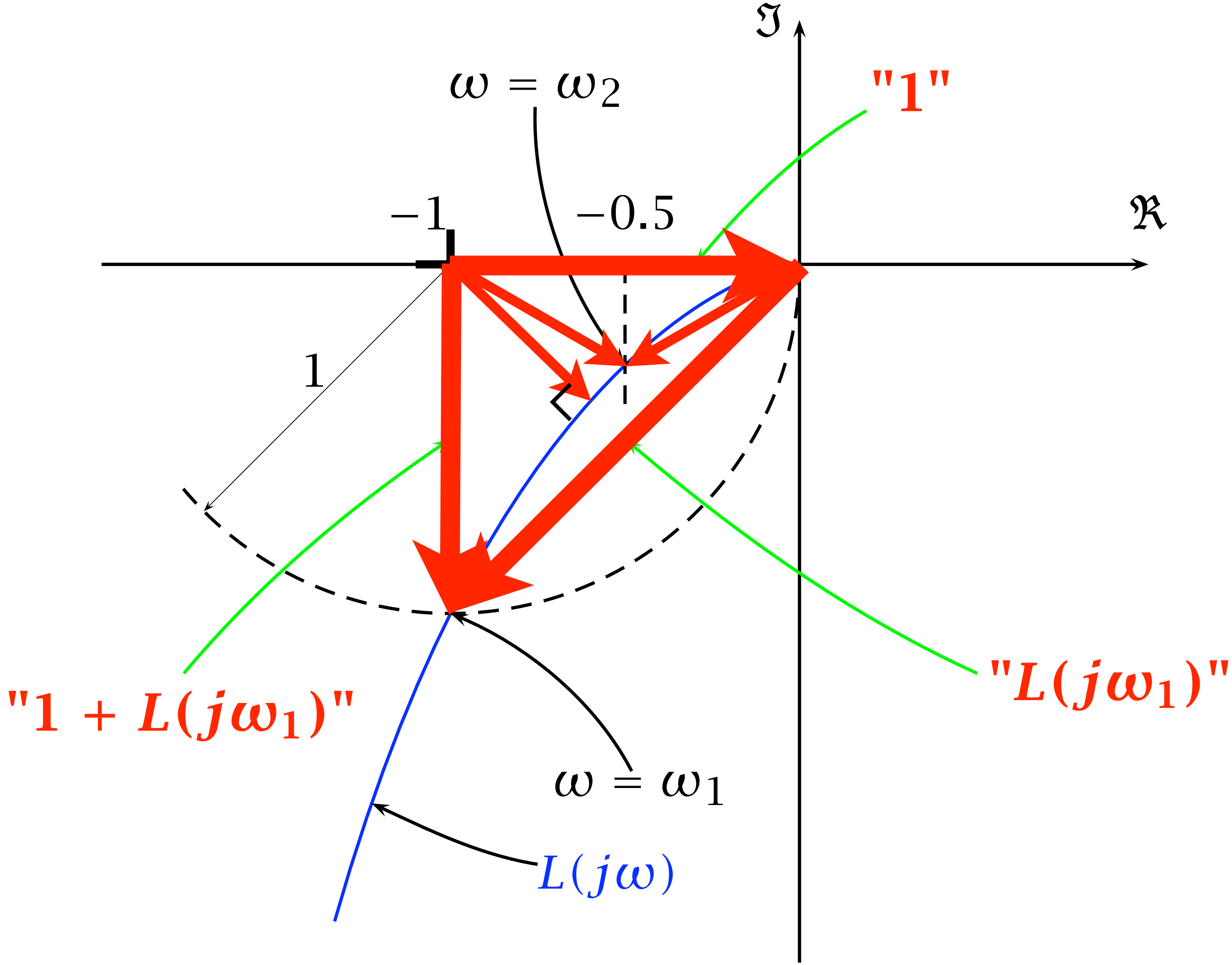
A: $|S(j\omega)| = \frac{1}{|1+L(j\omega)|} = 1$ when $|1 + L(j\omega)| = 1$, which is when the distance from the point -1 to the Nyquist locus equals 1 (this is the point $\omega = \omega_1$ overleaf).

B: $|T(j\omega)| = \frac{|L(j\omega)|}{|1+L(j\omega)|} = 1$ when $|L(j\omega)| = |1 + L(j\omega)|$, which is when the distance from the point -1 to the Nyquist locus equals the distance from the origin to the Nyquist locus (this is the point $\omega = \omega_2$ overleaf).

C: $|S(j\omega)| = \frac{1}{|1+L(j\omega)|}$ is maximized when $|1 + L(j\omega)|$ is minimized, that is when the distance from the point -1 to the Nyquist locus is at a minimum.

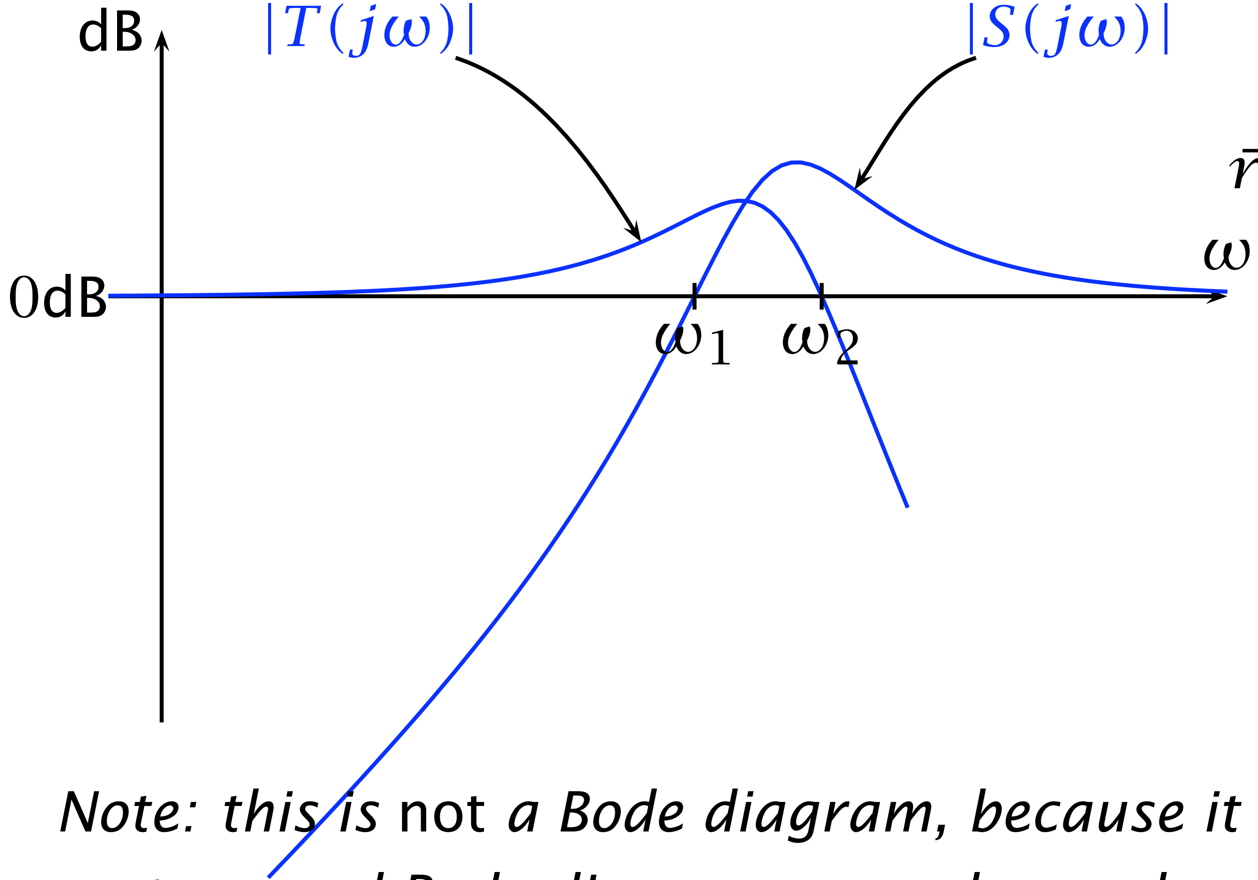
D: The easiest way to find the maximum value of $|T(j\omega)| = \frac{|L(j\omega)|}{|1+L(j\omega)|}$ is probably to try a few points around where $|1 + L(j\omega)|$ is minimized.

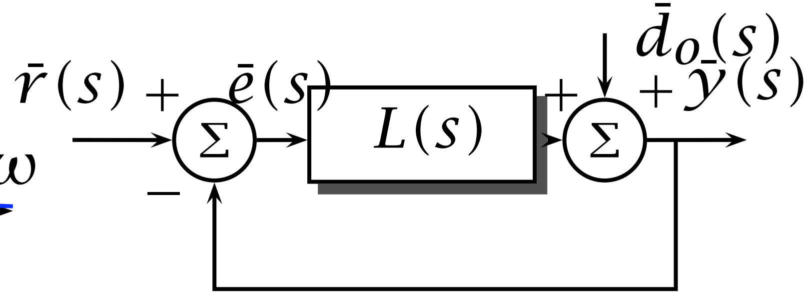
Nyquist diagram of the return ratio $L(s) = \frac{2}{s(1+s)}$



ω	$L(j\omega)$	$\frac{1}{1+L(j\omega)}$	$\frac{L(j\omega)}{1+L(j\omega)}$
1	$-1-j$	j	$1-j$
1.732	$-.5-.289j$	$1.5+.866j$	$-.5-.866j$

Closed-loop frequency responses: $S(j\omega) = \frac{1}{1+L(j\omega)}$, $T(j\omega) = \frac{L(j\omega)}{1+L(j\omega)}$





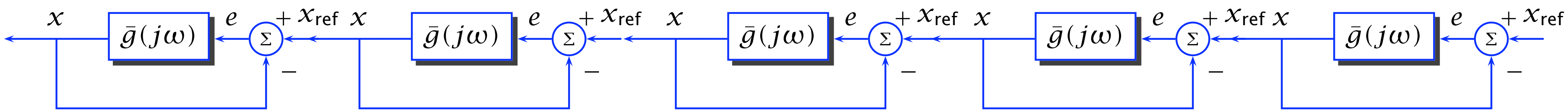
Note: this is not *a Bode diagram, because it is for a closed-loop system, and Bode diagrams are always drawn for open-loop systems (the plant, controller, return ratio etc).*

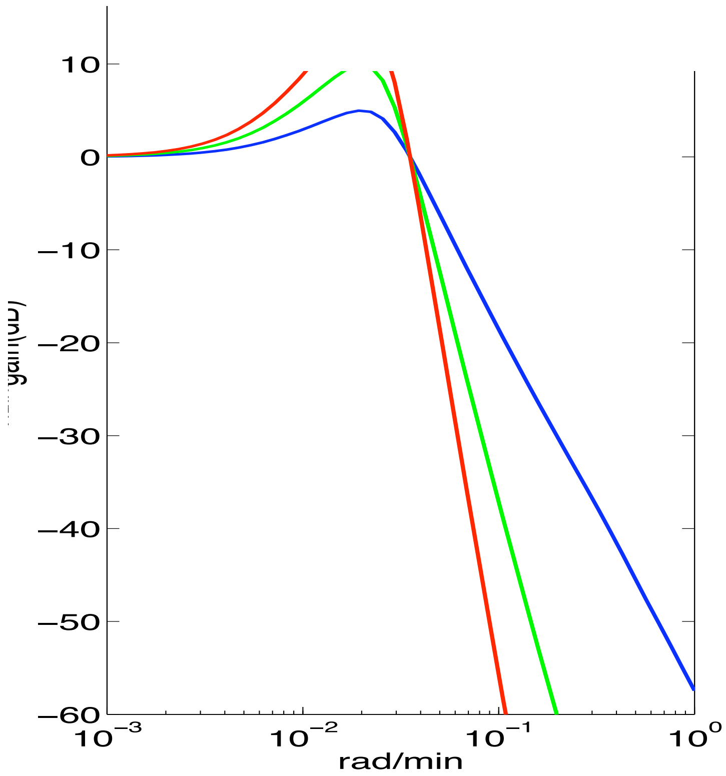
Small gain and/or phase margins correspond to there being frequencies at which $L(j\omega)$ comes close to the -1. We now see that this also corresponds to making $|1 + L(j\omega)|$ small and hence there being resonant peaks in the closed-loop transfer functions.

So,

Small gain and/or phase margins are - bad for robustness, and - bad for performance.

Block diagram





$T_{e_{i+3} \to e_i}$

6.6 The Nyquist stability theorem (for asymptotically stable $L(s)$)

On page 12 we gave an informal statement of the Nyquist stability criterion. The formal statement of the Nyquist stability theorem requires counting encirclements of the point -1:

As before, we take $L(s)$ to be the return ratio so that the closed-loop characteristic equation is $1 + L(s) = 0$.

We make the following simplifying assumption:

- $L(s)$ is asymptotically stable

This also guarantees that $L(j\omega)$ is finite for all ω (i.e. it has no $j\omega$-axis poles), and that $L(\infty)$ is finite (since $L(s)$ must be proper – see Handout 4).

Under this condition the "full" Nyquist diagram of $L(j\omega)$, for $-\infty < \omega < +\infty$, is a closed curve (since $L(j\infty) = L(-j\infty) = L(\infty)$).

Note that, since $(-j\omega) = (j\omega)^*$, it follows that $L(-j\omega) = L(j\omega)^*$. So the section of the Nyquist locus for $\omega < 0$ is the reflection in the real axis of the section for $\omega > 0$.

With this assumption we have:

The Nyquist Stability Theorem (for stable $L(s)$)

Consider a feedback system with an asymptotically stable return ratio $L(s)$. In this case, the feedback system is asymptotically stable (i.e. $\frac{L(s)}{1+L(s)}$ is asymptotically stable) if and only if the point $-1 + j0$ is *not* encircled by the "full" Nyquist diagram of $L(j\omega)$, for $-\infty < \omega < +\infty$.

6.6.1 Notes on the Nyquist Stability Theorem:

1. Encirclements must be ‘added algebraically’. If there is 1 clockwise and 1 anticlockwise encirclement then they ‘add up’ to 0 encirclements.

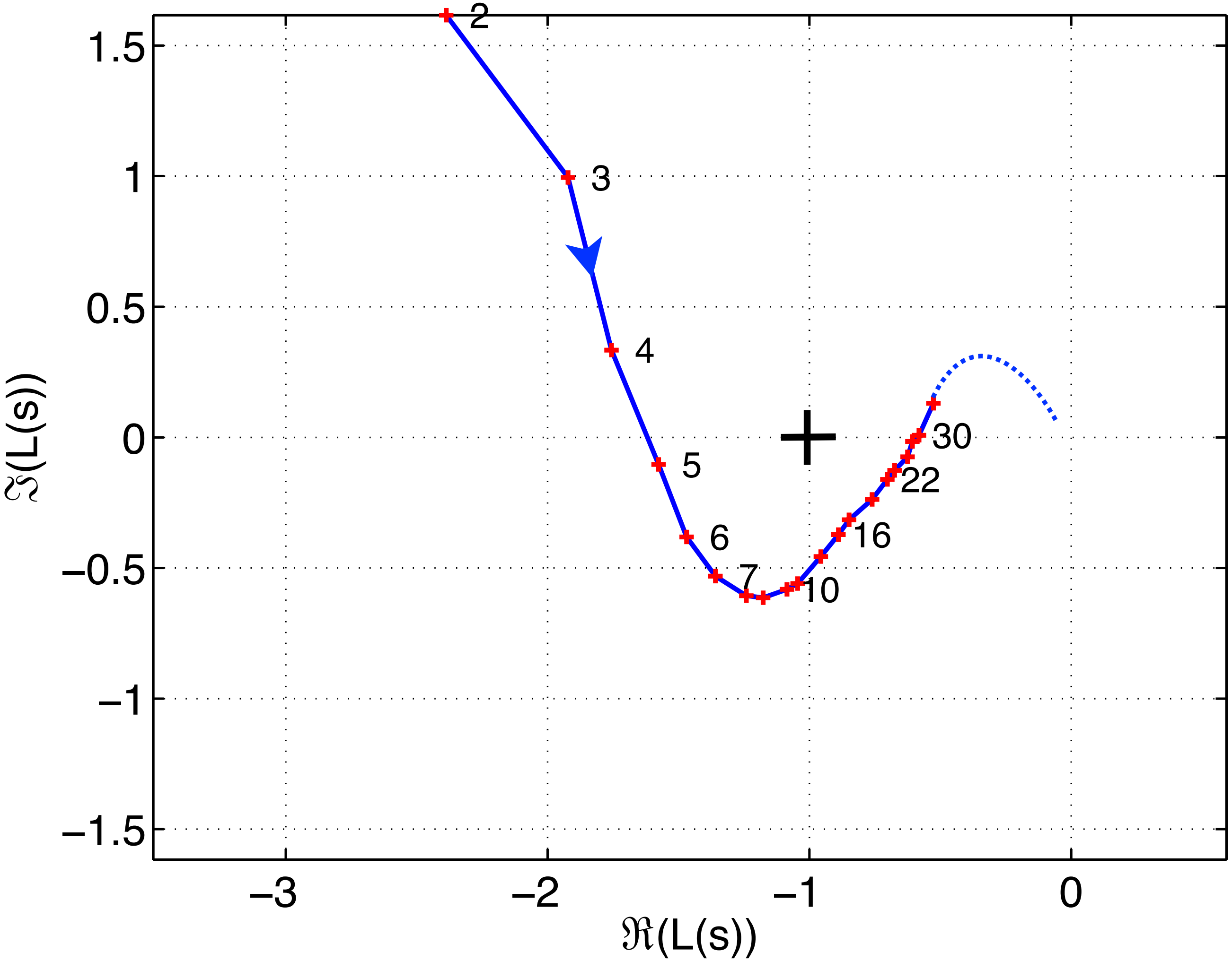
2. $L(s)$ often has one or more poles at 0 (due to integrators in the plant or the controller). The theorem still works, but one has to worry about what happens to the graph of $L(j\omega)$ at $\omega = 0$ – as the locus is no longer a closed curve. It can be shown that, if $L(s)$ has n poles at the origin, then the Nyquist locus should be completed by adding a large $n \times 180^\circ$ arc, in a clockwise direction.

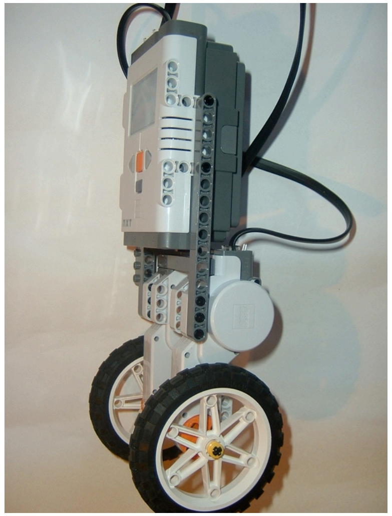
3. If $L(s)$ is unstable, and has n_p unstable poles, then the theorem must be modified as follows: “The feedback system is stable if and only if the ‘full’ Nyquist diagram encircles the point $-1 + j0$ n_p times in an anticlockwise direction.”

4. (This a repeat of the informal statement of the Nyquist stability criterion from page 12.)

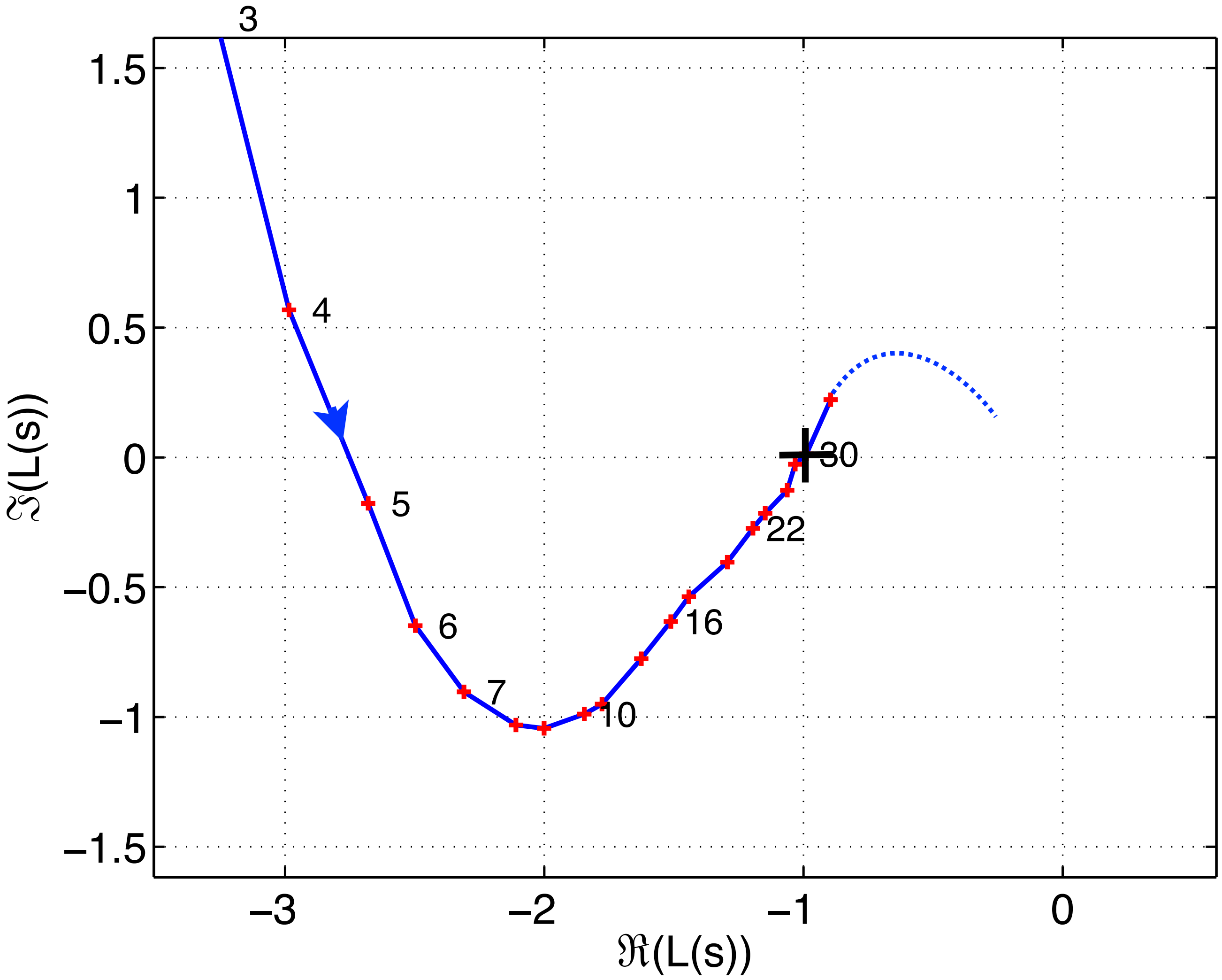
   A potentially ambiguous statement of the theorem, but one which almost always works, is: "The feedback system is stable if the Nyquist diagram of $L(j\omega)$ 'leaves the point $-1 + j0$ on its left' ". This still works (usually) if there are poles at the origin or even if $L(s)$ is unstable.

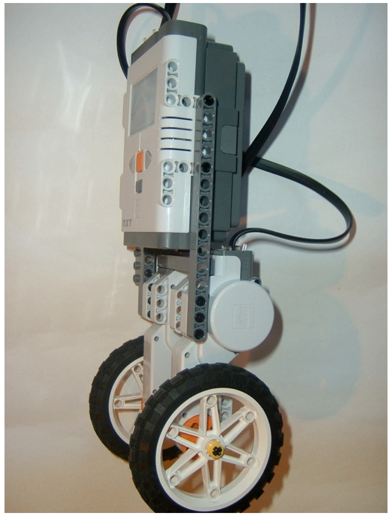
   This informal version of the theorem is adequate for the examples which follow, for any others that you will encounter on this course and indeed for any you are likely to encounter in practice.



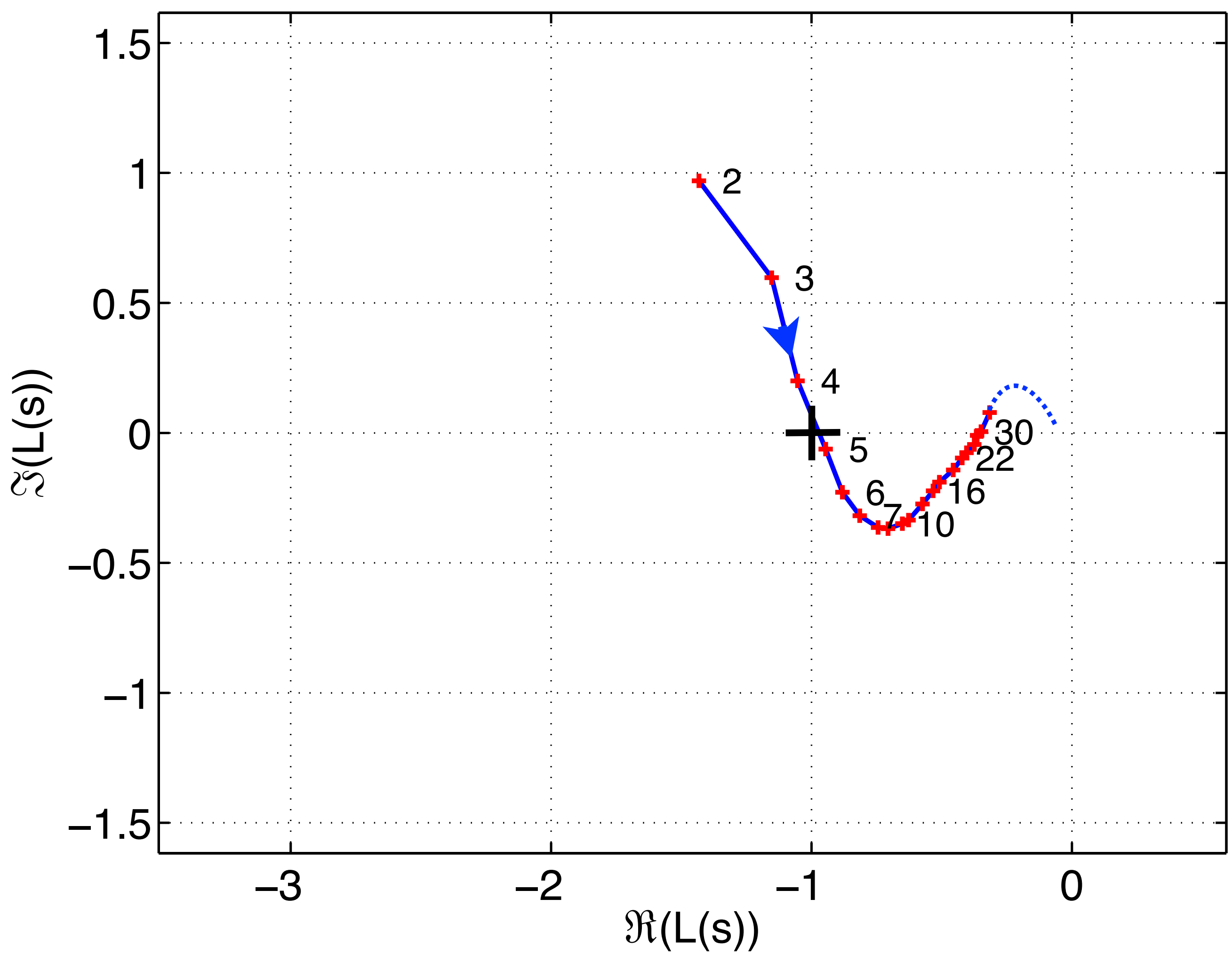


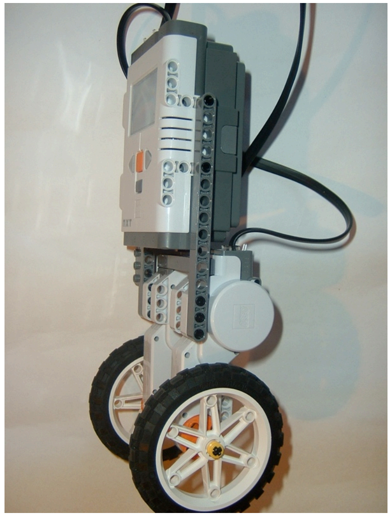
```
MotorPower = -35*WheelAngle - 0.3*WheelVelocity
                       + 0.8*LeanAngle + 0.35*GyroValue
```





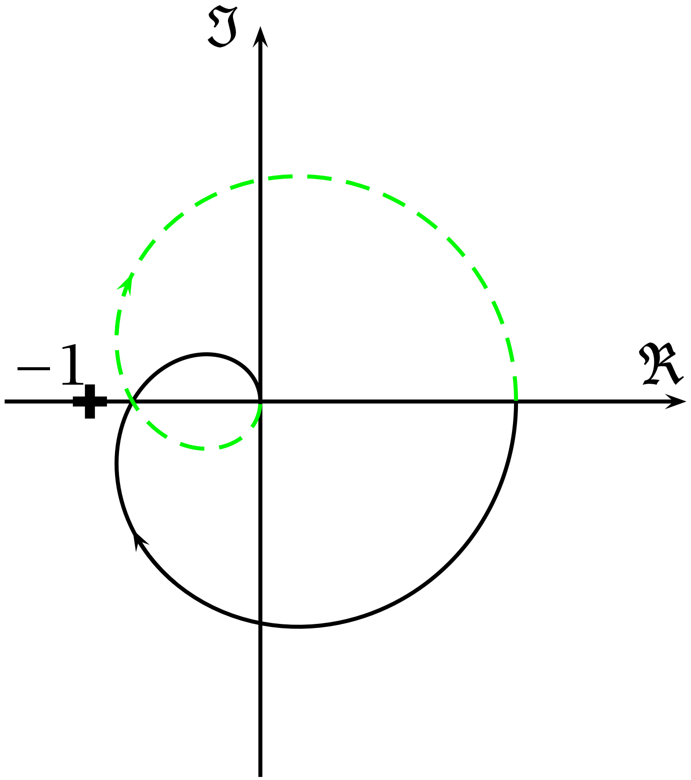
```
MotorPower = (-35*WheelAngle - 0.3*WheelVelocity
                 + 0.8*LeanAngle + 0.35*GyroValue) *1.7
```





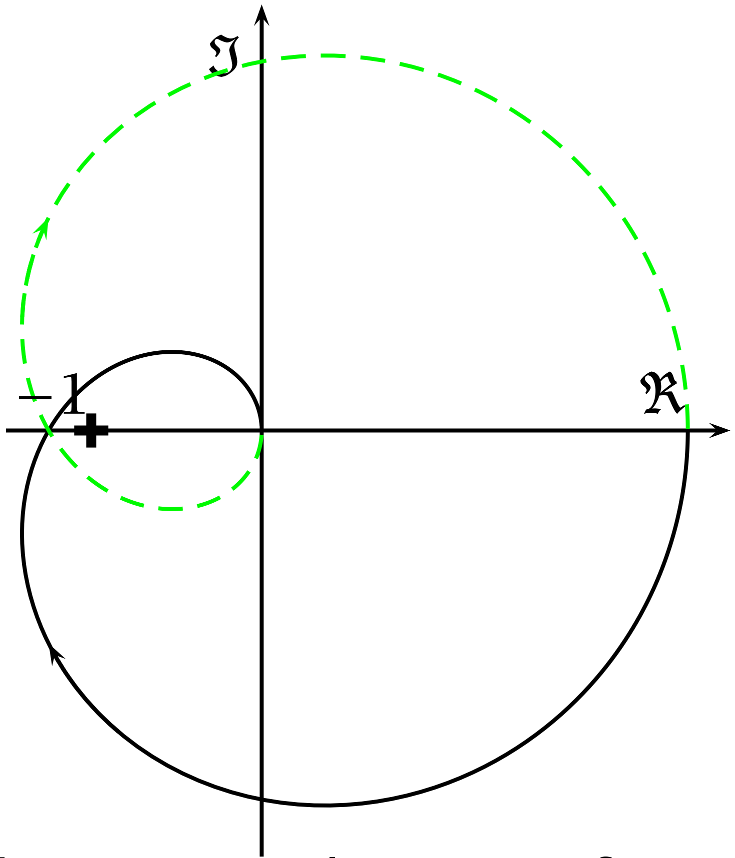
```
MotorPower = (-35*WheelAngle - 0.3*WheelVelocity
                 + 0.8*LeanAngle + 0.35*GyroValue) *0.6
```

Examples: Formal application of Nyquist stability theorem.



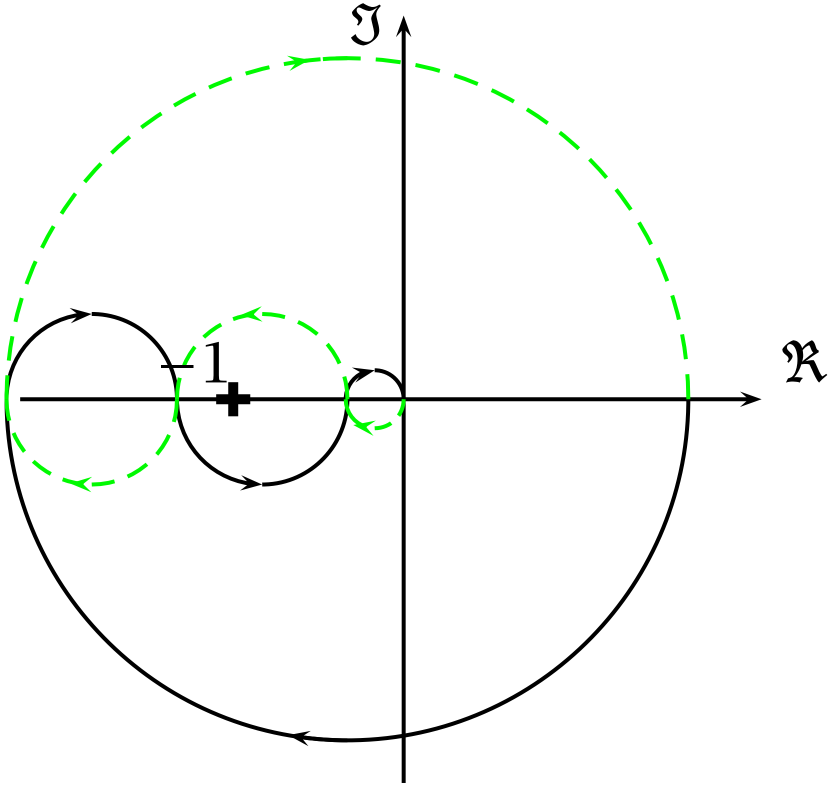
no encirlements of -1

$\Longrightarrow \quad \dfrac{L(s)}{1+L(s)}$ is asymptotically stable



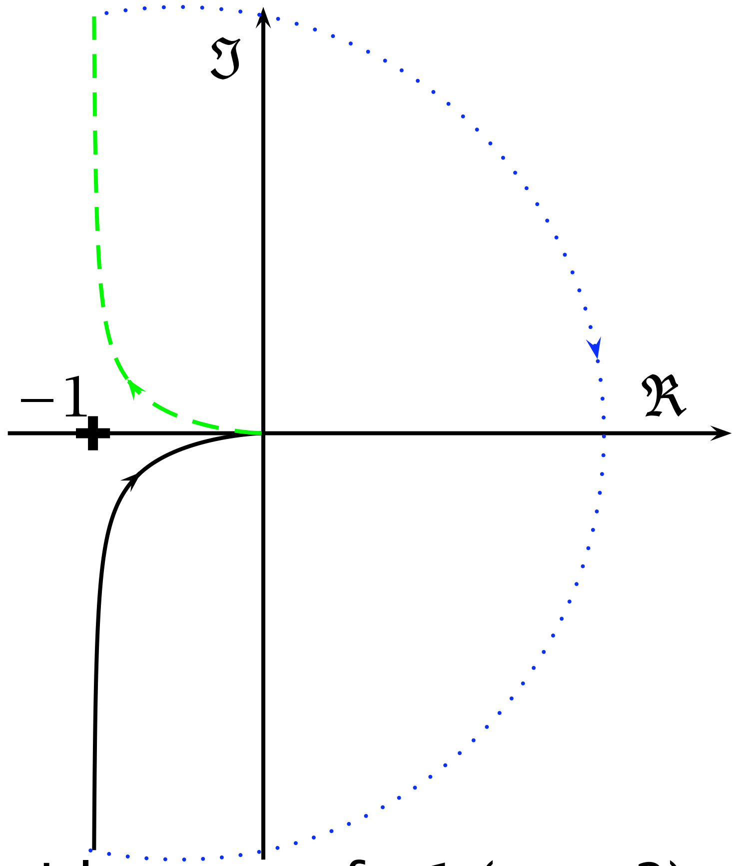
2 clockwise encirclements of -1

$\Longrightarrow \quad \dfrac{L(s)}{1+L(s)}$ is unstable



1 clockwise + 1 anticlockwise encirclement of -1
i.e. 0 net encirclements (note 1)

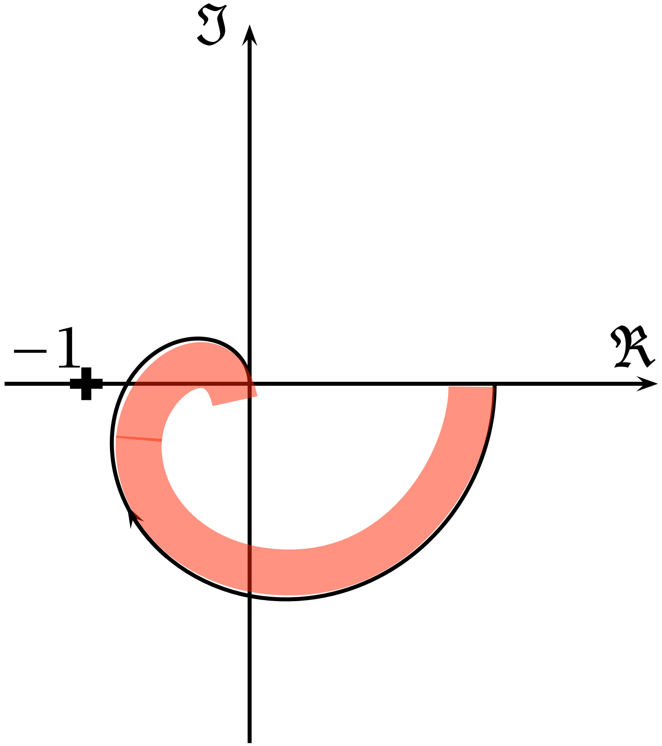
$\implies \dfrac{L(s)}{1+L(s)}$ is asymptotically stable



no encirlements of -1 (note 2)

$\implies \dfrac{L(s)}{1+L(s)}$ is asymptotically stable

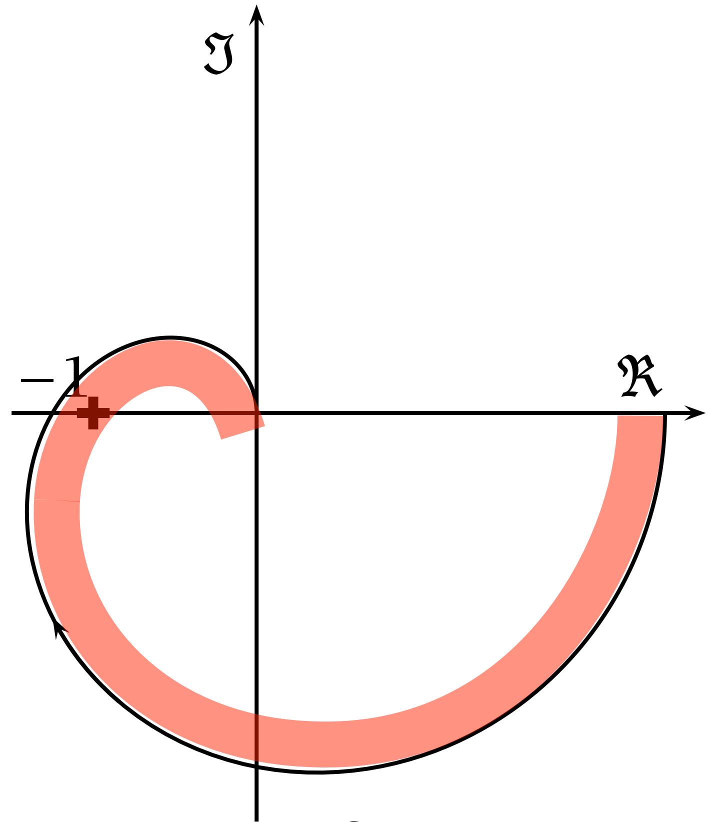
Examples: Informal application of the Nyquist stability theorem (based on note 4).



-1 to left of locus

$$\Longrightarrow \quad \frac{L(s)}{1+L(s)} \text{ is}$$

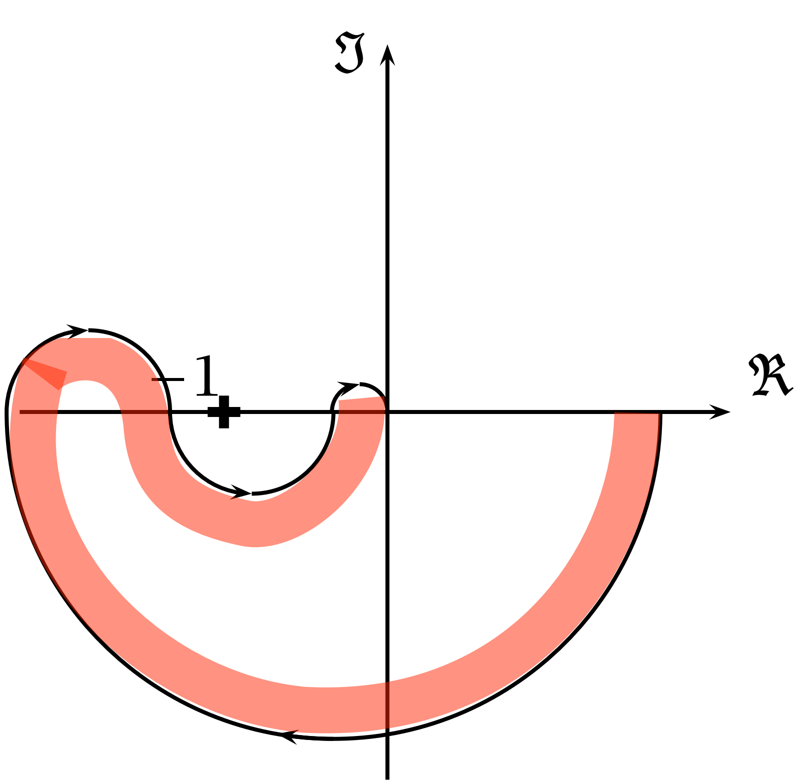
asymptotically stable



-1 to right of locus

$$\Longrightarrow \quad \frac{L(s)}{1+L(s)} \text{ is}$$

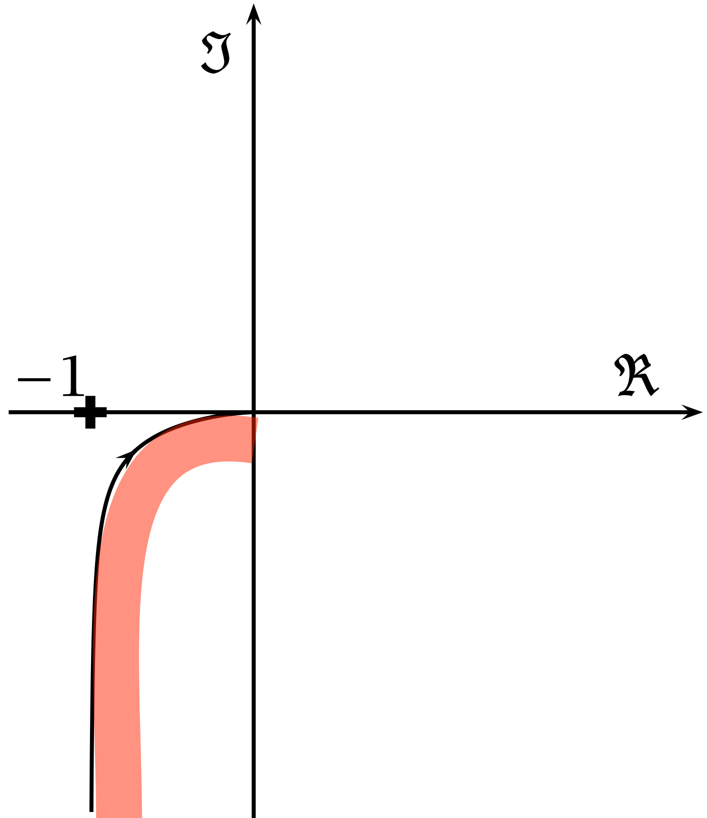
unstable



-1 to left of locus

$\implies \dfrac{L(s)}{1+L(s)}$ is

asymptotically stable



-1 to left of locus

$\implies \dfrac{L(s)}{1+L(s)}$ is

asymptotically stable

Hence: the informal application of the Nyquist stability criterion works for all these cases.