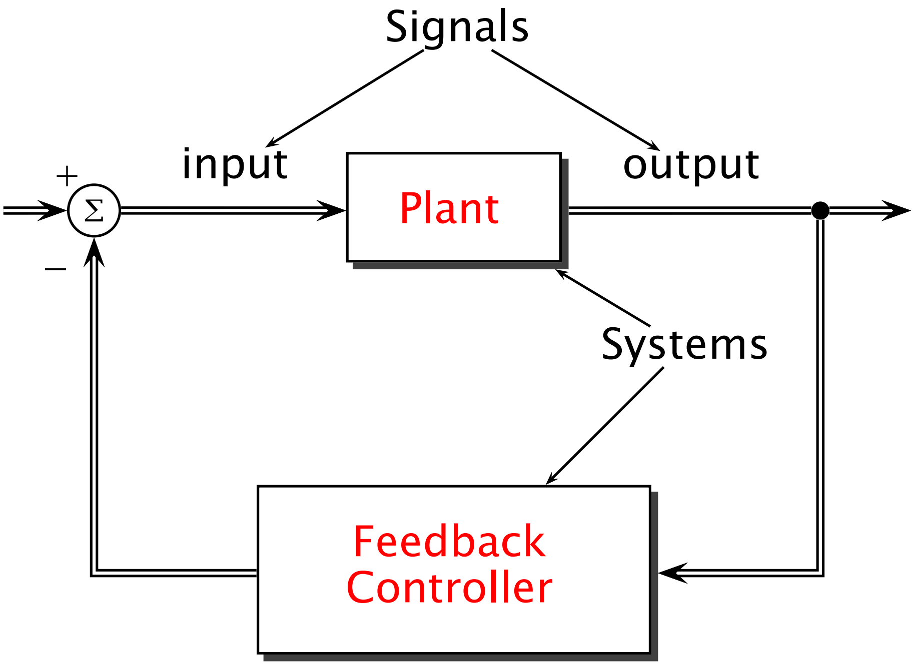
Part IB Paper 6: Information Engineering

LINEAR SYSTEMS AND CONTROL

Glenn Vinnicombe

HANDOUT 1

"Signals, systems and feedback"



(Filled-in version of notes at
`http://www-control.eng.cam.ac.uk/gv/p6`)

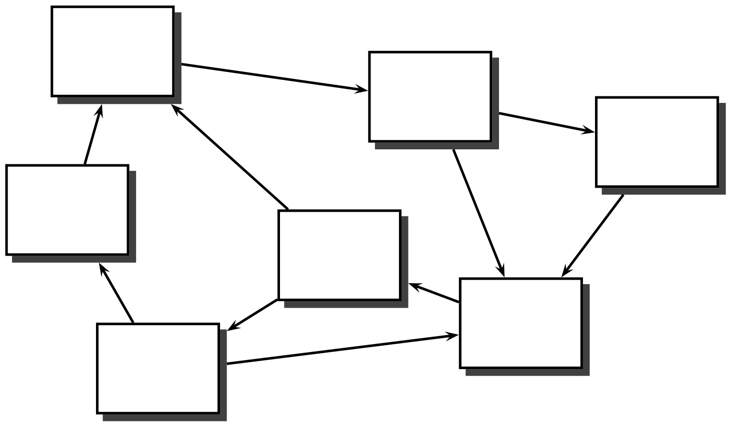
The **Aims** of the course are to:

- Introduce and motivate the use of feedback control systems.
- Introduce analysis techniques for linear systems which are used in control, signal processing, communications, and other branches of engineering.
- Introduce the specification, analysis and design of feedback control systems.
- Extend the ideas and techniques learnt in the IA Mechanical Vibrations course.

By the end of the course students should:

- Be able to develop and interpret block diagrams and transfer functions for simple systems.
- Be able to relate the time response of a system to its transfer function and/or its poles.
- Understand the term 'stability', its definition, and its relation to the poles of a system.
- Understand the term 'frequency response' (or 'harmonic response'), and its relation to the transfer function of a system.
- Be able to interpret Bode and Nyquist diagrams, and to sketch them for simple systems.
- Understand the purpose of, and operation of, feedback systems.
- Understand the purpose of proportional, integral, and derivative controller elements, and of velocity feedback.
- Possess a basic knowledge of how controller elements may be implemented using operational amplifiers, software, or mechanical devices.
- Be able to apply Nyquist's stability theorem, to predict closed-loop stability from open-loop Nyquist or Bode diagrams.
- Be able to assess the quality of a given feedback system, as regards stability margins and attenuation of uncertainty, using open-loop Bode and Nyquist diagrams.

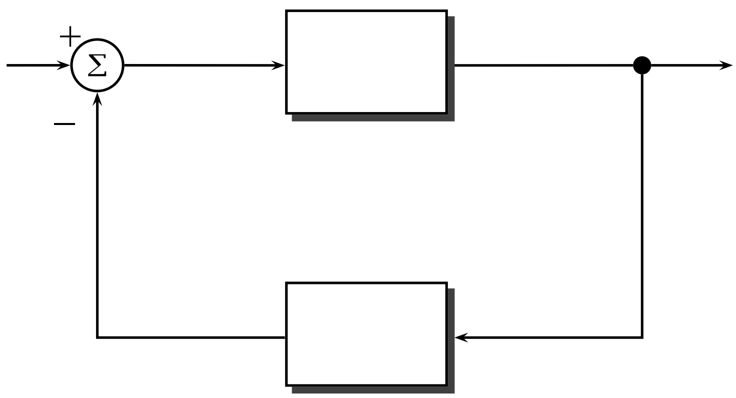
What the course is *really* about



- Understanding complex systems as an interconnection of simpler subsystems.
- Relating the behaviour of the interconnected system to the behaviour of the subsystems.
- (but we'll only consider the feedback interconnection in detail)

Part 1 (1st 7 lectures or so)

Part 2 (last 7 lectures or so)



SYLLABUS

Course material	Section numbers book 1	Section numbers book 2
Examples of feedback control systems. Use of block diagrams. Differential equation models. Meaning of 'Linear System'.	1.1-1.13 2.2-2.3	1.1-1.3 2.1-2.5
Review of Laplace transforms. Transfer functions. Poles (characteristic roots) and zeros. Impulse and step responses. Convolution integral. Block diagrams of complex systems.	2.4-2.6	3.1-3.2
Definition of stability. Pole locations and stability. Pole locations and transient characteristics.	6.1 5.6	3.3-3.6
Frequency response (harmonic response). Nyquist (polar) and Bode diagrams.	8.1-8.3	6.1-6.3
Terminology of feedback systems. Use of feedback to reduce sensitivity. Disturbances and steady-state errors in feedback systems. Final value theorem.	4.1-4.2 4.4-4.5	4.1 3.1.6
Proportional, integral, and derivative control. Velocity (rate) feedback. Implementation of controllers in various technologies.	7.7 12.6	4.3
Nyquist's stability theorem. Predicting closed-loop stability from open-loop Nyquist and Bode plots.	9.1-9.3	6.3
Performance of feedback systems: Stability margins, Speed of response, Sensitivity reduction.	8.5 9.4-9.6 12.5	6.4,6.6 6.9

References

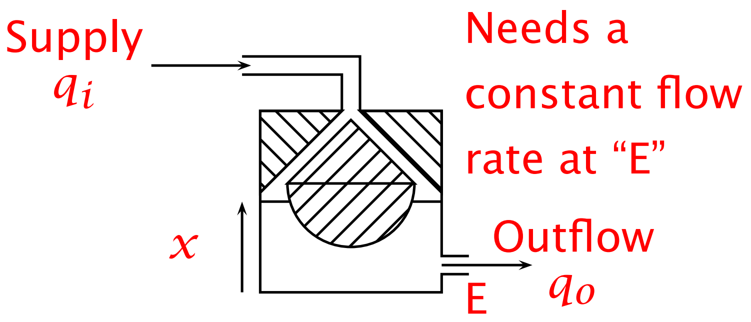
1. Dorf,R.C, and Bishop,R.H, Modern Control Systems, 10th ed., (Addison-Wesley), 2005.

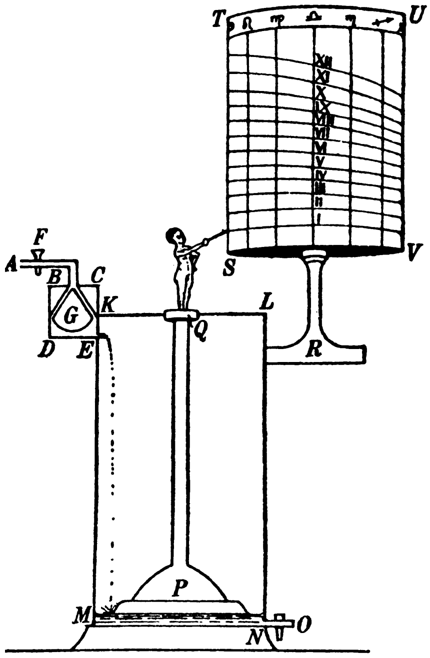
2. Franklin,G.F, Powell,J.D, and Emami-Naeini,A, Feedback Control of Dynamic Systems, 5th ed., (Addison-Wesley), 2006.

Contents

1.1 Examples of feedback systems

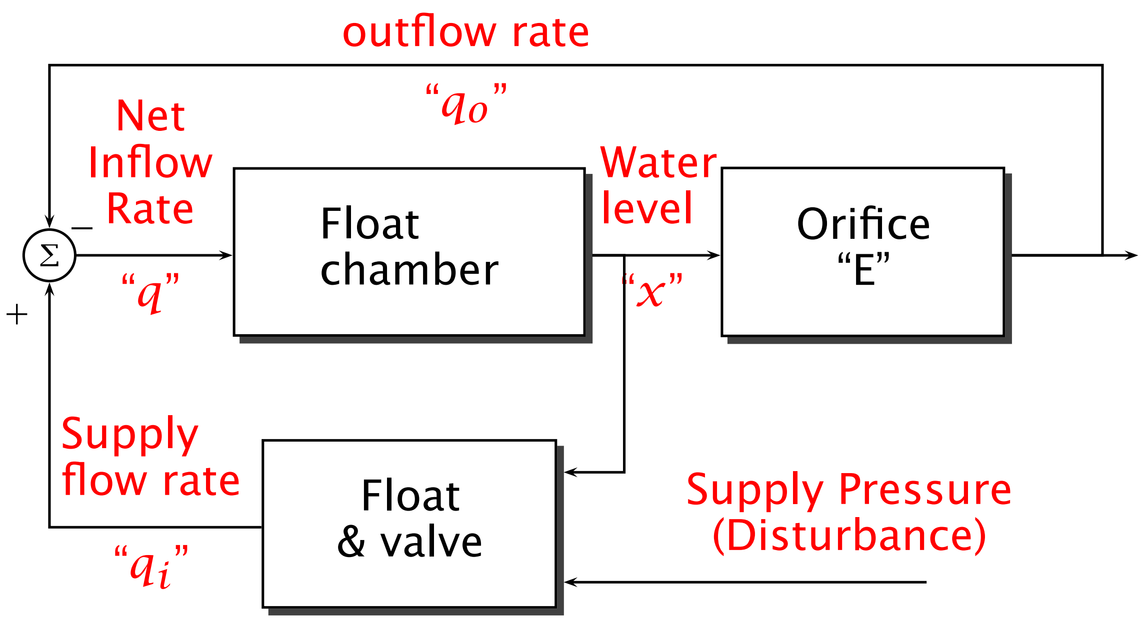
1.1.1 Ktesibios' Float Valve regulator (Water-clock, Alexandria 250BC)





is a feedback control system.

Block Diagram:



Signals have units (usually), are functions of time, and are represented by the *connections*:

e.g. Net inflow "$q(t)$" is measured in m^3/s
Water level "$x(t)$" is measured in m

Systems have equations, and are represented by the *blocks*:

e.g. the Float chamber is described by

$$x(t) = \frac{\mathbf{1}}{A} \int_{\mathbf{0}}^{t} q(\tau)\, d\tau$$

cross-sectional area

1.1.2 Watt's Governor

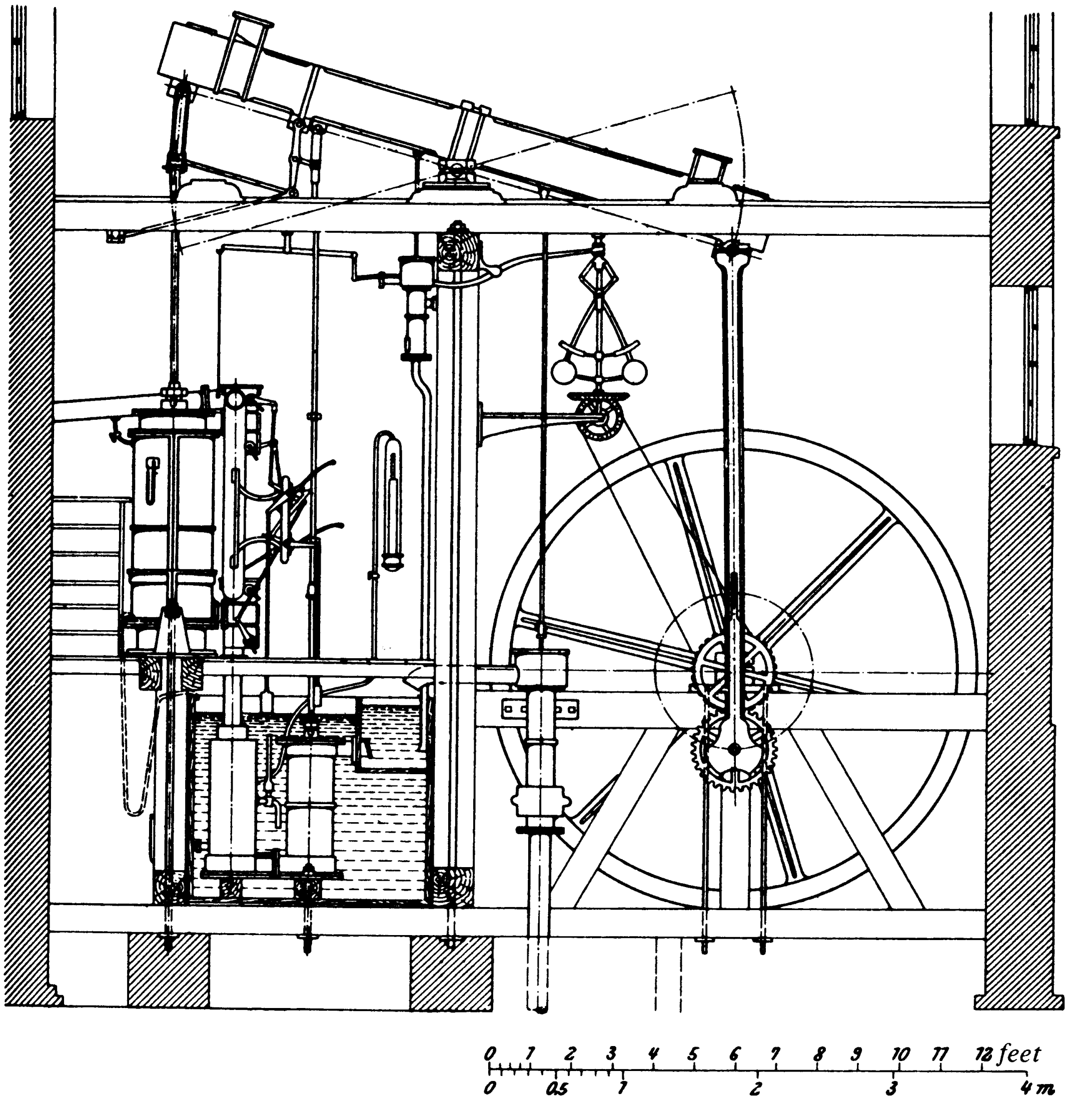
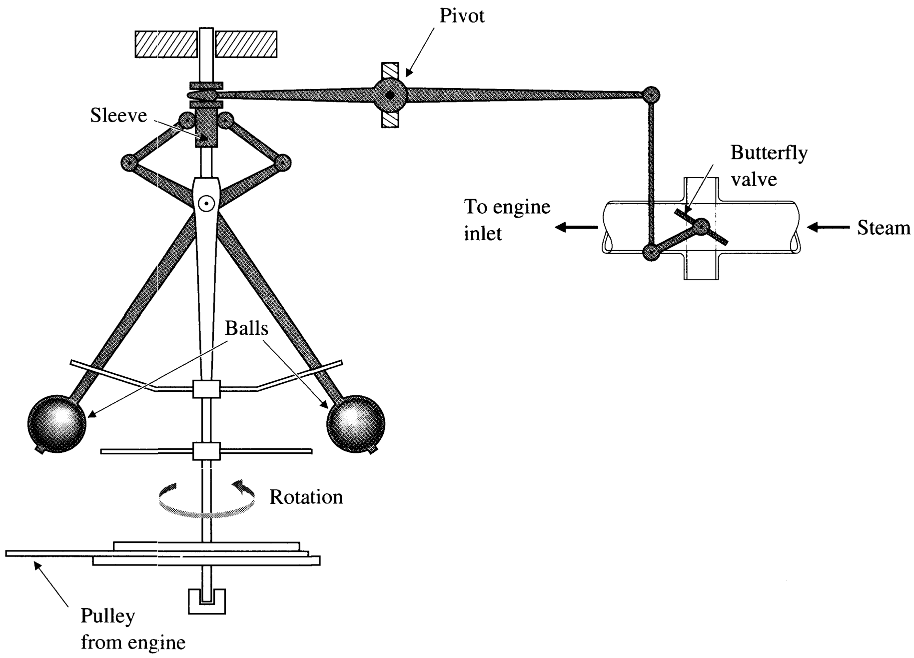


Figure 1 Watt steam engine (1789-1800) with centrifugal governor.

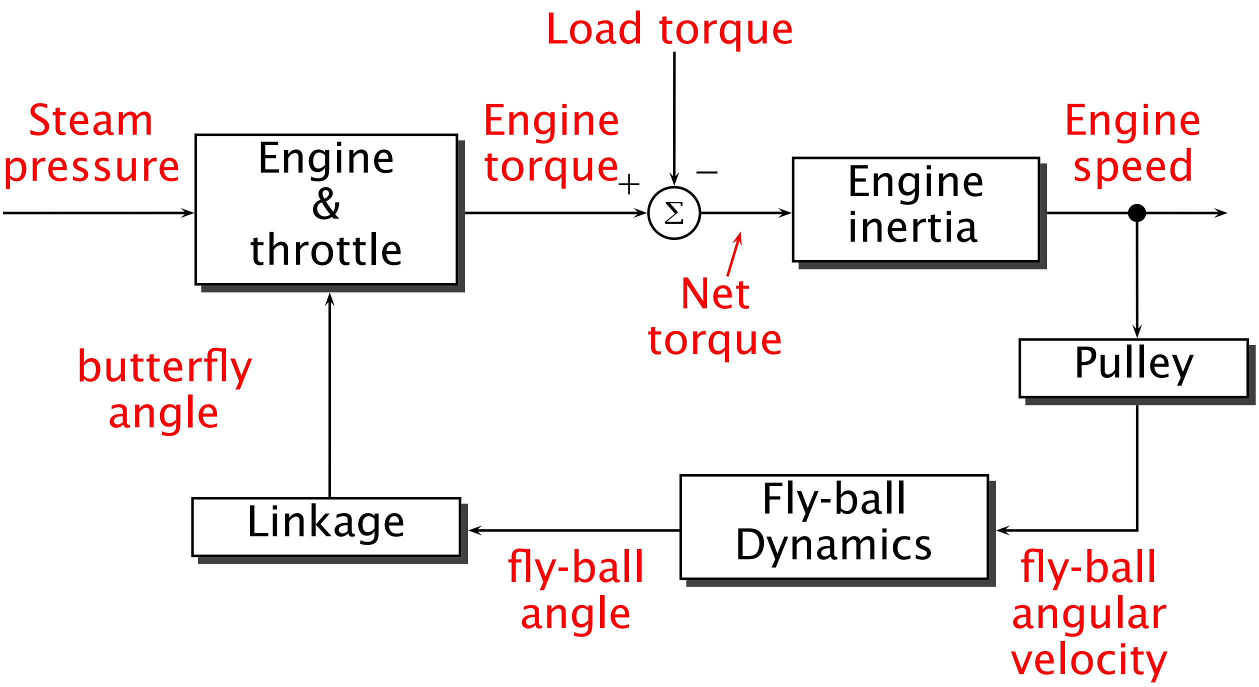
If the load upon the running engine is suddenly increased, its speed will decrease. The flyweights will swing back, and the sleeve will slide upward, causing the steam valve to open. The increase in the flow rate of steam, and hence in torque, will accelerate the engine. The centrifugal weights will fly outward again, reducing the aperture of the valve. Ultimately, the engine will reach an equilibrium at a new speed that lies somewhat below the equilibrium speed prior to the load increase. This *offset* due to lasting disturbances or changes in the command signal is a characteristic of all *proportional* control systems. The increased load requires an increased

Watt's Governor



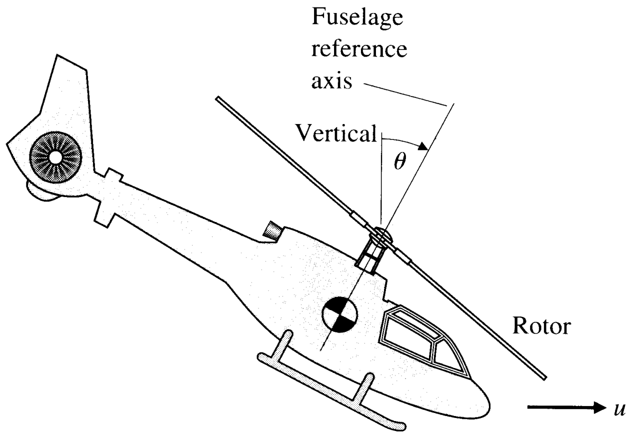
Is a feedback control system.

Block diagram:



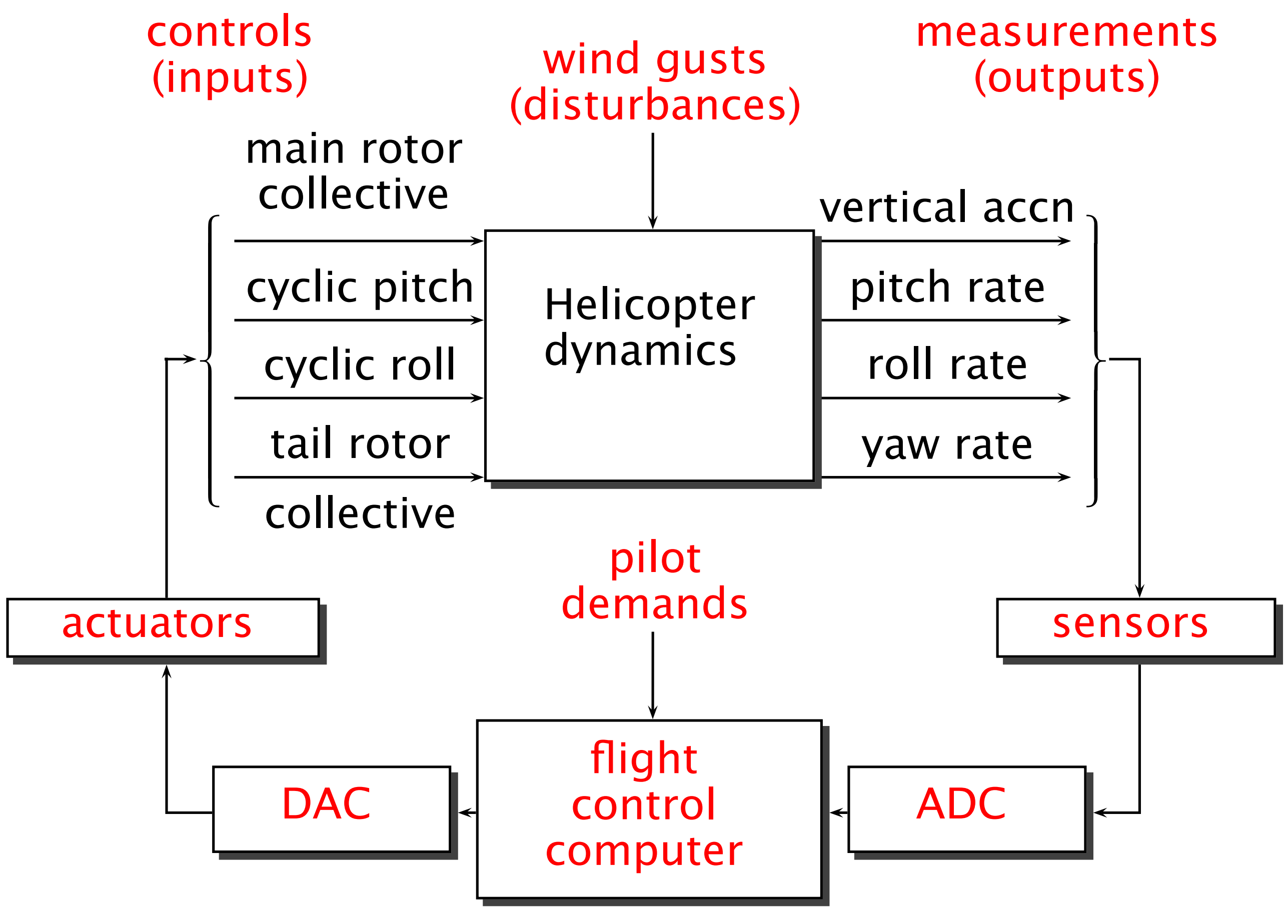
Note: it would be wrong to label the input to the feedback system as simply "steam" rather than "steam pressure". Steam in itself is not a quantity (although its pressure, temperature or flow rate is).

1.1.3 A Helicopter Flight Control System

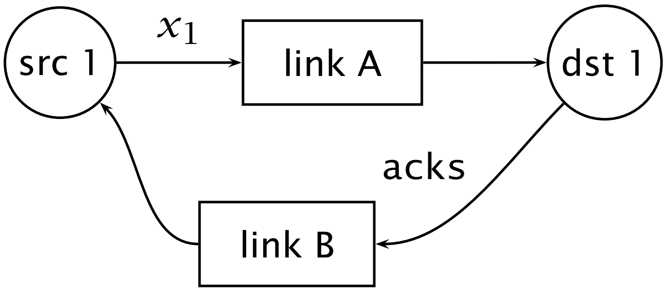


Is a feedback control system

Block Diagram:



1.1.4 Internet congestion control (TCP)



Is a feedback control system

– in fact, the largest man made one in the world.

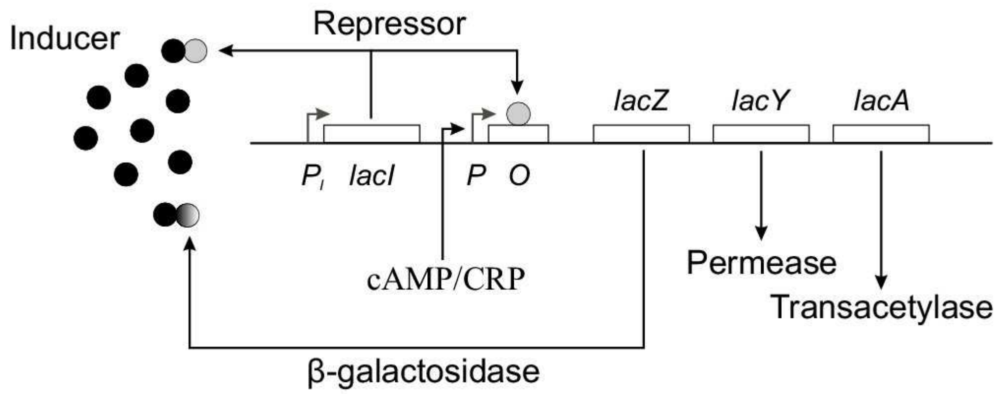
(in reality, of course, there are many source/destination pairs competing for bandwidth over many links)

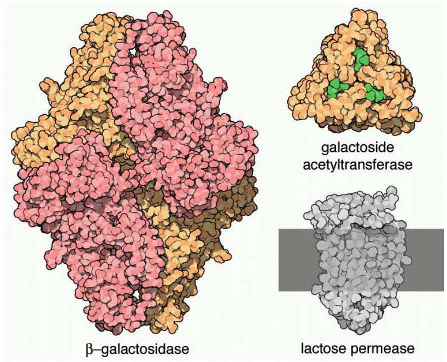
Note: This is NOT a block diagram

– it shows the flow of "stuff" (in this case packets) not information.

Files to be transferred across the Internet using the Transmission Control Protocol (TCP) - eg a download from the web - are broken into packets of size typically around 1500bytes, with headers specifying the destination and the number of the packet amongst other information. These packets are sent one by one into the network, with the recipient sending acknowledgements back to the source whenever one is received. Routers in the network typically operate a drop tail queue. If a packet is received when the queue is full then it is simply discarded. Packet loss thus indicates congestion. If a packet is received out of order, it is assumed that intervening packets have been lost. The recipient sends a duplicate acknowledgement to signal this and the source lowers its rate (in response to the congestion) and resends the lost packet(s). Whilst a steady stream of successive acknowledgements is being received the source gradually increases its sending rate. In normal operation sources are thus constantly increasing and decreasing their rates in an attempt to make use of the available bandwidth. Congestion (ie full queues and the resulting packet loss) can occurr anywhere in the network - at the edges (eg your adsl modem, or at the exchange), in the core (eg a big transatlantic link) or, very often, at peering points, which are the connections between the networks that make up the Internet.

1.1.5 The *lac* operon – E.Coli (≈ 130 million years BC!)





(Only read this if you're interested!) The diagram above illustrates 4 genes and some control regions along the DNA of E.Coli. E.Coli's favourite sugar is glucose, but it will quite happily "eat" lactose if there's no glucose around. If there *is* glucose around *or* if there is *no* lactose around then there is no need to produce β-galactosidase (the enzyme which breaks down lactose, first into allolactose and then glucose) or the permease (which transports lactose into the cell). In addition, when it is metabolising lactose, it wants to regulate the amount of enzyme production to match the available lactose. This is the control system which achieves this: The lacI gene codes for a protein (the repressor) which binds to the operator (O) and stops the lacZ,Y and A genes being transcribed (ie "read"). If there's lactose in the cell, and at least some β-galactosidase, then there will also be allolactose (the inducer). In this case the repressor binds with it instead, and falls off the DNA. In the absence of glucose, the cAMP/CRP complex binds at the promoter (P), this encourages RNA polymerase to bind and initiate transcription of lacZ,Y and A.

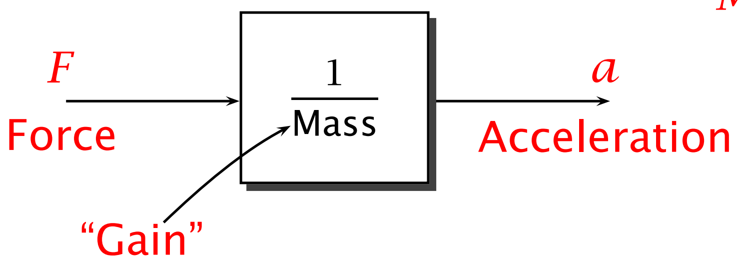
– for more details, see 3G1 next year …

1.2 Block Diagrams

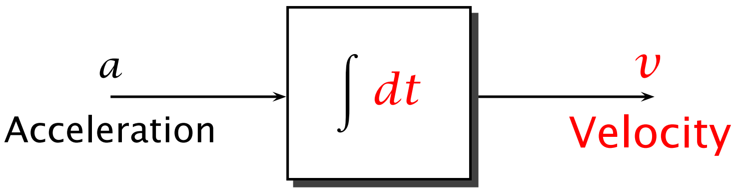
1.2.1 What goes in the blocks?

Some of them act like "amplifiers" or "attenuators"

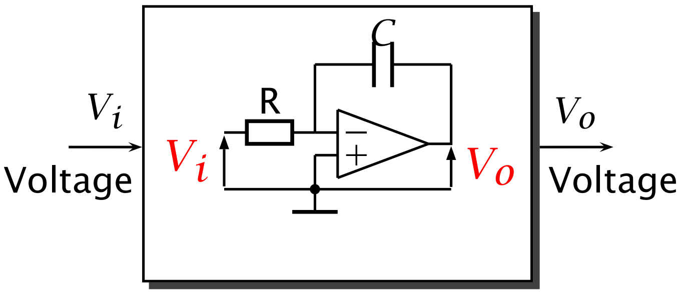
$$a = \frac{1}{Mass} \times F$$



But many are dynamic processes described by Ordinary Differential Equations (ODEs).



$$\dot{v} = a$$



$$\dot{V}_o = -\frac{1}{RC} V_i$$

(We shall (later) describe these by *transfer functions*.)

Note: By drawing this circuit as a block, we are implicitly assuming that any current it draws has negligible effect on the preceding block and that the following block draws insignificant current from it (i.e. that R is large and the op-amp is close to ideal).

1.2.2 Signals and systems

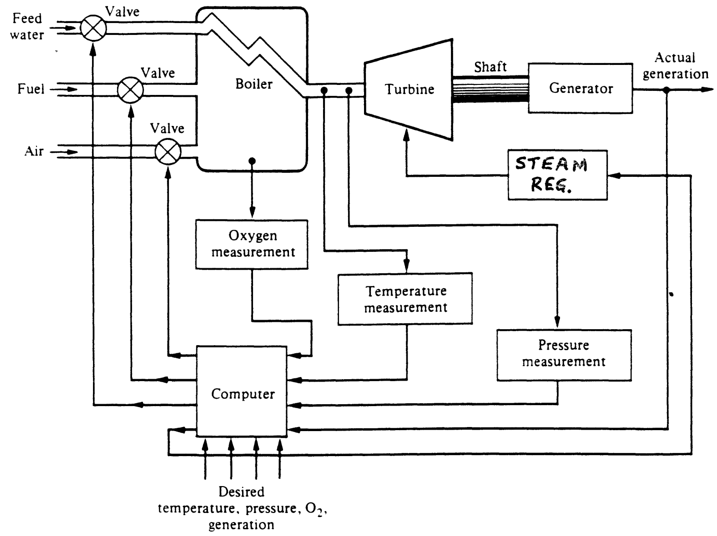
Block diagrams represent the flow of information, not the flow of "stuff".

taking a numeric value as a function of time

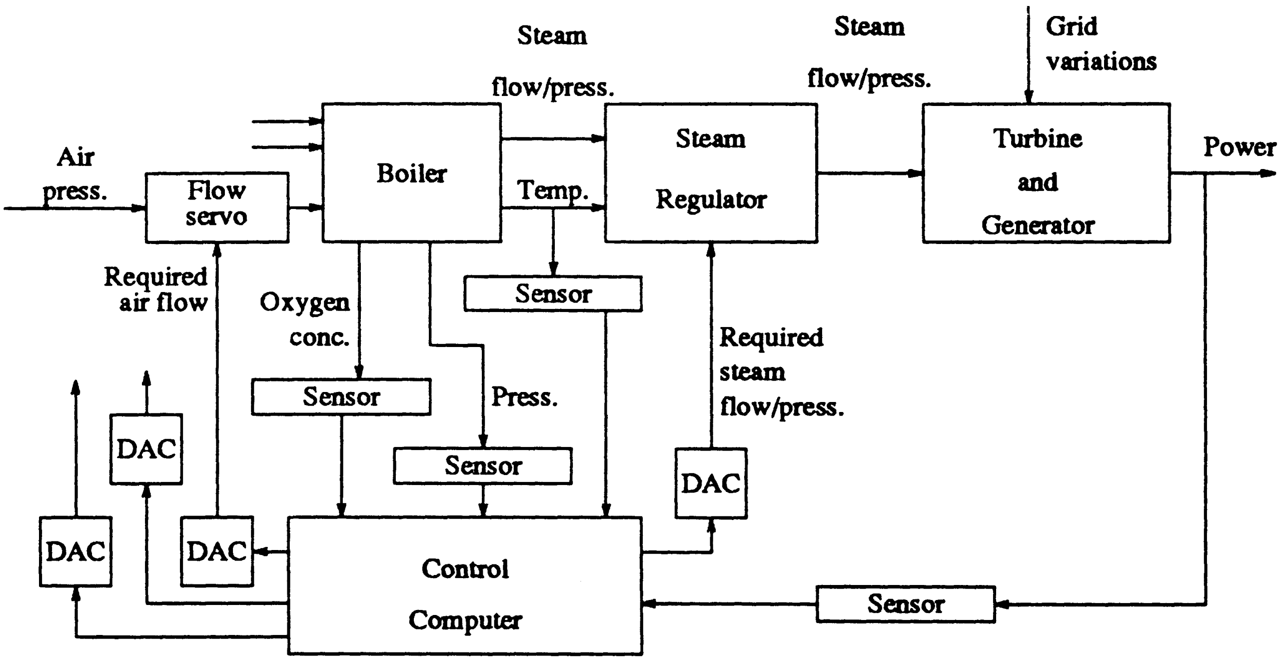
Blocks represent "systems ", whose inputs and outputs are "signals ".

equations mapping inputs into outputs

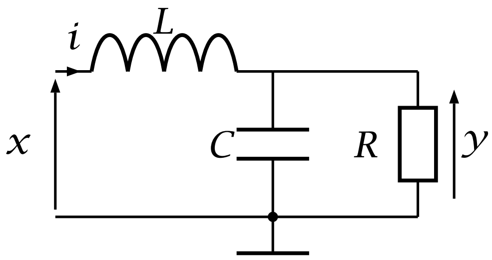
This is **NOT** a block diagram (in our sense)



This **IS** a block diagram



1.2.3 ODE models – A circuits example



$$x - y = L\frac{di}{dt}$$

$$i = C\dot{y} + \frac{y}{R}$$

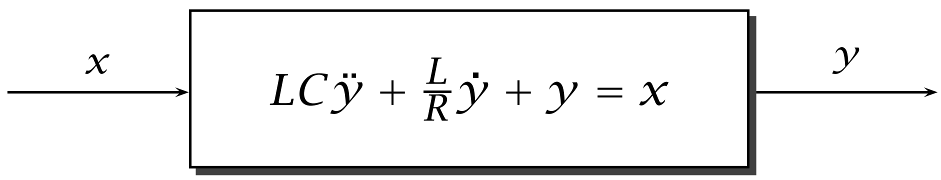
$\Longrightarrow$

$$x - y = L\left(C\ddot{y} + \frac{\dot{y}}{R}\right)$$

which gives a 2nd-order *linear* Ordinary Differential Equation:

$\Longrightarrow$

$$LC\ddot{y} + \frac{L}{R}\dot{y} + y = x$$



1.2.4 Block diagrams and the control engineer

For the control engineer:

Some blocks are given (fixed)

eg

- Steam Engine Dynamics
- Aircraft Dynamics

(the “plant”)

while other blocks are to be designed

eg

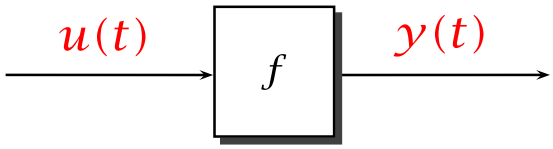
- Geometry of fly-ball mechanism in Watt governor.
- The program in an aircraft’s flight control computer.

(the “controller”)

1.3 Linear Systems

1.3.1 What is a "linear system"

Consider a "system" f mapping dynamic inputs u into outputs y



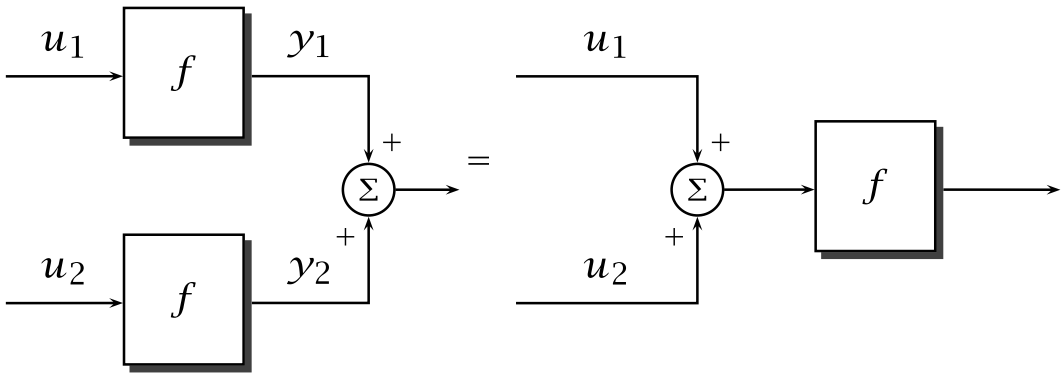
$$y = f\left(u\right)$$

the "system" f is *linear* if superposition holds, that is, if

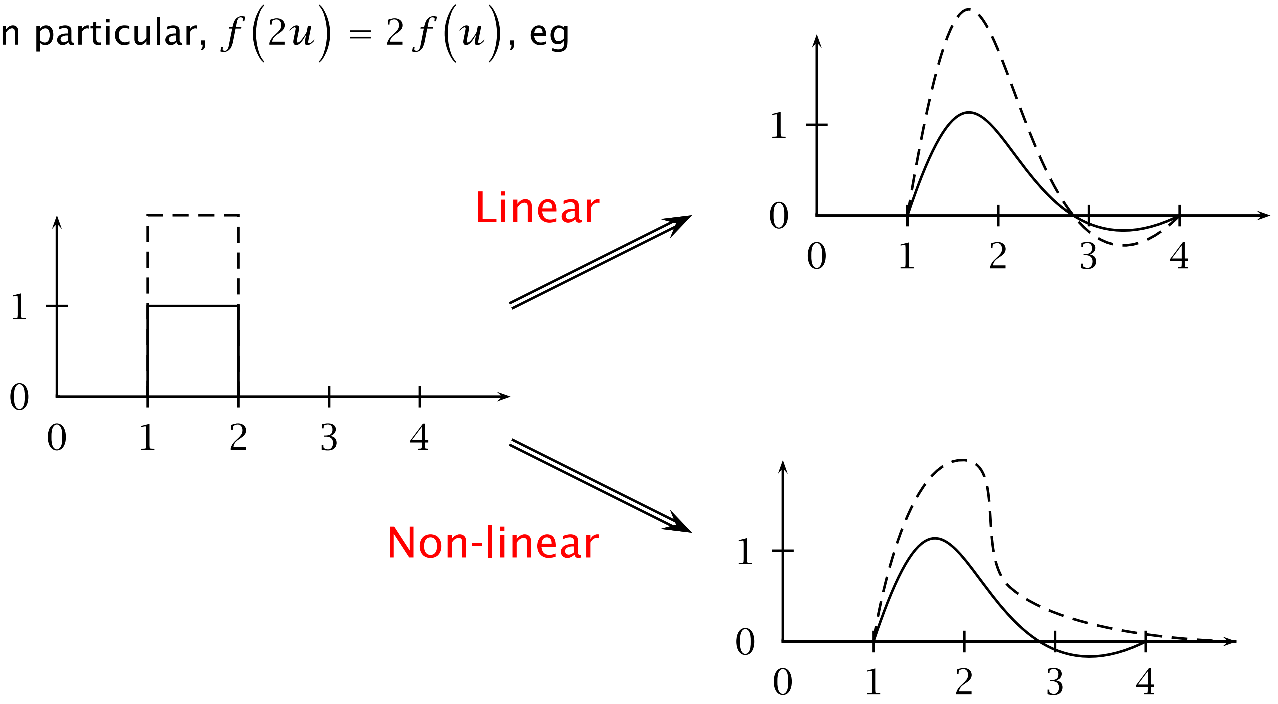
$$\underbrace{f\left(u_1\right)}_{} + \underbrace{f\left(u_2\right)}_{} = f\left(\mathbf{u_1} + \mathbf{u_2}\right)$$

for any u_1 and u_2.

In terms of block diagrams. If f is a linear system,

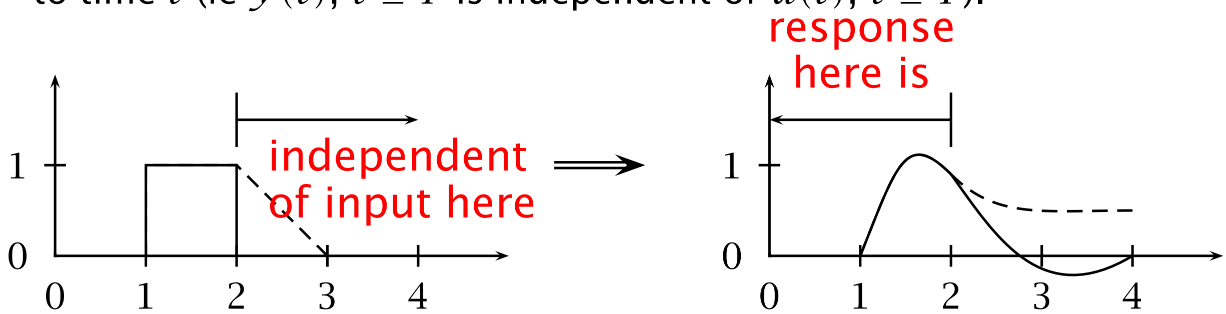


In particular, $f\left(2u\right) = 2\,f\left(u\right)$, eg



In addition, we shall also assume that all systems are:

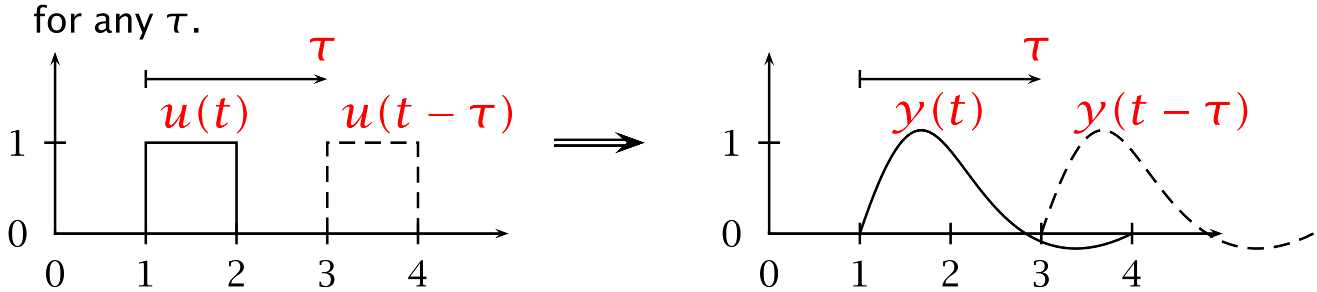
- *causal* – the output at time T, $y(T)$, depends only on the input up to time t (ie $y(t)$, $t \le T$ is independent of $u(t)$, $t \ge T$).



- *time-invariant* – the response of the system to a particular input doesn't depend on when that input is applied, ie if

$$u(t) \to y(t), \text{ then } u(t-\tau) \to y(t-\tau)$$

for any τ.



Almost all the linear systems we will consider in this course can be described as linear differential equations with constant coefficients and, possibly, delays. For example

$$\frac{d^2x(t)}{dt^2} + x(t-T) = \frac{du(t)}{dt} + 2u(t)$$

describes a linear system, as if

$$\frac{d^2x_1(t)}{dt^2} + x_1(t-T) = \frac{du_1(t)}{dt} + 2u_1(t)$$

and

$$\frac{d^2x_2(t)}{dt^2} + x_2(t-T) = \frac{du_2(t)}{dt} + 2u_2(t)$$

then

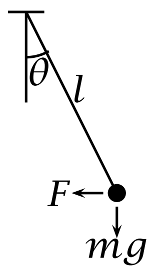
$$\begin{aligned} \frac{d^2}{dt^2}(x_1(t) + x_2(t)) + (x_1(t-T) + x_2(t-T)) \\ = \frac{d}{dt}(u_1(t) + u_2(t)) + 2(u_1(t) + u_2(t)) \end{aligned}$$

which is just the *superposition* of solutions. If there are x^2 terms or $\sin(x)$ terms, for example, then this doesn't work.

1.3.2 Linearization

All real systems are actually nonlinear, but many of these behave approximately linearly for small perturbations from equilibrium.

e.g. Pendulum:



$$Fl\cos\theta + mlg\sin\theta = -ml^2\ddot{\theta}$$

But, for *small* θ

$$Fl + mlg\theta \approx -ml^2\ddot{\theta}$$

or

$\boxed{l\ddot{\theta} + g\theta \approx -F/m}$ which is a linear ODE

General case

Suppose a system is described by an ODE of the form

$$\dot{x} = f(x, u)$$

where f is a smooth function. Assume that this system has an *equilibrium* at (x_0, u_0), by which we mean that

$$f(x_0, u_0) = 0.$$

where x_0 and u_0 are constants.

Let $x = x_0 + \delta x$, $u = u_0 + \delta u$,

and use a Taylor series expansion to obtain:

$$\cancelto{0}{\dot{x}_0} + \dot{\delta x} = f(x_0 + \delta x, u_0 + \delta u)$$

$$= \cancelto{0}{f(x_0, u_0)} + \underbrace{\left.\frac{\partial f}{\partial x}\right|_{x_0,u_0}}_{A} \delta x + \underbrace{\left.\frac{\partial f}{\partial u}\right|_{x_0,u_0}}_{B} \delta u + \overset{\text{neglect}}{\text{higher order terms}}$$

which results in the linear ODE

$$\boxed{\dot{\delta x} = A\delta x + B\delta u}$$

This is a simple example of a state-space model*. This procedure can be generalized to higher order systems with many inputs and outputs - see 3F2 next year*.

As an example of a higher order *state-space model*, consider the differential equation

$$\ddot{y} + \dot{y} + y = u$$

which we will regard as representing a linear system with input u and output y. If we write $x_1 = y$ and $x_2 = \dot{y}$ then this equation can be rewritten as the pair of equations

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = u - x_2 - x_1$$

or, in matrix form

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ -1 & -1\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix} u, \quad y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix}$$

which is usually written as

$$\dot{x} = Ax + Bu, \quad y = Cx.$$

1.3.3 When can we use linear systems theory?

Linearity is often desirable:

- Hi-Fi audio system (non-linearities are called distortion).
- Aircraft fly-by-wire system (for predictable response)

If we are going to design a controller to keep a system near equilibrium then we can ensure that perturbations are small (and hence that behaviour is approximately linear). This justifies the use of linear theory for the design!

- so linear systems theory is often very useful even when the underlying systems are actually nonlinear

However: some systems are designed to behave nonlinearly:

- Switch or relay (because it is either on or off).
- Automated air traffic control system.
  (either have a collision or not) .

In such cases linear theory is of little use in itself.

(flying along a trajectory is often a linear problem, but *choosing* that trajectory is usually a nonlinear problem)

1.4 Laplace Transforms

Laplace transforms are an essential tool for the analysis of linear, time-invariant, causal systems. We shall now briefly review some pertinent facts that you learnt at Part IA and introduce some new ideas.

DEFINITION:

$$\bar{y}(s) = \int_{0^-}^{\infty} y(t)e^{-st}\,dt$$

(provided the integral converges for sufficienty large and positive values of s.)

Note, a Laplace transform

- is *NOT* a function of t
- *IS* a function of s.

Various notations:

$$\mathcal{L}\{y(t)\} = \mathcal{L}y = \bar{y}(s) = \int_{0^-}^{\infty} y(t)e^{-st}\,dt$$

Notation for the *inverse transform*:

$$y(t) = \mathcal{L}^{-1}\bar{y}(s)$$

EXAMPLES

Find $\bar{y}(s)$ if $y(t) = C$ (a constant)

$$\bar{y}(s) = \int_0^{\infty} Ce^{-st}dt = C\left[\frac{-e^{-st}}{s}\right]_0^{\infty} = \frac{C}{s} \quad \text{(taking Real}(s) > 0\text{).}$$

Find $\bar{y}(s)$ if $y(t) = e^{-at}$

$$\bar{y}(s) = \int_0^{\infty} e^{-(s+a)t}dt = \left[\frac{e^{-(s+a)t}}{-(s+a)}\right]_0^{\infty} = \frac{1}{s+a} \quad \text{(taking Real}(s) > -a\text{).}$$

Addition or Superposition property

If

$$y(t) = Ay_1(t) + By_2(t)$$

then

$$\bar{y}(s) = A\bar{y}_1(s) + B\bar{y}_2(s)$$

(A, B constants)

Proof:

$$\begin{aligned}
\bar{y} &= \int_0^\infty (Ay_1 + By_2)e^{-st}dt \\
&= A\int_0^\infty y_1 e^{-st}dt + B\int_0^\infty y_2 e^{-st}dt \\
&= A\bar{y}_1 + B\bar{y}_2
\end{aligned}$$

$\Longrightarrow$ The operation of taking a Laplace transform is linear.

Transforms of derivatives

$$\begin{aligned}
\mathcal{L}\dot{y}(t) &= \int_0^\infty \frac{dy}{dt}e^{-st}dt \\
&= \left[y(t)e^{-st}\right]_0^\infty + s\int_0^\infty y(t)e^{-st}dt \\
&= s\bar{y} - y(0)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}\ddot{y} &= \int_0^\infty \frac{d^2y}{dt^2}e^{-st}dt \\
&= \left[\frac{dy}{dt}e^{-st}\right]_0^\infty + s\int_0^\infty \frac{dy}{dt}e^{-st}dt \\
&= -\dot{y}(0) + s(s\bar{y} - y(0)) \\
&= s^2\bar{y} - sy(0) - \dot{y}(0)
\end{aligned}$$

Obvious pattern:

$$\boxed{\begin{aligned} \mathcal{L}y &= \bar{y} \\ \mathcal{L}\dot{y} &= s\bar{y} - y(0) \\ \mathcal{L}\ddot{y} &= s^2\bar{y} - sy(0) - \dot{y}(0) \\ \vdots \;\; & \vdots \;\; \vdots \\ \mathcal{L}\frac{d^n y}{dt^n} &= s^n\bar{y} - s^{n-1}y(0) - s^{n-2}\dot{y}(0) - \\ & \quad - \ldots - \left(\tfrac{d^{n-1}y}{dt^{n-1}}\right)(0) \end{aligned}}$$

In particular, if $y(0) = \dot{y}(0) = \ddot{y}(0) = \cdots = 0$, then

$$\begin{aligned} \mathcal{L}y &= \bar{y} \\ \mathcal{L}\dot{y} &= s\bar{y} \\ \mathcal{L}\ddot{y} &= s^2\bar{y} \\ \vdots \;\; & \vdots \;\; \vdots \\ \mathcal{L}\frac{d^n y}{dt^n} &= s^n\bar{y} \end{aligned}$$

differentiation (in the time domain) corresponds to multiplication by s (in the s domain)

Laplace Transform of t^n

Define $\bar{y}_n(s) = \mathcal{L}\frac{t^n}{n!}$.

$$\begin{aligned}
\bar{y}_n &= \int_0^\infty \frac{t^n}{n!}e^{-st}dt \\
&= \left[-\frac{1}{s}\frac{t^n}{n!}e^{-st}\right]_0^\infty + \frac{1}{s}\int_0^\infty \frac{nt^{n-1}}{n!}e^{-st}dt \\
&= \frac{1}{s}\int_0^\infty \frac{t^{n-1}}{(n-1)!}e^{-st}dt \\
&= \frac{1}{s}\bar{y}_{n-1},
\end{aligned}$$

(since for Real$(s) > 0$, and as $t \to \infty$, then $|e^{-st}| \to 0$ faster than $t^n \to \infty$).

Thus we have

$$\begin{aligned}
\bar{y}_0 &= \mathcal{L}\,1 = \frac{1}{s} \\
\bar{y}_1 &= \mathcal{L}\,t = \frac{1}{s^2} \\
\bar{y}_2 &= \mathcal{L}\frac{t^2}{2} = \frac{1}{s^3} \\
\bar{y}_3 &= \mathcal{L}\frac{t^3}{3\times 2} = \frac{1}{s^4}
\end{aligned}$$

Similarly $\quad \bar{y}_n = \mathcal{L}\frac{t^n}{n!} = \frac{1}{s^{n+1}}$

integration (in the time domain) corresponds to division by s (in the s domain)

Poles and Zeros

Suppose $G(s)$ is a *rational* function of s, by which we mean

$$G(s) = \frac{n(s)}{d(s)}$$

where $n(s)$ and $d(s)$ are polynomials in s.

Then the roots of $n(s)$ are called the *zeros* of $G(s)$

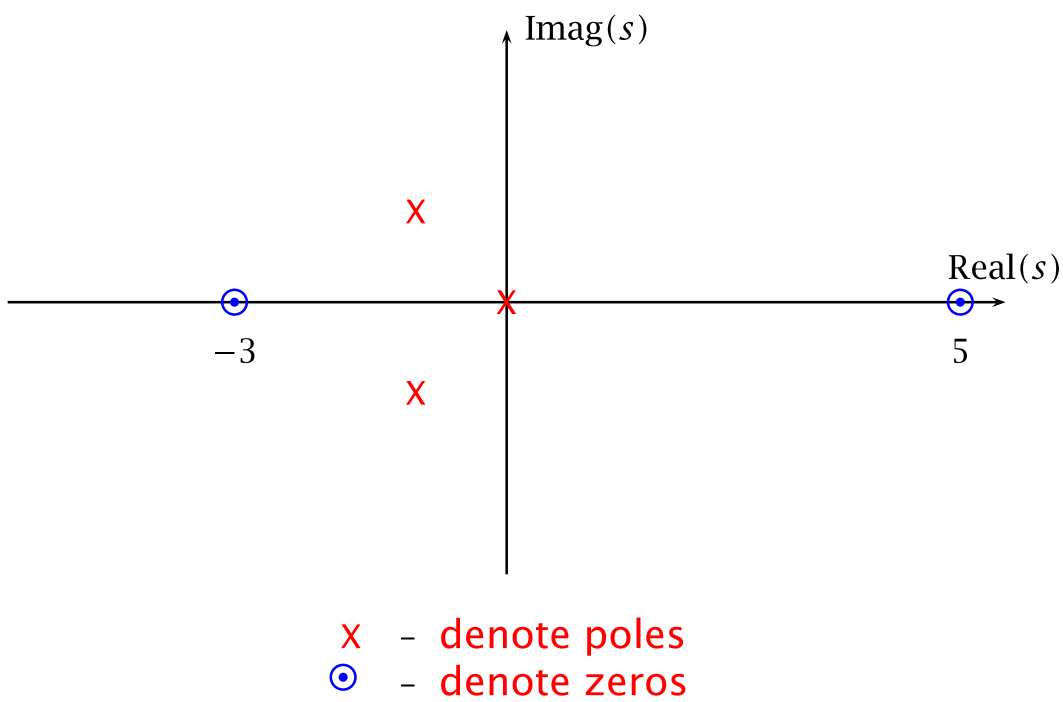
and the roots of $d(s)$ are called the *poles* of $G(s)$

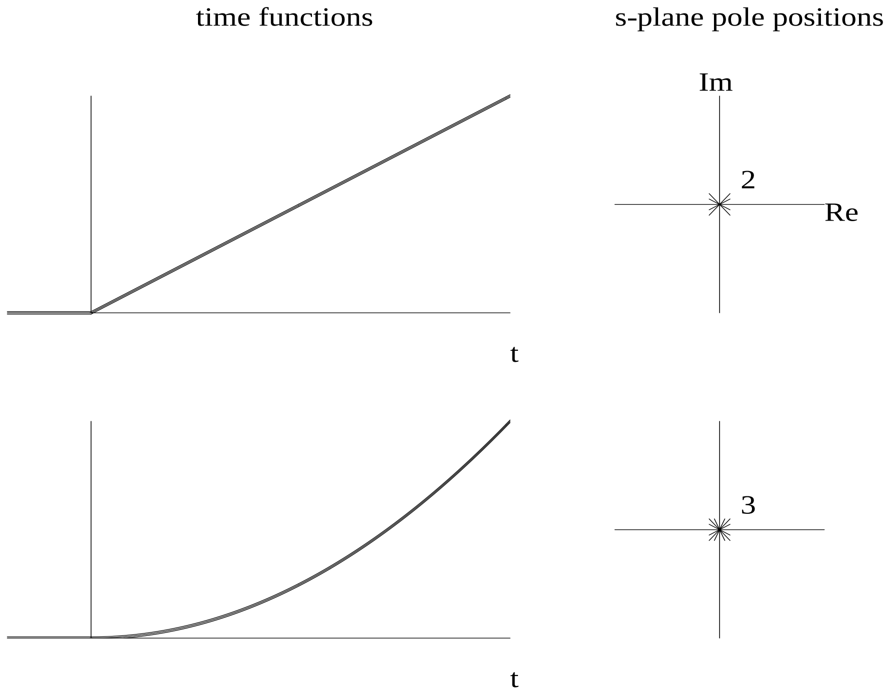
Example:

$$\begin{aligned} G(s) &= \frac{4s^2 - 8s - 60}{s^3 + 2s^2 + 2s} \\ &= \frac{\mathbf{4}(s + \mathbf{3})(s - \mathbf{5})}{s(s + \mathbf{1} + j)(s + \mathbf{1} - j)} \end{aligned}$$

Zeros of $G(s)$ are $\mathbf{-3, +5.}$

Poles of $G(s)$ are $\mathbf{-1} - j, \mathbf{-1} + j, \mathbf{0}$





Time functions and pole positions for $y(t) = t$ and $y(t) = t^2$

Laplace Transforms of Sines and Cosines

$$y = e^{i\omega t} = \cos\omega t + i\sin\omega t$$

$$\begin{aligned}\bar{y} &= \frac{1}{s - i\omega} = \mathcal{L}\cos\omega t + i\mathcal{L}\sin\omega t \\ &= \frac{s + i\omega}{s^2 + \omega^2}\end{aligned}$$

Equating reals : $\boxed{\mathcal{L}\cos\omega t = \frac{s}{s^2 + \omega^2}}$

and similarly : $\boxed{\mathcal{L}\sin\omega t = \frac{\omega}{s^2 + \omega^2}}$

poles at $s = \pm\omega i$ in both cases

NOTE: Results like this are tabulated in the Maths and Electrical Data Books.

Shift in s theorem

$$\boxed{\begin{aligned} \text{If } \mathcal{L}y(t) &= \bar{y}(s) \\ \text{then } \mathcal{L}e^{at}y(t) &= \bar{y}(s-a). \end{aligned}}$$

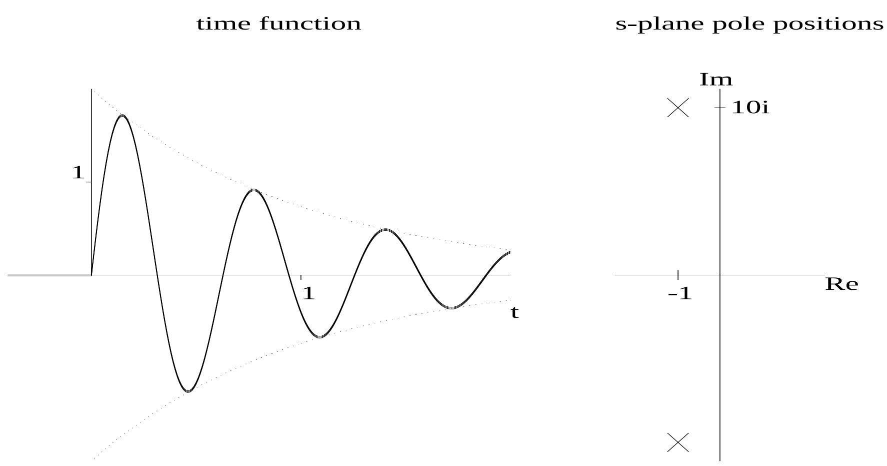
Proof:

$$\begin{aligned} \mathcal{L}e^{at}y(t) &= \int_0^\infty e^{-(s-a)t}y(t)\,dt \\ &= \bar{y}(s-a), \end{aligned}$$

Example of use:

$$\begin{aligned} \mathcal{L}^{-1}\frac{20}{s^2+2s+101} &= \mathcal{L}^{-1}\frac{20}{(s+1)^2+100} \\ &= 2e^{-t}\sin 10t \end{aligned}$$

because $\mathcal{L}^{-1}\dfrac{10}{s^2+100} = \sin 10t$



Time functions and pole positions for $y(t) = 2e^{-t}\sin 10t$

Initial and Final Value Theorems

If $\bar{y}(s) = \mathcal{L}\, y(t)$ then *whenever the indicated limits exist* we have

Final Value Theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\bar{y}(s)$$

Initial Value Theorem:

$$\lim_{t\to 0^+} y(t) = \lim_{s\to\infty} s\bar{y}(s)$$

Proofs omitted (as it's a little tricky to prove these properly.)

However, for rational functions of s it is easy to demonstrate that these relationships hold:

Let a partial fraction of $\bar{y}(s)$ be given as:

$$\bar{y}(s) = \frac{b_0}{s} + \sum_{i=1}^{n} \frac{b_i}{s+a_i} \quad \text{and so} \quad y(t) = b_0 + \sum_{i=1}^{n} b_i e^{-a_i t}.$$

Hence

$$\underline{y(0) = b_0 + \sum_{i=1}^{n} b_i} \quad \text{and, provided } a_i > 0, \quad \underline{y(\infty) = b_0}.$$

On the other hand,

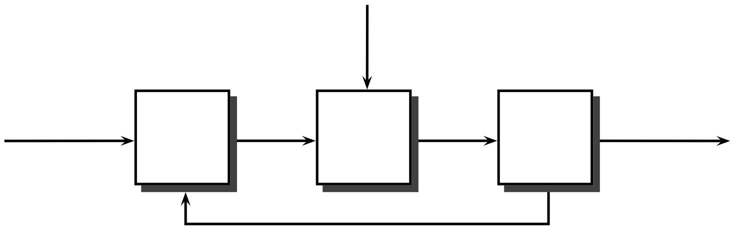
$$s\bar{y}(s) = b_0 + \sum_{i=1}^{n} \frac{sb_i}{s+a_i}$$

hence

$$\underline{s\bar{y}(s)|_{s=\infty} = b_0 + \sum_{i=1}^{n} b_i} \quad \text{and, provided } a_i \neq 0, \quad \underline{s\bar{y}(s)|_{s=0} = b_0}$$

which are the same expressions as above.

1.5 Key points



- Feedback is used to reduce sensitivity.
- We use block diagrams to represent feedback interconnections.
- Each block represents a “system”.
- Each connection carries a “signal”.
- We shall assume that systems are described by *linear*, *time-invariant* and *causal* ODE’s.

- We distinguish between *causes* (the input signals) and *effects* (the output signals).
- Large and complex systems can be constructed by connecting together simpler sub-systems.
- Laplace transforms are central to the study of linear, time-invariant systems.