Tracking beyond the Nyquist frequency in sampled-data systems
- going beyond the sampling limit

Dedicated to Malcolm Smith
on the occasion of his 60^{th} birthday

Yutaka Yamamoto
Kyoto University, Emeritus
CentraleSupelec

Our encounters

- Many conferences
- 1997, few days visit to Cambridge

Malcolm playing piano at my home,
after CDC 1996 Kobe

Control 2000

MTNS 2006

Joint work with

Kaoru Yamamoto,
Lund University

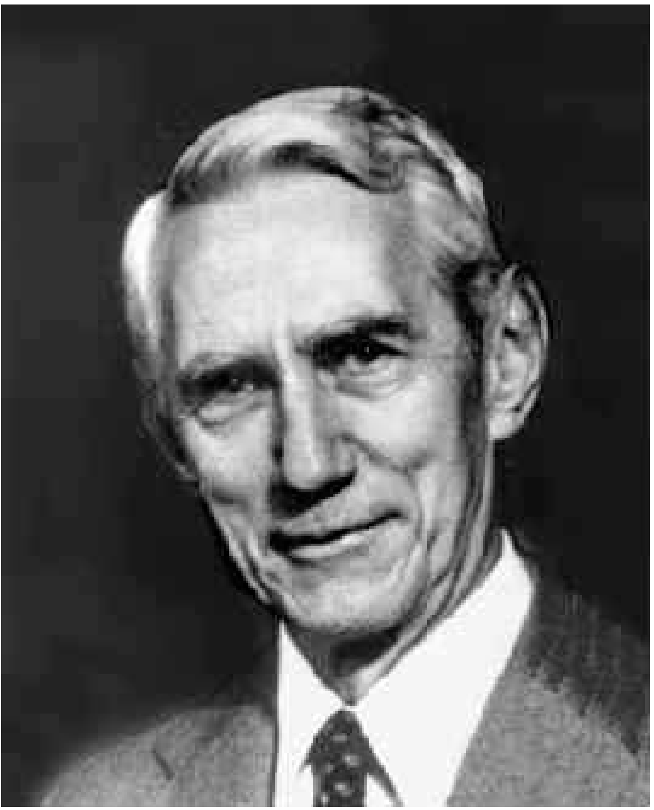
Masaaki Nagahara,
U. Kitakyshu

Topic of this talk

- Tracking to signals beyond the Nyquist frequency

WHAT'S SO SPECIAL ABOUT IT?

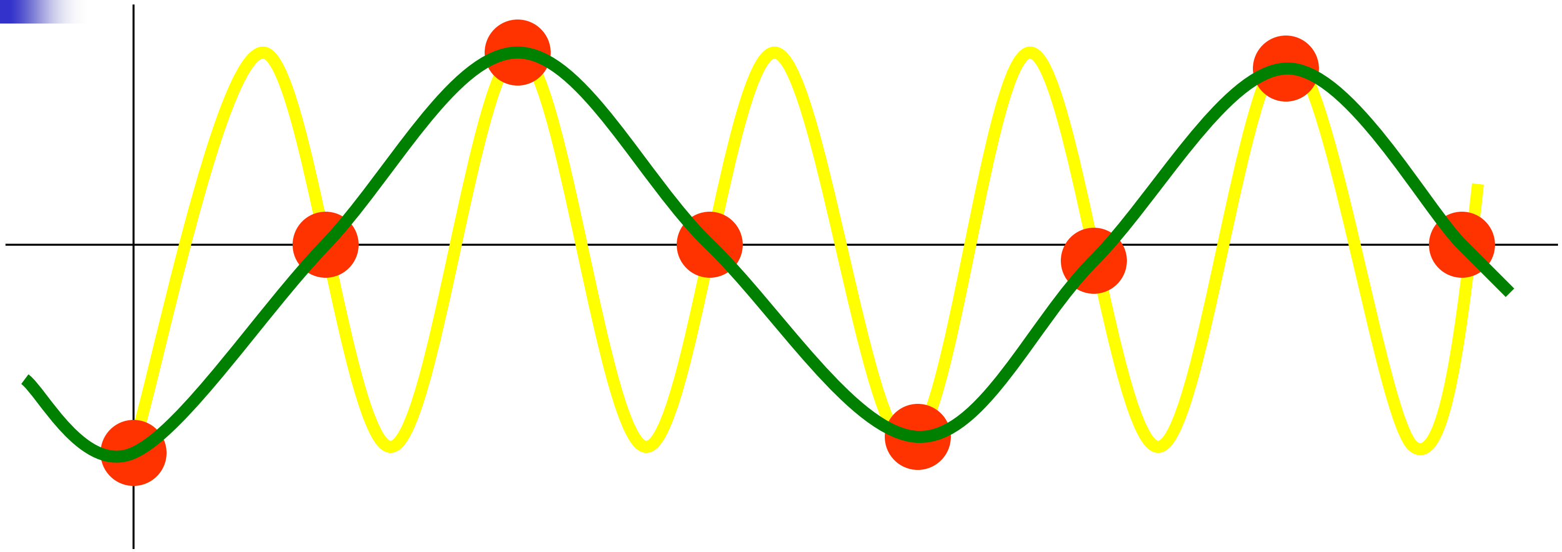
Claude Shannon



Claude Elwood Shannon (1916-2001)

- Communication in the Presence of Noise, C. E. Shannon, Proc. IRE, vol. 37, 1949, pp. 10–21.
- *How fast should we sample in transmitting data through a channel?*
- ⇒ *Unique reconstruction below the Nyquist frequency*

Typical difficulty
Sampling → Aliasing



- High-freq. intersample information can be lost
- If no high-freq. components beyond the Nyquist frequency (=1/2 of sampling freq.) → unique restoration

→ Whittaker-Shannon-Someya sampling theorem

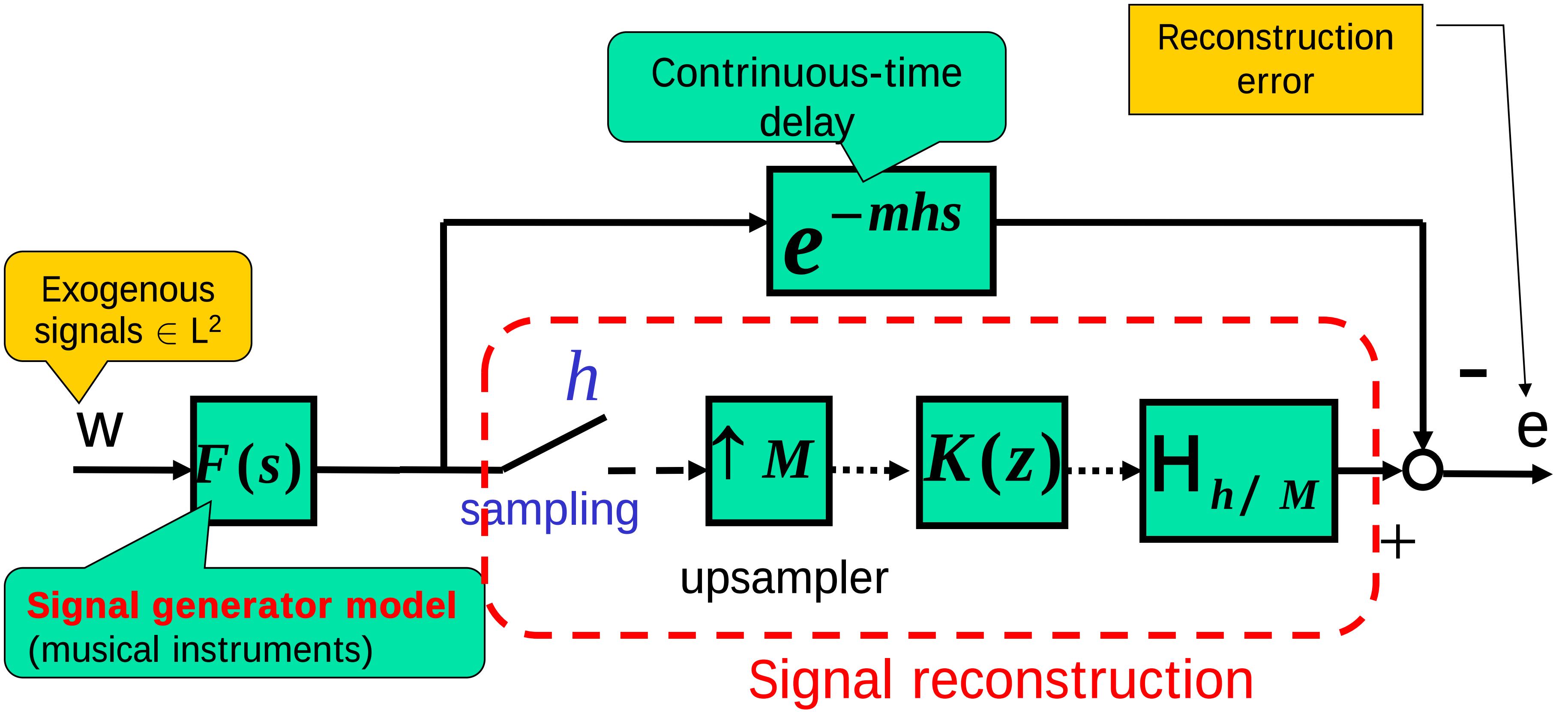
Observation

- Low and high frequencies cannot be distinguished.
- $\Rightarrow$ truncate high frequency and concentrate on low frequency
- Bound = π/h, Nyquist frequency
- This "Shannon paradigm" has been prevalent for the past 60+ years

Popular (but wrong) understanding of the Shannon paradigm

- *We can do nothing for signals beyond the Nyquist frequency*
- This is wrong
- Hidden assumption: no information on high-freq. ⇒ Band limiting hypothesis

Sampled-data Design Model



Problem: Find K[z] satisfying

$$\|T_{ew}\| < \gamma$$

Sampled-data H^∞ control problem

Moving image demo

Left: Bicubic – no high freq. beyond Nyquist freq
Right: yy (reconstructed high freq.)

Why success?

- Signal generator model $F(s)$
- This contains information on high frequency
- Allows us to recover high freq. components

New question

Can we control signals beyond the Nyquist freq.?

- Dates back to an old question in 1993
- “*Can we control high-freq. through aliases?*”

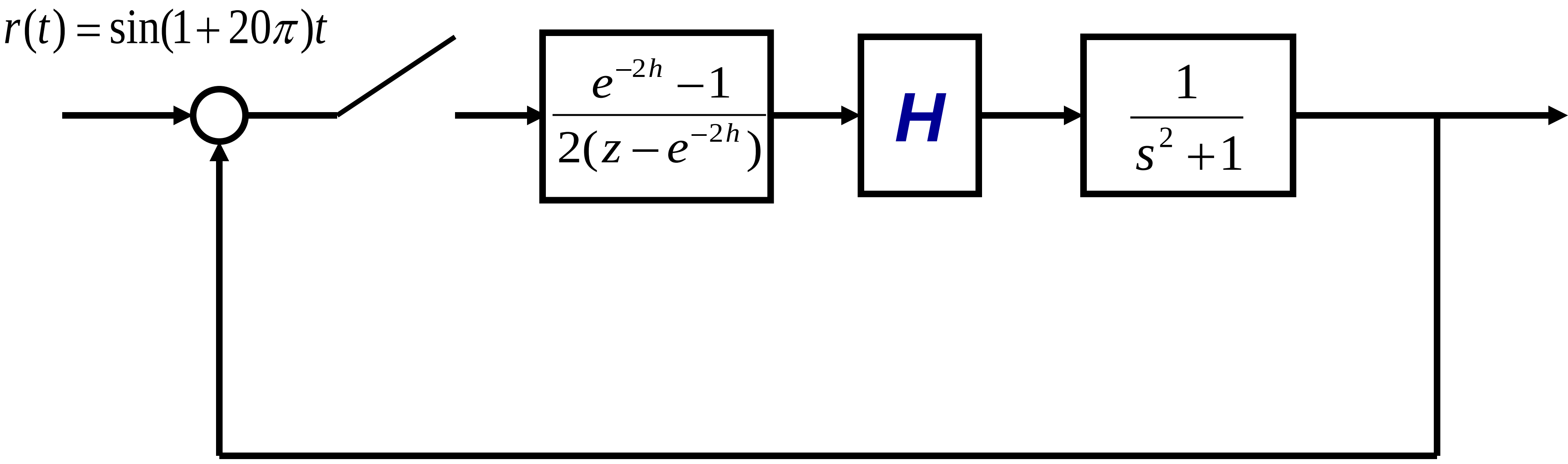
Demands

- Hard disk drives
- Often high-freq. disturbances (due to winds)
- Sampling frequency is limited by a physical limitation
- Can we reject such high-freq. disturbances? (beyond the Nyquist frequency)

1st Tracking Problem

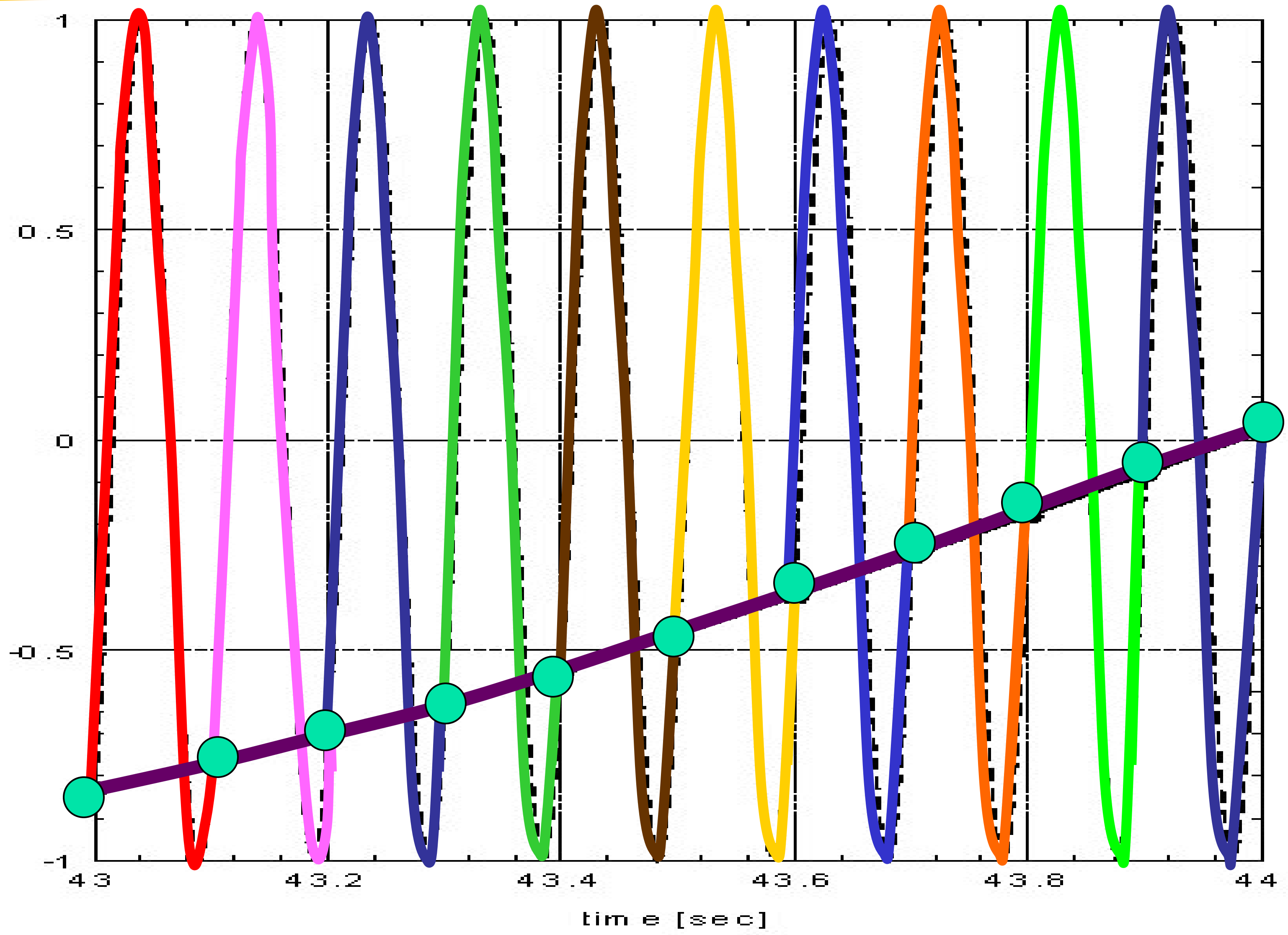
- Sampled-data system
- Sampling period h
- Nyquist frequency: π/h [rad/sec]
- Can we track a reference with frequency higher than π/h ?

Counterexample?



Response

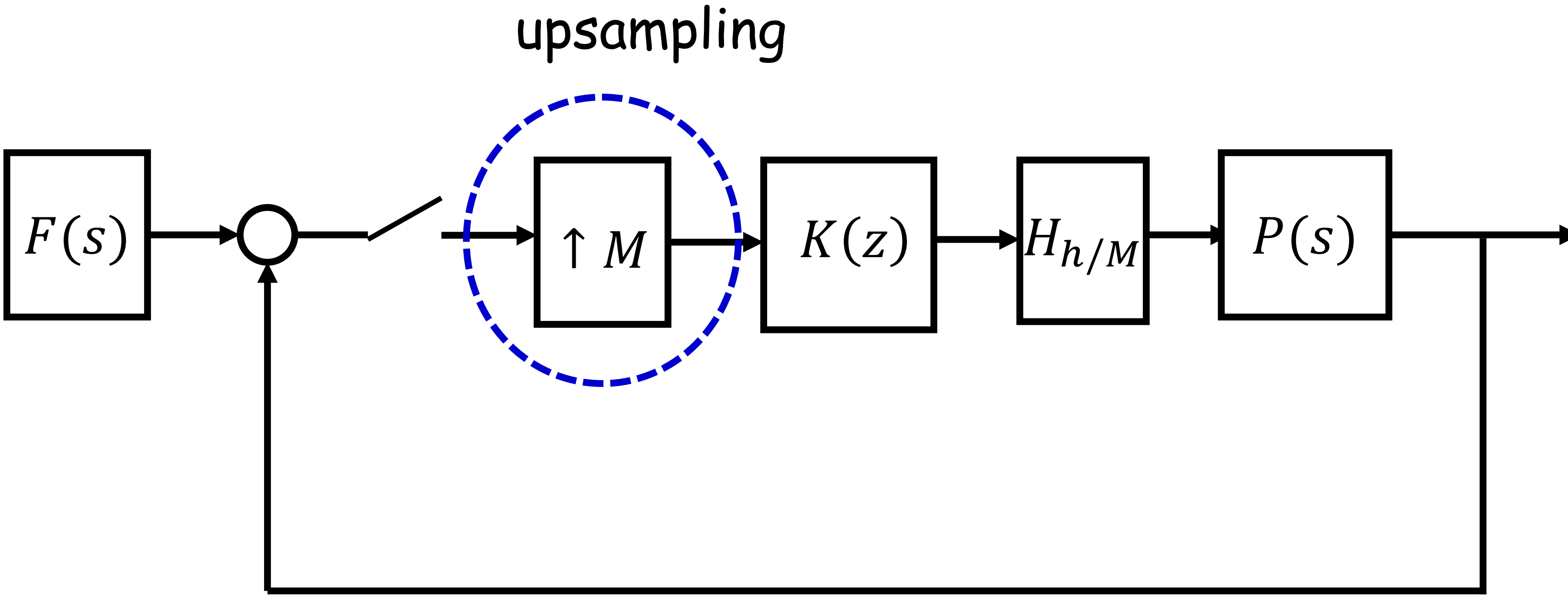
$$v(\theta) = \sin(1 + 20\pi)\theta$$



Recipe

- We need upsampling to take care of the intersampling behavior
- We need a proper weighting in the high frequency range (i.e., right signal model)

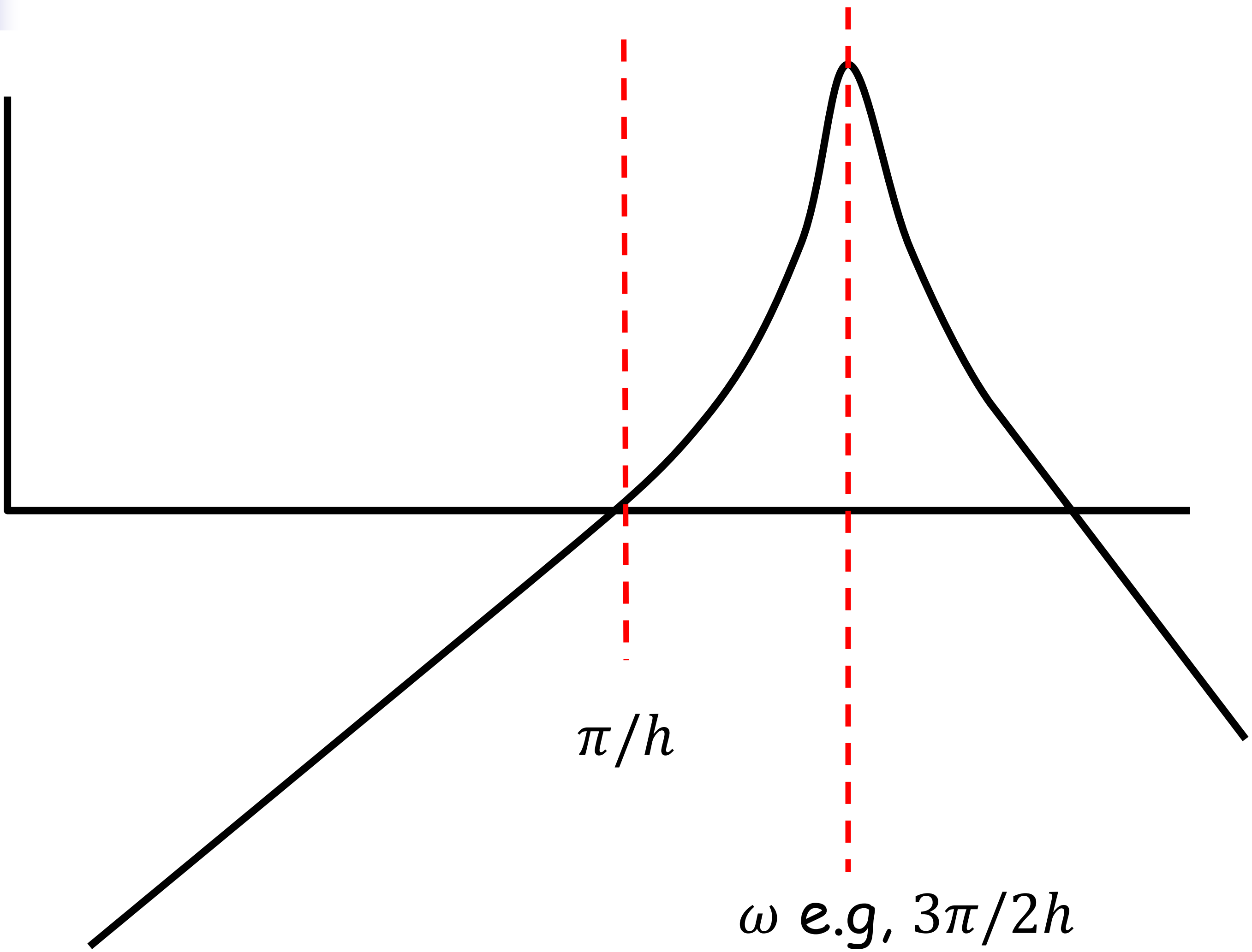
Basic construction

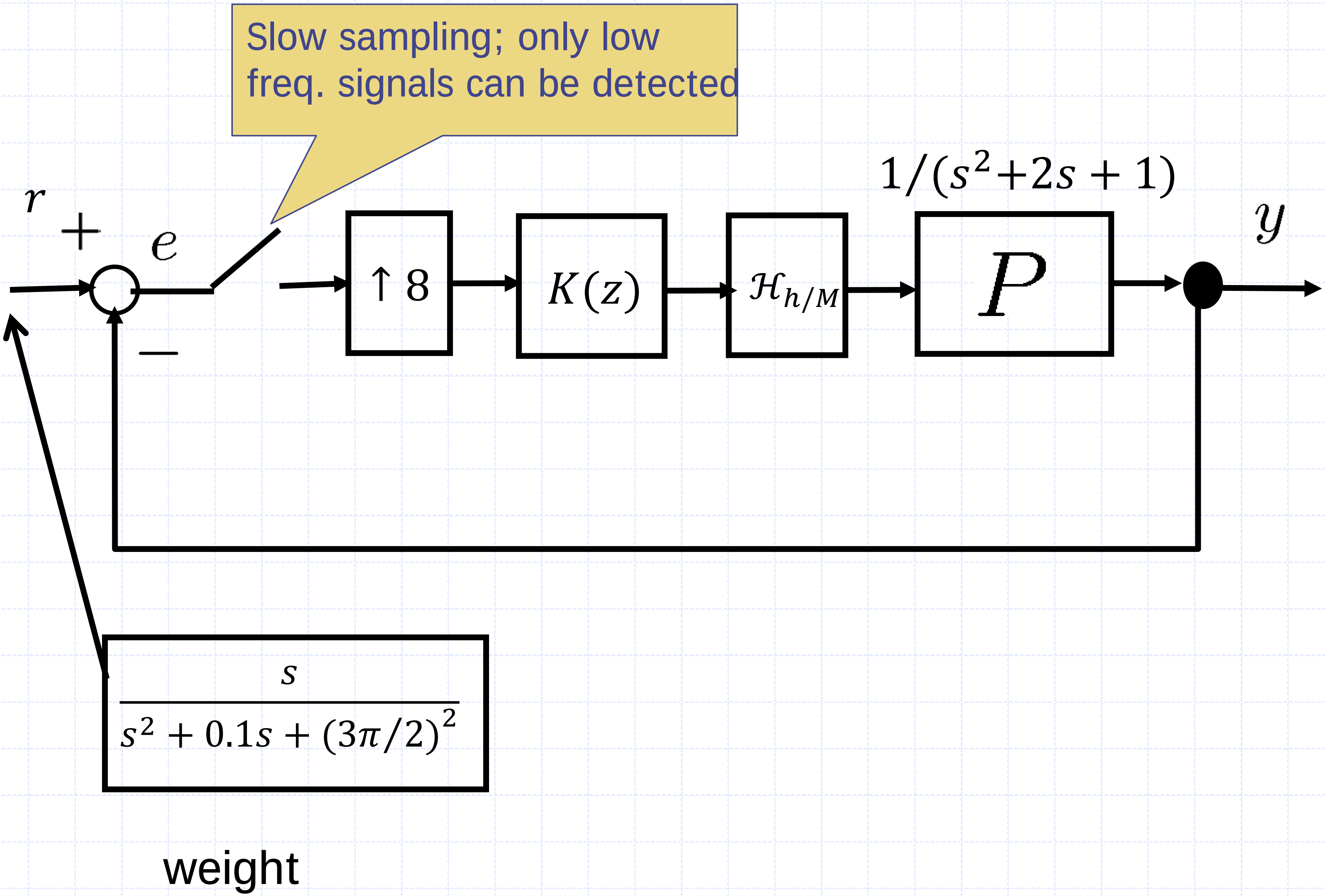


Example

- $P(s) = \frac{1}{s^2+2s+1}$
- $h = 1$, Nyquist freq. $=\pi$
- $r(t) = (3\pi/2)t$
- $F(s) = \frac{s}{s^2+0.1s+(3\pi/2)^2}$, peak at $(3\pi/2)$

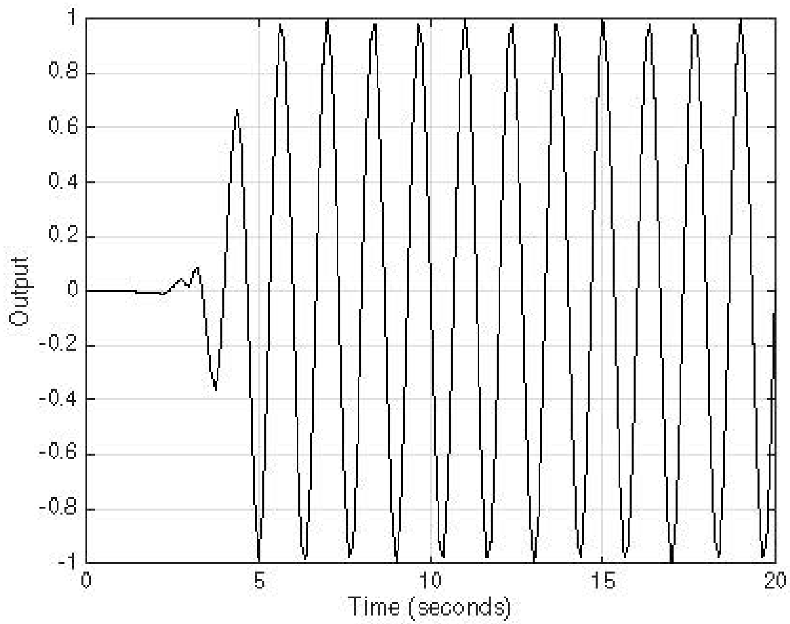
signal model $F(s)$



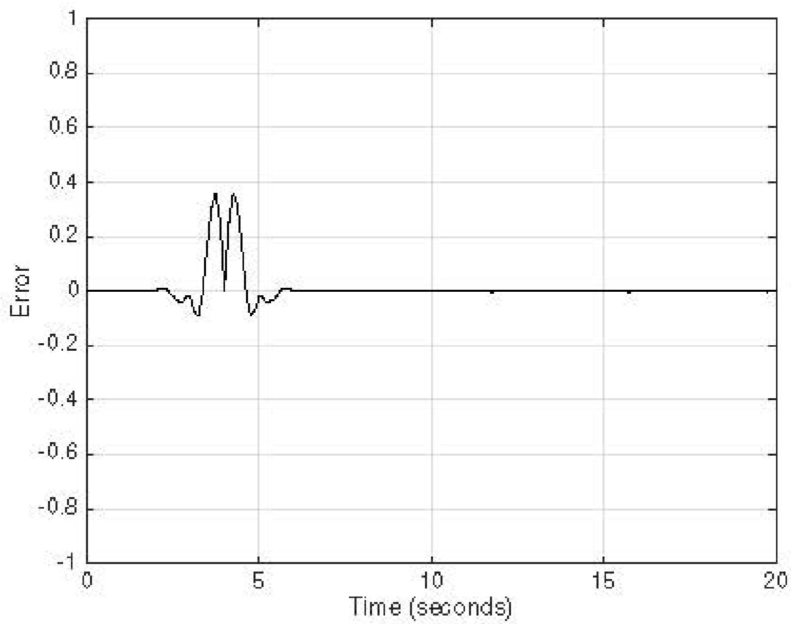


Slow sampling; only low freq. signals can be detected
$1/(s^2+2s+1)$
r
$+$
e
$-$
$\uparrow 8$
$K(z)$
$\mathcal{H}_{h/M}$
P
y
$\frac{s}{s^2+0.1s+(3\pi/2)^2}$
weight

Example - results



$y(t)$

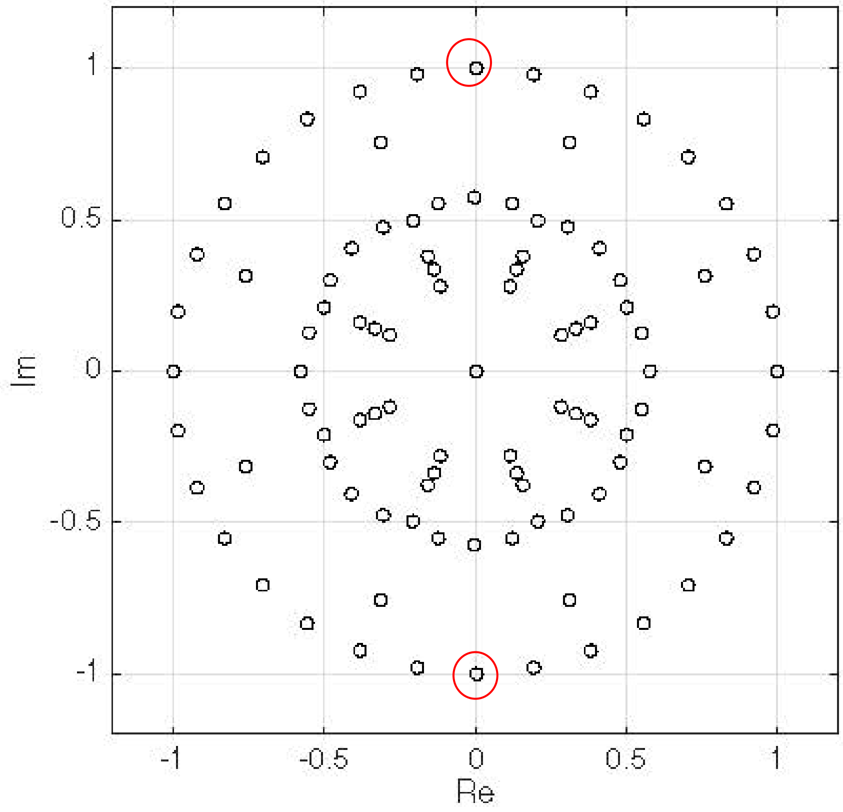


$e(t)$

Presented at the
CDC 2016, Las Vegas

Poles in the controller

This gives rise to an approx. internal model along with the upsampler+fast hold



Poles at $e^{\pm j(3/2\pi)} = \pm j$.

Next questions

- Disturbance rejection
- More than one reference or disturbance signals?
- robustness

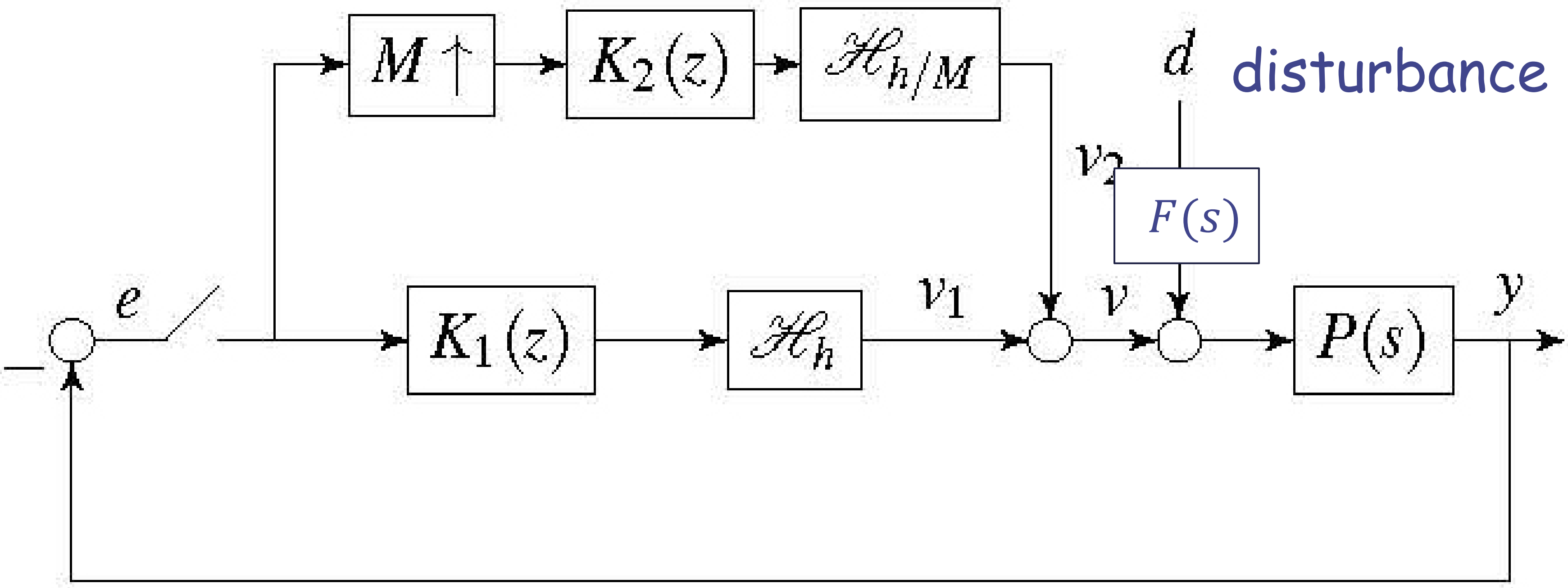
More than one signals

- Standard recipe: weights at two frequencies
  - May work, but
- Fails if they are separated by π
- Sampling cannot distinguish two signals

Two step design:

1. Design K_1 for low frequency
2. Then design K_2

$\sin \pi t/2 + \sin 3\pi t/2$



Two step design configuration

$$F(s) = {}^{50s}\!/\!_{(s^2 + 0.2s + \omega_1{}^2)(s^2 + 0.1s + \omega_2{}^2)}$$

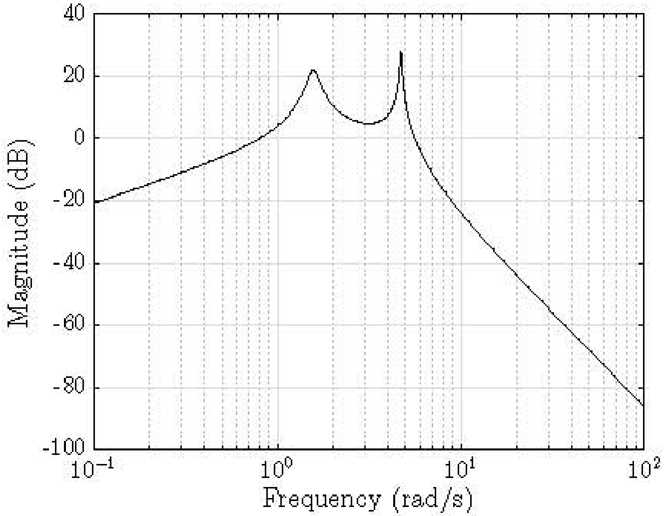


Fig. 4. Weighting function $F(s)$.

Weighting function

To be presented at the 1st IEEE CCTA, Hawaii, 2017

Output $y(t)$

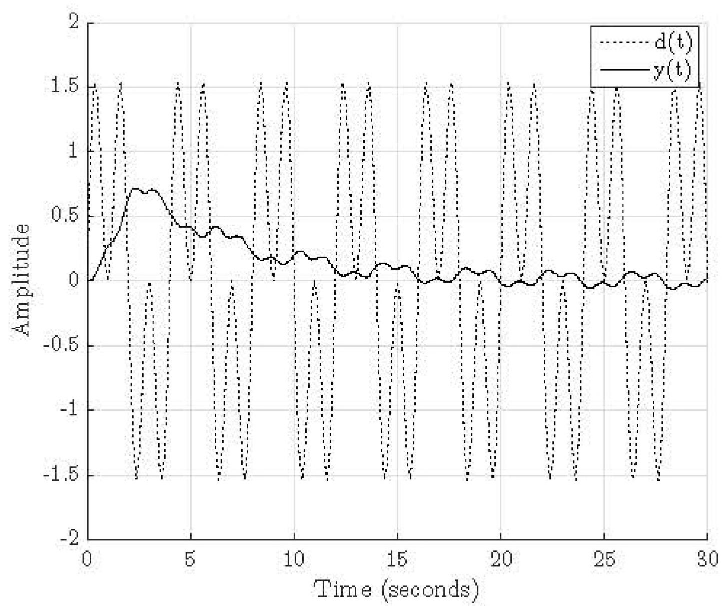


Fig. 8. System output (solid) against disturbance $\sin(\pi/2)t + \sin(3\pi/2)t$ (dotted).

Conclusion

- Tracking/rejection are possible for signals beyond the Nyquist frequency
- Not limited by the Shannon paradigm
- Crucial elements:
  - a. Physical model
  - b. appropriate weighting
  - c. upsampling (multirate processing)
- Many more applications expected

Happy 60th Malcolm!

YYFest, 2010

At YYFest 2010

YYFest, 2010

MTNS 2006, Kyoto

I want to thank you for yet another:

- Malcolm was Kaoru’s Ph. D. supervisor for 2011-2015
- Kaoru got a Ph. D. under his supervision
- Thesis title: “Disturbance Attenuation in Mass Chains with Passive Interconnection”

At the graduation ceremony at Caius

April, 2016


Thank you Malcolm for your
Long-term friendship,
Superb supervision of Kaoru,
Many happy returns
Of the day!