Malcolm Smith **University of Cambridge**

A new look at the Ladenheim catalogue

In Memoriam R. Kalman

The talk commemorates Rudolf Kalman, who was a principal speaker at the first three workshops in this series. The talk will describe Kalman's work during his last decade on network synthesis and realisation, and discuss the connections he outlined with the theory of algebraic invariants. The talk also provides an introduction to the Ladenheim catalogue (i.e. the set of 5-element electrical networks with at most two reactive elements and at most three resistors). The work of Jiang and the speaker, and the recent work of A. Morelli and the speaker on the catalogue will be described. The talk concludes with a discussion on the status of some outstanding conjectures of Kalman in connection with the catalogue.

Arjan Van der Schaft **University of Groningen**

Towards nonlinear network synthesis

It is well-known that a impedance/admittance matrix can be realized by an RLC network with transformers if and only is the transfer matrix is passive as well as reciprocal (gyratorless synthesis). The 'if' direction follows from the fact that a reciprocal transfer matrix defines a unique (indefinite) inner product on the minimal state space, and that there always exists by passivity a storage function that is 'compatible' with this inner product. The resulting state space realization is a linear port-Hamiltonian system with a special structure, which allows to formulate the system (in different coordinates) as a gradient system as well. An important part of this reasoning can be extended to the nonlinear case by the definition of a reciprocal nonlinear port-Hamiltonian system which can be also formulated as a gradient system with respect to an indefinite Hessian Riemannian structure. This turns out to be closely related to the classical Brayton-Moser formulation of nonlinear RLC circuits.

Venkat Chandrasekaran **California Institute of Technology**

Finding Planted Subgraphs using the Schur-Horn Relaxation

Extracting structured subgraphs inside large graphs – often known as the planted subgraph problem – is a fundamental question that arises in a range of application domains. This problem is NP-hard in general, and as a result, significant efforts have been directed towards the development of tractable procedures that succeed on specific families of problem instances. We describe a new computationally efficient convex relaxation for solving the planted subgraph problem; our approach is based on tractable semidefinite descriptions of majorization inequalities on the spectrum of a symmetric matrix. This procedure is effective at finding planted subgraphs that consist of few distinct eigenvalues, and it generalizes previous convex relaxation techniques for finding planted cliques. Our analysis relies prominently on the notion of spectrally comonotone matrices, which are pairs of symmetric matrices that can be transformed to diagonal matrices with sorted diagonal entries upon conjugation by the same orthogonal matrix. (Joint work with Utkan Candogan).

David Limebeer **University of Oxford**

Passive Networks in the Control of Aeroelastic Phenomena in Long-Span Suspension Bridges

Long-Span Suspension Bridges are subject to wind buffeting and aeroelastic instabilities. These difficulties are particularly important in the early stages of erection when the bridge particularly vulnerable. This study presents a control system based on passively controlled leading- and trailing-edge flaps that has been designed to suppress wind induced instabilities such as flutter and torsional divergence. The utility of the approach is demonstrated on a three-dimensional bridge model as well as with wind tunnel testing. The bridge's full multimodal response is considered and numerical predictions show very good agreement against experimental data. It is shown that optimised compensator network parameters, and optimum hinge locations, result in a substantially improved deck aerodynamic performance. Particular importance is given to ensuring that the controlled system has good closed-loop 'robustness' properties.

Jason Jiang **University of Bristol**

Mechanical Network Synthesis of Biquadratic and Bicubic Impedances

Interest in classical circuit synthesis has been revived due to the introduction of an ideal mechanical modelling element, the inerter, and the deployment of its vibration control capability in racing car suspensions. For mechanical structures, minimising the complexity of vibration suppression systems is critically important due to space and weight constraints. This makes the efficient synthesis of low-order impedances a crucial problem to tackle for wider industrial applications. This talk will summarise and review recent progresses on network synthesis of 2nd and 3rd order impedance functions.

Timothy Hughes **University of Cambridge**

Behavioral network synthesis. Part I: passivity.

Classical circuit analysis and synthesis techniques focus on the system's transfer function. However, some of the best known synthesis techniques result in networks with uncontrollable driving-point behaviors. For example, the Bott-Duffin network for realizing the impedance $(s^2+s+1)/(s^2+s+4)$ actually realizes a driving-point behavior equal to the solutions of the differential equation $(\frac{d}{dt}+1)(\frac{d^2}{dt^2}+\frac{d}{dt}+4)v = (\frac{d}{dt}+1)(\frac{d^2}{dt^2}+\frac{d}{dt}+1)i$. This is uncontrollable. Notably, the transfer function is insufficient for describing the driving-point behavior of this network as it disregards the effects of the differential operator $\frac{d}{dt}+1$.

In this talk, we extend some fundamental results on passivity and circuit synthesis to include such uncontrollable behaviors. We will answer the following question: given the system $P_n\frac{d^n i}{dt^n} + \cdots + P_0 i = Q_m\frac{d^m v}{dt^m} + \cdots + Q_0 v$, *what are the necessary and sufficient conditions for this system to be passive.* The answer is well known for the case in which the square polynomial matrices $P(s) := P_n s^n + \cdots P_0$ and $Q(s) := Q_m s^m + \cdots + Q_0$ are coprime and Q is invertible. Namely: $Q^{-1}P$ is positive-real. For the general case, we provide new necessary and sufficient conditions on the pair (P, Q). Unlike the positive-real condition, these conditions distinguish between the system $(\frac{d}{dt}+1)(\frac{d^2}{dt^2}+\frac{d}{dt}+4)v = (\frac{d}{dt}+1)(\frac{d^2}{dt^2}+\frac{d}{dt}+1)i$ (which is passive), and $(\frac{d^2}{dt^2}+1)(\frac{d^2}{dt^2}+\frac{d}{dt}+4)v = (\frac{d^2}{dt^2}+1)(\frac{d^2}{dt^2}+\frac{d}{dt}+1)i$ (which is not). An equivalent condition will also be obtained in terms of a linear matrix inequality, which we use to obtain a realization of a (not necessarily controllable) passive behavior as the driving-point behavior of a circuit containing resistors, inductors, capacitors, transformers, and gyrators.

Nicos Karcanias **City University London**

Passive Network Redesign and Implicit Network Descriptions

Redesigning systems by changing elements, topology, organization, augmenting the system by the addition of subsystems, or removing parts, is a major challenge and systems and control theory have a major role to play. A special case is the redesign of passive electric networks which aims to change the natural dynamics of the network (natural frequencies) by the above operations leading to a modification of the network. This problem is different to a standard control problem since it involves changing the system to achieve the desirable natural frequencies. In fact this problem involves the selection of alternative values for dynamic elements and non-dynamic elements within a fixed interconnection topology and/or alteration of the interconnection topology and possible evolution of the network (increase of elements, branches). We use impedance and admittance modelling, which introduce new implicit descriptions for electrical networks described by integral differential symmetric linear operators and the system properties of these implicit descriptions are examined. We identify the natural topologies expressing the structured transformations which are linked to the impedance graph-, or the admittance graph-topology of the network. Integral part of the study is the mathematical characterization and the representation of the different types of transformations that may be applied on the structured network models. Such transformations are characterized on the impedance, or admittance model description and appropriate representations are introduced which in turn provide a suitable set up for the study of the natural frequencies assignment. Within this structured framework we consider the effect of changes of dynamic, or non-dynamic element on the natural frequencies. A general control theoretic framework for natural frequency shaping by network redesign is presented using the algebro-geometric framework of the Determinantal Frequency Assignment.

Alessandro Morelli **University of Cambridge**

Group action, equivalence and realisation in electrical networks

This talk discusses further the joint work with M.C. Smith on the classification of the positive-real biquadratic functions which can be realised by five-element RLC networks with at most two reactive elements—the networks of the "Ladenheim catalogue". The classification of the catalogue is formalised by group actions and equivalences. It will be shown that the set of impedances that can be realised may be derived in explicit form as a semialgebraic set. Examples of the derivation of the semialgebraic sets will be provided and general conclusions presented.

Paul Fuhrmann **Ben-Gurion University of the Negev**

Control and Interpolation

In Memory of My Friend and Colleague Uwe Helmke

The central theme of the talk is to understand the connection between control and interpolation theory. To keep the scope of the talk under control, we will focus on the theme of reachability. We will do this on two levels, the first being the characterization of (partial) reachability, whereas the second is open loop control, that is given a state in the state space, finding a/all control sequences that drive the system from rest to that state. The Kalman and Hautus criteria seem to be the answer to that. However, when the system to be studied is a large interconnected network, the computational complexity can be overwhelming.

In order to simplify the computations, we would like to use both local reachability information as well as the interconnection data. To start on this journey, we shall focus on two simple, albeit important, special cases, namely parallel and series interconnections. Characterizations of reachability of parallel and series connection of linear systems have been known for some time, see [Callier, and Nahum (1975)], [Fuhrmann (1975)] and, in an infinite dimensional setting, [Fuhrmann(1976a)]. The case of parallel connection of systems is the simplest one to analyze as there is no dynamic interconnection between the systems. This case has been worked out in [Fuhrmann and Helmke(2015)] and shown to be related to a polynomial matrix version of the Chinese remainder theorem and, consequently, to tangential Lagrange interpolation. The analysis of the series connection centers on deriving recursive computation for the inversion of a reduced reachability map. This turns out to be related to a tangential, Newton like, interpolation problem. The analysis of these two cases provides a glimpse into the complexity of controlling interconnected systems.

[Callier, and Nahum (1975)] F.M. Callier, and C.D. Nahum. *Necessary and sufficient conditions for the complete controllability and observability of systems in series using the coprime decomposition of a rational matrix. IEEE Trans. Circuits Syst.*, vol. 22, (1975), 90–95.

[Fuhrmann (1975)] P.A. Fuhrmann, "On controllability and observability of systems connected in parallel", *IEEE Trans. Circuits and Systems*, CAS-22 (1975), 57.

[Fuhrmann(1976a)] P.A. Fuhrmann, "On series and parallel coupling of a class of discrete time infinite-dimensional systems", *SIAM J. Control and Optim.*, 14 (1976), 339-358.

[Fuhrmann and Helmke(2015)] P.A. Fuhrmann and U. Helmke, *The Mathematics of Networks of Linear Systems*, Springer, New York, 2015 (to appear).

Patrick Dewilde **Technical University of Munich**

Between inner-outer and spectral factorization

Inner-outer factorization can be considered as one of the main if not the main workhorse in both dynamical system theory and network theory. It works in very varied circumstances, including time variant and non-linear environments. Up until recently it was doubtful or not known whether it could handle spectral factorization (another central workhorse) as well. The talk will show that this is indeed the case and how it works. In short: one spectral factorization = two consecutive inner-outers. Also the consequences of this new insight will be considered.

Rodolphe Sepulchre **University of Cambridge**

Localised network behaviours

Balancing positive and negative feedback provides a versatile mechanism to localise the sensitivity of a behaviour in a given window. Resonant systems are localised in frequency while excitable systems are localised in amplitude. The emphasis of this talk will be on network interconnections of localised systems and how the network topology can contribute to network localisation.

Fabien Seyfert **INRIA France**

Analytical Techniques for Identification and Design of Microwave Filters

Microstrip or cavity microwave filters are formally infinite dimensional dynamical systems. We will first explain how local approximations in frequency yield a low pass equivalent electrical network, made of resonators coupled by ideal impedance inverters, that describes in a satisfactory manner the behaviour of these devices. We will detail some circuital design procedures for this kind of network, and in particular an algebraic framework used to solve exhaustively realisation problems by means of Groebner basis computations. We will also consider the identification problem, that is the extraction of a circuital realisation at hand of scattering data of the filter, and present an approach based on rational approximation and the resolution of extremal problems in Hardy spaces.

Michael Chen **University of Hong Kong**

Realization of Biquadratic Impedances as Five-Element Bridge Networks

The birth of a new mechanical element named inerter completes the analogy between passive mechanical and electrical circuit elements. As a result, research interest in passive network synthesis has been renewed, very much motivated by passive mechanical control containing inerters.

This talk will introduce a recent result on passive network synthesis problem of biquadratic impedances as five-element bridge networks. A necessary and sufficient condition is obtained for biquadratic impedances to be realizable as two-reactive five-element bridge networks. Based on the types of the two reactive elements, the discussion is divided into two parts, and a canonical form for biquadratic impedances is utilized to simplify and combine the conditions. Moreover, the realizability result for the biquadratic impedance is extended to the general five-element bridge networks. Numerical examples will be presented for illustration.

Paolo Rapisarda **University of Southampton**

From data to state model via duality

The mirroring of external (at the level of inputs and outputs) and internal (at the level of state) properties is a Leitmotiv in systems and control theory. I will illustrate a variation on such theme, focusing on identification of linear time-invariant 1- and 2-D systems, and of linear time-varying systems.

By factorizing a matrix computed from external trajectories of the dual- and the primal system, state trajectories corresponding to the external ones can be constructed. To complete the identification of a state model, a system of linear equations must be solved. This identification procedure is analogous to subspace identification, where time-invariance is exploited. By making use of duality we can derive state equations also when time-invariance is absent (as in the case of time-varying systems) or it is cumbersome to take advantage of (as in the case of 2-D systems).

Richard Pates **University of Lund**

Decomposing Analysis and Synthesis Problems in Electrical Networks

Passivity theory is a powerful tool for analysing and designing large networks, such as electrical power systems, since it allows for a component oriented approach. This is a major advantage because typically the analysis and synthesis problems associated with the components are tractable, while their network counterparts are not. In this talk we investigate other structural features present in electrical networks, such as fundamental circuits and cutsets, that can also be exploited in decomposing analysis and synthesis problems. This allows for analysis and design on the basis of less restrictive dynamical properties than passivity, whilst still retaining many of the benefits of the component oriented approach.

Tim Hughes **University of Cambridge**

Behavioral network synthesis. Part II: reciprocity.

In Part I, we extended some fundamental results on passivity and circuit synthesis to include uncontrollable behaviors. In this part, we will consider a second property possessed by many important physical systems: reciprocity. In particular, we answer the question *what are the necessary and sufficient conditions for the behavior* $P_n\frac{d^n i}{dt^n} + \cdots + P_0 i = Q_m\frac{d^m v}{dt^m} + \cdots + Q_0 v$ *to be realized as the driving-point behavior of a reciprocal circuit (a circuit containing resistors, inductors, capacitors, and transformers)*. Also, we will provide lower bounds on the numbers of inductors and capacitors required to realize such a behavior.

This theory is well established for the case in which the square polynomial matrices $P(s) := P_n s^n + \cdots P_0$ and $Q(s) := Q_m s^m + \cdots + Q_0$ are coprime and Q is invertible. In this case, we require $Q^{-1}P$ to be positive-real and symmetric; and the number of capacitors (resp., inductors) must be greater than or equal to the number of positive (resp., negative) eigenvalues of the Bezoutian of P and Q. In this talk, we extend these results to the cases in which P and Q are not co-prime and/or Q is not invertible. In particular, we obtain the curious result that, for a reciprocal network, *both* an inductor *and* a capacitor are require to realize each uncontrollable mode (corresponding to a root of the determinant of the greatest common divisor of P and Q). As an interesting corollary, we find that the driving-point behavior of any circuit containing only resistors, transformers, and capacitors is necessarily controllable.