Passive Network Synthesis Revisited

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Semi-Plenary Lecture
OUTLINE OF TALK

1. Motivating example (vehicle suspension).
3. Positive-real functions and Brune synthesis.
5. Darlington synthesis.
6. Minimum reactance synthesis.
7. Synthesis of resistive $n$-ports.
8. Vehicle suspension.
10. Motorcycle steering instabilities.
Motivating Example – Vehicle Suspension

Performance Objectives

1. Control vehicle body in the face of variable loads.
2. Insulate effect of road undulations (ride).
3. Minimise roll, pitch under braking, acceleration and cornering (handling).
QUARTER-CAR VEHICLE MODEL (CONVENTIONAL SUSPENSION)

- **Load Disturbances**
- **Spring**
- **Damper**
- **Road Disturbances**

Diagram with:
- Tyre
- Damper
- Spring
- Load Disturbances
- Road Disturbances

Variables:
- $m_s$
- $m_u$
- $k_t$
The Most General Passive Vehicle Suspension

Replace the spring and damper with a general positive-real impedance \( Z(s) \).

But is \( Z(s) \) physically realisable?
ELECTRICAL-MECHANICAL ANALOGIES

1. Force-Voltage Analogy.

   voltage $\leftrightarrow$ force
   current $\leftrightarrow$ velocity

Oldest analogy historically, cf. electromotive force.

2. Force-Current Analogy.

   current $\leftrightarrow$ force
   voltage $\leftrightarrow$ velocity
   electrical ground $\leftrightarrow$ mechanical ground

Independently proposed by: Darrieus (1929), Hähnle (1932), Firestone (1933).

Respects circuit “topology”, e.g. terminals, through- and across-variables.
**Standard Element Correspondences (Force-Current Analogy)**

\[ v = Ri \quad \text{resistor} \leftrightarrow \text{damper} \quad cv = F \]
\[ v = L \frac{di}{dt} \quad \text{inductor} \leftrightarrow \text{spring} \quad kv = \frac{dF}{dt} \]
\[ C \frac{dv}{dt} = i \quad \text{capacitor} \leftrightarrow \text{mass} \quad m \frac{dv}{dt} = F \]

What are the **terminals** of the mass element?
The Exceptional Nature of the Mass Element

Newton's Second Law gives the following network interpretation of the mass element:

- One terminal is the centre of mass,
- Other terminal is a fixed point in the inertial frame.

Hence, the mass element is analogous to a grounded capacitor.

Standard network symbol for the mass element:
## Table of usual correspondences

<table>
<thead>
<tr>
<th>Mechanical</th>
<th>Electrical</th>
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<tbody>
<tr>
<td>( F ) ( v_2 ) ( F ) ( v_1 )</td>
<td>( i ) ( v_2 ) ( i ) ( v_1 )</td>
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<tr>
<td><strong>spring</strong></td>
<td><strong>inductor</strong></td>
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<tr>
<td>( F ) ( v_2 ) ( v_1 = 0 )</td>
<td>( i ) ( v_2 ) ( i ) ( v_1 )</td>
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<td><strong>mass</strong></td>
<td><strong>capacitor</strong></td>
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<td>( F ) ( v_2 ) ( F ) ( v_1 )</td>
<td>( i ) ( v_2 ) ( i ) ( v_1 )</td>
</tr>
<tr>
<td><strong>damper</strong></td>
<td><strong>resistor</strong></td>
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CONSEQUENCES FOR NETWORK SYNTHESIS

Two major problems with the use of the mass element for synthesis of “black-box” mechanical impedances:

- An electrical circuit with ungrounded capacitors will not have a direct mechanical analogue,
- Possibility of unreasonably large masses being required.

QUESTION

Is it possible to construct a physical device such that the relative acceleration between its endpoints is proportional to the applied force?
ONE METHOD OF REALISATION

Suppose the flywheel of mass $m$ rotates by $\alpha$ radians per meter of relative displacement between the terminals. Then:

$$F = (m\alpha^2) (\dot{v}_2 - \dot{v}_1)$$

(Assumes mass of gears, housing etc is negligible.)
THE IDEAL INERTER

We define the Ideal Inerter to be a mechanical one-port device such that the equal and opposite force applied at the nodes is proportional to the relative acceleration between the nodes, i.e.

\[ F = b(\dot{v}_2 - \dot{v}_1). \]

We call the constant \( b \) the inertance and its units are kilograms.

The ideal inerter can be approximated in the same sense that real springs, dampers, inductors, etc approximate their mathematical ideals.

We can assume its mass is small.

**A NEW CORRESPONDENCE FOR NETWORK SYNTHESIS**

<table>
<thead>
<tr>
<th>Mechanical</th>
<th>Electrical</th>
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<tbody>
<tr>
<td><img src="image" alt="Mechanical Circuit" /></td>
<td><img src="image" alt="Electrical Circuit" /></td>
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<tr>
<td>$F$</td>
<td>$i$</td>
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<tr>
<td>$v_2$</td>
<td>$v_1$</td>
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<tr>
<td>$Y(s) = \frac{k}{s}$</td>
<td>$Y(s) = \frac{1}{Ls}$</td>
</tr>
<tr>
<td>$\frac{dF}{dt} = k(v_2 - v_1)$</td>
<td>$\frac{di}{dt} = \frac{1}{L}(v_2 - v_1)$</td>
</tr>
<tr>
<td><strong>Inverter</strong></td>
<td><strong>Inductor</strong></td>
</tr>
<tr>
<td>$F = b\frac{d(v_2 - v_1)}{dt}$</td>
<td>$i = Cs$</td>
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<tr>
<td>$v_2$</td>
<td>$v_1$</td>
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<tr>
<td>$Y(s) = bs$</td>
<td>$i = C\frac{d(v_2 - v_1)}{dt}$</td>
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<tr>
<td><strong>Damper</strong></td>
<td><strong>Capacitor</strong></td>
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<td>$F = c(v_2 - v_1)$</td>
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<td>$Y(s) = c$</td>
<td>$i = \frac{1}{R}(v_2 - v_1)$</td>
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<td>$F$</td>
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<td>$v_2$</td>
<td>$v_1$</td>
</tr>
<tr>
<td><strong>Y(s) = admittance = \frac{1}{impedance}</strong></td>
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</tbody>
</table>
Rack and pinion inerter
made at
Cambridge University
Engineering Department

mass \approx 3.5 \text{ kg}
inertance \approx 725 \text{ kg}
stroke \approx 80 \text{ mm}
Damper-inerter series arrangement with centring springs
ALTERNATIVE REALISATION OF THE INERTER
Ballscrew inerter made at Cambridge University Engineering Department

Mass $\approx 1$ kg, Inertance (adjustable) $= 60$–$180$ kg
ELECTRICAL EQUIVALENT OF QUARTER CAR MODEL

\[ Y(s) = \text{Admittance} = \frac{\text{Force}}{\text{Velocity}} = \frac{\text{Current}}{\text{Voltage}} \]
**Positive-real functions**

**Definition.** A function $Z(s)$ is defined to be **positive-real** if one of the following two equivalent conditions is satisfied:

1. $Z(s)$ is analytic and $Z(s) + Z(s)^* \geq 0$ in $\text{Re}(s) > 0$.

2. $Z(s)$ is analytic in $\text{Re}(s) > 0$, $Z(j\omega) + Z(j\omega)^* \geq 0$ for all $\omega$ at which $Z(j\omega)$ is finite, and any poles of $Z(s)$ on the imaginary axis or at infinity are simple and have a positive residue.
Passivity Defined

Definition. A network is passive if for all admissible $v, i$ which are square integrable on $(-\infty, T],$

\[ \int_{-\infty}^{T} v(t)i(t) \, dt \geq 0. \]

Proposition. Consider a one-port electrical network for which the impedance $Z(s)$ exists and is real-rational. The network is passive if and only if $Z(s)$ is positive-real.

O. Brune showed that any (rational) positive-real function could be realised as the impedance or admittance of a network comprising resistors, capacitors, inductors and transformers. (1931)
Minimum functions

A **minimum function** $Z(s)$ is a positive-real function with no poles or zeros on $j\mathbb{R} \cup \{\infty\}$ and with the real part of $Z(j \omega)$ equal to 0 at one or more frequencies.
FOSTER PREAMBLE FOR A POSITIVE-REAL $Z(s)$

Removal of poles/zeros on $j\mathbb{R} \cup \{\infty\}$. e.g.

$$\frac{s^2 + s + 1}{s + 1} = s + \frac{1}{s + 1}$$

↑

**lossless**

$$\downarrow$$

$$\frac{s^2 + 1}{s^2 + 2s + 1} = \left(\frac{2s}{s^2 + 1} + 1\right)^{-1}$$

Can always reduce a positive-real $Z(s)$ to a minimum function.
The Brune cycle

Let $Z(s)$ be a minimum function with $Z(j\omega_1) = jX_1$ ($\omega_1 > 0$).
Write $L_1 = X_1/\omega_1$ and $Z_1(s) = Z(s) - L_1s$.

Case 1. ($L_1 < 0$)

$Z_1(s)$ is positive-real. Let $Y_1(s) = 1/Z_1(s)$. Therefore, we can write

$$Y_2(s) = Y_1(s) - \frac{2K_1s}{s^2 + \omega_1^2}$$

for some $K_1 > 0$. 
The Brune cycle (cont.)

Then:

\[ Z(s) = \frac{1}{sL_2} \]

where \( L_2 = 1/2K_1 \), \( C_2 = 2K_1/\omega_1^2 \) and \( Z_2 = 1/Y_2 \).

Straightforward calculation shows that

\[ Z_2(s) = sL_3 + Z_3(s) \]

where \( L_3 = -L_1/(1 + 2K_1L_1) \). Since \( Z_2(s) \) is positive-real, \( L_3 > 0 \) and \( Z_3(s) \) is positive-real.
THE BRUNE CYCLE (cont.)

Then:

\[ Z(s) \]

\[ L_1 < 0 \quad L_3 > 0 \]

\[ L_2 > 0 \]

\[ C_2 > 0 \]

To remove negative inductor:

\[ L_p = L_1 + L_2 \]

\[ L_s = L_2 + L_3 \]

\[ M = L_2 \]

Some algebra shows that: \( L_p, L_s > 0 \) and \( \frac{M^2}{L_p L_s} = 1 \) (unity coupling coefficient).
The Brune cycle (cont.)

Realisation for completed cycle:

\[ Z(s) \]

\[ L_p \quad L_s \quad C_2 \]

\[ M \]

\[ Z_3(s) \]
THE BRUNE CYCLE (cont.)

**Case 2.** $(L_1 > 0)$. As before $Z_1(s) = Z(s) - L_1 s$

\[ L_1 > 0 \]

(no need for negative inductor!)

\[ Z(s) \]
\[ Z_1(s) \]

Problem: $Z_1(s)$ is not positive-real!
Let’s press on and hope for the best!!
As before let $Y_1 = 1/Z_1$ and write

\[ Y_2(s) = Y_1(s) - \frac{2K_1 s}{s^2 + \omega_1^2}. \]

Despite the fact that $Y_1$ is not positive-real we can show that $K_1 > 0$. 
**The Brune cycle (cont.)**

Hence:

\[ Z(s) = \frac{L_1}{s + L_1} + \frac{L_2}{s + L_2} + C_2 > 0 \]

But still \( Z_2(s) \) is not positive-real. Again we can check that

\[ Z_2(s) = sL_3 + Z_3(s) \]

where \( L_3 = -L_1/(1 + 2K_1L_1) \).

This time \( L_3 < 0 \) and \( Z_3(s) \) is positive-real.
THE BRUNE CYCLE (cont.)

So:

\[ Z(s) \]

\[ L_1 > 0 \quad L_2 > 0 \quad L_3 < 0 \]

\[ C_2 > 0 \]

As before we can transform to:

\[ Z(s) \]

\[ M \]

\[ L_p \quad L_s \]

\[ C_2 \]

\[ Z_3(s) \]

where \( L_p, L_s > 0 \) and \( \frac{M^2}{L_p L_s} = 1 \) (unity coupling coefficient).
R. Bott and R.J. Duffin showed that transformers were unnecessary in the synthesis of positive-real functions. (1949)
RICHARDS’S TRANSFORMATION

Theorem. If $Z(s)$ is positive-real then

$$R(s) = \frac{kZ(s) - sZ(k)}{kZ(k) - sZ(s)}$$

is positive-real for any $k > 0$.

Proof.

$Z(s)$ is p.r. $\Rightarrow$ $Y(s) = \frac{Z(s) - Z(k)}{Z(s) + Z(k)}$ is b.r. and $Y(k) = 0$

$\Rightarrow$ $Y'(s) = \frac{k + s}{k - s}Y(s)$ is b.r.

$\Rightarrow$ $Z'(s) = \frac{1 + Y'(s)}{1 - Y'(s)}$ is p.r.

$R(s) = Z'(s)$ after simplification.
**BOTT-DUFFIN CONSTRUCTION** (cont.)

Idea: use Richards’s transformation to eliminate transformers from Brune cycle.

As before, let $Z(s)$ be a minimum function with $Z(j\omega_1) = jX_1$ ($\omega_1 > 0$).

Write $L_1 = X_1/\omega_1$.

**Case 1.** $(L_1 > 0)$

Since $Z(s)$ is a minimum function we can always find a $k$ s.t. $L_1 = Z(k)/k$.

Therefore:

$$R(s) = \frac{kZ(s) - sZ(k)}{kZ(k) - sZ(s)}$$

has a zero at $s = j\omega_1$. 
BOTT-DUFFIN CONSTRUCTION (cont.)

We now write:

\[
Z(s) = \frac{kZ(k)R(s) + Z(k)s}{k + sR(s)}
\]

\[
= \frac{kZ(k)R(s)}{k + sR(s)} + \frac{Z(k)s}{k + sR(s)}
\]

\[
= \frac{1}{Z(k)R(s)} + \frac{s}{kZ(k)} + \frac{1}{kZ(k)s + \frac{R(s)}{Z(k)}}.
\]
Bott-Duffin construction (cont.)

\[ Z(s) = \frac{1}{Z(k)R(s)} + \frac{s}{kZ(k)} + \frac{1}{kZ(k)s + \frac{R(s)}{Z(k)}} \]
**Bott-Duffin construction (cont.)**

We can write: \( \frac{1}{Z(k)R(s)} = \text{const} \times \frac{s}{s^2 + \omega_1^2} + \frac{1}{R_1(s)} \) etc.

\[
Z(s) = \frac{1}{Z(k)R(s)} = \text{const} \times \frac{s}{s^2 + \omega_1^2} + \frac{1}{R_1(s)} \text{ etc.}
\]

\[ R_1 \]

\[ R_2 \]

\[ Z(s) \]
EXAMPLE — RESTRICTED DEGREE

**Proposition.** Consider the real-rational function

\[ Y_b(s) = k \frac{a_0 s^2 + a_1 s + 1}{s(d_0 s^2 + d_1 s + 1)} \]

where \( d_0, d_1 \geq 0 \) and \( k > 0 \). Then \( Y_b(s) \) is positive real if only if the following three inequalities hold:

\[ \beta_1 := a_0 d_1 - a_1 d_0 \geq 0, \]
\[ \beta_2 := a_0 - d_0 \geq 0, \]
\[ \beta_3 := a_1 - d_1 \geq 0. \]
**Brune Realisation Procedure for** $Y_b(s)$

Foster preamble always sufficient to complete the realisation if $\beta_1, \beta_2 > 0$. (No Brune or Bott-Duffin cycle is required).

A continued fraction expansion is obtained:

$$Y_b(s) = k \frac{a_0 s^2 + a_1 s + 1}{s(d_0 s^2 + d_1 s + 1)} = \frac{k}{s} + \frac{1}{s \frac{k_b}{k}} + \frac{1}{c_3 + \frac{1}{\frac{1}{c_4} + \frac{1}{b_2 s}}}$$

where $k_b = \frac{k\beta_2}{d_0}$, $c_3 = k/\beta_3$, $c_4 = \frac{k\beta_4}{\beta_1}$, $b_2 = \frac{k\beta_4}{\beta_2}$ and $\beta_4 := \beta_2^2 - \beta_1\beta_3$. 
Darlington Synthesis

Realisation in Darlington form: a lossless two-port terminated in a single resistor.

For a lossless two-port with impedance:

\[
Z = \begin{pmatrix}
Z_{11} & Z_{12} \\
Z_{12} & Z_{22}
\end{pmatrix}
\]

we find

\[
Z_1(s) = Z_{11} \frac{R^{-1}(Z_{11}Z_{22} - Z_{12}^2)/Z_{11} + 1}{R^{-1}Z_{22} + 1}.
\]
Writing

\[ Z_1 = \frac{m_1 + n_1}{m_2 + n_2} = \frac{n_1}{m_2} \frac{m_1/n_1 + 1}{n_2/m_2 + 1}, \]

where \( m_1, m_2 \) are polynomials of even powers of \( s \) and \( n_1, n_2 \) are polynomials of odd powers of \( s \), suggests the identification:

\[ Z_{11} = \frac{n_1}{m_2}, \quad Z_{22} = R \frac{n_2}{m_2}, \quad Z_{12} = \sqrt{R} \frac{\sqrt{n_1 n_2 - m_1 m_2}}{m_2}. \]

Augmentation factors are necessary to ensure a rational square root.

Once \( Z(s) \) has been found, we then write:

\[ Z(s) = sC_1 + \frac{s}{s^2 + \alpha^2} C_2 + \cdots \]

where \( C_1 \) and \( C_2 \) are non-negative definite constant matrices.
DARLINGTON SYNTHESIS (cont.)
Each term in the sum is realised in the form of a T-circuit and a series connection of all the elementary two-ports is then made:
ELECTRICAL AND MECHANICAL REALISATIONS OF THE ADMITTANCE $Y_b(s)$
**MINIMUM REACTANCE SYNTHESIS**

Let $L_1 = \cdots = C_1 = \cdots = 1$. If

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

is the hybrid matrix of $X$, i.e.

$$\begin{pmatrix} v_1 \\ i_2 \end{pmatrix} = M \begin{pmatrix} i_1 \\ v_2 \end{pmatrix},$$

then

$$Z(s) = M_{11} - M_{12}(sI + M_{22})^{-1}M_{21}.$$
MINIMUM REACTANCE SYNTHESIS

Conversely, if we can find a state-space realisation \( Z(s) = C(sI - A)^{-1}B + D \) such that the constant matrix

\[
M = \begin{bmatrix}
D & -C \\
B & -A
\end{bmatrix}
\]

has the properties

\[
M + M' \geq 0, \quad \text{diag}\{I, \Sigma\}M = M\text{diag}\{I, \Sigma\}
\]

where \( \Sigma \) is a diagonal matrix with diagonal entries +1 or −1. Then \( M \) is the hybrid matrix of the nondynamic network terminated with inductors or capacitors, which realises \( Z(s) \).

A construction is possible using the positive-real lemma and matrix factorisations.

SYNTHESIS OF RESISTIVE $n$-PORTS

Let $R$ be a symmetric $n \times n$ matrix.

A necessary and sufficient condition for $R$ to be realisable as the driving-point impedance of a network comprising resistors and transformers only is that it is non-negative definite.

No necessary and sufficient condition is known in the case that transformers are not available.

A general necessary condition is known: that the matrix is paramount.$^1$

A matrix is defined to be the paramount if each principal minor of the matrix is not less than the absolute value of any minor built from the same rows.

It is also known that paramountcy is sufficient for the case of $n \leq 3$.\(^2\)

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Simple Suspension Struts

Layout S1

Layout S2

Layout S3

Layout S4

parallel

series
Performance Measures

Assume:

Road Profile Spectrum = \( \kappa |n|^{-2} \) \( \text{m}^3/\text{cycle} \)

where \( \kappa = 5 \times 10^{-7} \text{ m}^3\text{cycle}^{-1} = \text{road roughness parameter} \). Define:

\[
J_1 = E \left[ \ddot{z}_s(t) \right] \quad \text{ride comfort}
\]

= r.m.s. body vertical acceleration
**Optimisation of $J_1$ (ride comfort)**

(a) Optimal $J_1$

(b) Percentage improvement in $J_1$

Key: layout S1 (bold), layout S2 (dashed), layout S3 (dot-dashed), and layout S4 (solid).

**CONTROL SYNTHESIS FORMULATION**

\[ F_s \]

\[ m_s \]

\[ k_s \]

\[ K(s) \]

\[ m_u \]

\[ k_t \]

\[ z_s \]

\[ z_u \]

\[ z_r \]

\[ G(s) \]

\[ F \]

\[ K(s) \]

\[ w \]

\[ z \]

\[ \dot{z}_s - \dot{z}_u \]

**Ride comfort:** \( w = z_r, \ z = \dot{z}_s \)

**Performance measure:** \( J_1 = \text{const.} \times \| T_{\dot{z}_r \rightarrow s \dot{z}_s} \|_2 \)
**Bilinear Matrix Inequality (BMI) formulation**

Let \( K(s) = C_k(sI - A_k)^{-1}B_k + D_k \) and \( T_{\hat{z}_r \rightarrow \hat{z}_s} = C_{cl}(sI - A_{cl})^{-1}B_{cl} \).

**Theorem.** There exists a positive real controller \( K(s) \) such that 
\[ \|T_{\hat{z}_r \rightarrow \hat{z}_s}\|_2 < \nu \] and \( A_{cl} \) is stable, if and only if the following problem is feasible for some \( X_{cl} > 0, X_k > 0, Q, \nu^2 \) and \( A_k, B_k, C_k, D_k \) of compatible dimensions:

\[
\begin{bmatrix}
A_{cl}^T X_{cl} + X_{cl} A_{cl} & X_{cl} B_{cl} \\
B_{cl}^T X_{cl} & -I
\end{bmatrix} < 0, \quad \begin{bmatrix}
X_{cl} & C_{cl}^T \\
C_{cl} & Q
\end{bmatrix} > 0, \quad \text{tr}(Q) < \nu^2,
\]

\[
\begin{bmatrix}
A_k^T X_k + X_k A_k & X_k B_k - C_k^T \\
B_k^T X_k - C_k & -D_k^T - D_k
\end{bmatrix} < 0.
\]

A SPECIAL PROBLEM

What class of positive-real functions $Z(s)$ can be realised using one damper, one inerter, any number of springs and no transformers?

Leads to the question: when can $X$ be realised as a network of springs?
**Theorem.** Let

\[ Y(s) = \frac{(R_2R_3 - R_6^2) s^3 + R_3 s^2 + R_2 s + 1}{s \left( \det R \right) s^3 + (R_1R_3 - R_5^2) s^2 + (R_1R_2 - R_4^2) s + R_1}, \quad (1) \]

where \( R := \begin{bmatrix} R_1 & R_4 & R_5 \\ R_4 & R_2 & R_6 \\ R_5 & R_6 & R_3 \end{bmatrix} \) is non-negative definite.

A positive-real function \( Y(s) \) can be realised as the driving-point admittance of a network comprising one damper, one inerter, any number of springs and no transformers if and only if \( Y(s) \) can be written in the form of (1) and there exists an invertible diagonal matrix \( D = \text{diag}\{1, x, y\} \) such that \( DRD \) is paramount.

An explicit set of inequalities can be found which are necessary and sufficient for the existence of \( x \) and \( y \).

M.Z.Q. Chen and M.C. Smith, Mechanical networks comprising one damper and one inerter, in preparation.
COLLABORATION WITH IMPERIAL COLLEGE

Application to motorcycle stability.

At high speed motorcycles can experience significant steering instabilities. Observe: Paul Orritt at the 1999 Manx Grand Prix
**Weave and Wobble Oscillations**

Steering dampers improve wobble (6–9 Hz) and worsen weave (2–4 Hz). Simulations show that steering inacters have, roughly, the opposite effect to the damper. Root-loci (with speed the varied parameter):

Can the advantages be combined?

SOLUTION — A STEERING COMPENSATOR

... consisting of a network of dampers, ineters and springs.
Needs to behave like an inerter at weave frequencies and like a damper at wobble frequencies.

Prototype designed by N.E. Houghton and manufactured in the Cambridge University Engineering Department.
Conclusions

- A new mechanical element called the “inertor” was introduced which is the true network dual of the spring.

- The inertor allows classical electrical network synthesis to be mapped exactly onto mechanical networks.

- Applications of the inertor: vehicle suspension, motorcycle steering and vibration absorption.

- Economy of realisation is an important problem for mechanical network synthesis.

- The problem of minimal realisation of positive-real functions remains unsolved.
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                    (Imperial College)