Transactions of the Institute of Measurement and Control 28, 3 (2006) pp. 1–10

OFS model-based adaptive control for block-oriented non-linear Systems

Haitao Zhang[1,2], Chi Zhang[3], Zonghai Chen[2], Ming Li[2] and Wei Xiang[2]

[1]Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, 430074, P.R.C
[2]Department of Automation, University of Science and Technology of China, Hefei, 230027, P.R.C
[3]Department of Modern Physics, University of Science and Technology of China, Hefei, 230027, P.R.C

A non-linear state feedback algorithm based on an OFS (orthonormal functional series) model combined with Volterra Functional Series is proposed to solve block-oriented non-linear systems. This algorithm has excellent approximation ability for the variance of system time-delay, and can guarantee the closed-loop robust stability and the zero steady-state error property. In addition, it has lower online optimization computational load when dealing with the hard input constraints in contrast to traditional NMPC (non-linear model predictive control) based on the OFS model. The analyses of the stability and robustness of this algorithm are put forward systemically, and the steady-state performance analysis is also given. Case studies involving simulations on a Wiener-type non-linear system and temperature experiments on a simulation chemical reactor are used to validate the efficiency and superiority of this proposed algorithm.

Key words: block-oriented non-linearity; OFS model; stability; Volterra functional series.

Address for correspondence: Haitao Zhang, Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, 430074, P.R.C. E-mail: zht@mail.ustc.edu.in

 10.1191/0142331206tm170oa

1. Introduction

During many real industrial processes such as distillation, pH neutralization control, hydro-control and chemical reactions, there widely exists a type of so-called block-oriented non-linear systems such as Wiener-type and Hammerstein-type non-linear systems (Gómez and Baeyens, 2004; Henson, 1997). Wiener-type systems consist of a linear dynamic element followed by a memory-less non-linear element, whereas, Hammerstein-type systems contain the same elements in reverse order (Gómez and Baeyens, 2004). In recent years, the control of these types of systems has become one of the most important and difficult tasks in non-linear control field (Gómez and Baeyens, 2004; Henson, 1997).

Combined with Volterra series, the OFS model, which has some advantages such as excellent approximation ability for the variances of the control plant's time-delay (Mäkilä, 1990) due to its similarity to Páde approximants, can be easily extended to the field of non-linear predictive control of block-oriented non-linear systems with time-delay. Zervos and Dumont (1988) proposed a novel linear model predictive control (MPC) based on a Laguerre series and successfully applied the scheme to pH control in an industrial bleach plant extraction stage in 1990 (Dumont *et al.*, 1990), which was the first successful industrial application of OFS-based control algorithms. From 2000 to 2006, Zhang presented many successful industrial applications of Laguerre functional series-based control algorithms on a high temperature semiconductor diffusion furnace (Zhang *et al.,* 2002), double water tanks (Zhang *et al.,* 2004a) and a heavy oil distillation column (Zhang *et al.,* 2004b). Meanwhile, he has also made some theoretical progresses in this field (Zhang *et al.,* 2002, 2004a,b). Olivera *et al.* (2000) extended the Laguerre functional series-based control algorithm to a constrained robust one.

However, some strict conditions, which cannot always be fulfilled in real industrial processes, must be imposed on most of the former OFS model-based non-linear model predictive control (NMPC) algorithms (Campello *et al.*, 2004; Parker and Doyle 1998; Parker, 2002) to guarantee closed-loop stability. Beside, these algorithms cannot guarantee zero steady-state error property. Therefore, they must be improved to satisfy the requirements of modern industrial production.

The main contributions of this paper are: 1) to propose an OFS model-based adaptive non-linear state feedback control algorithm for block-oriented non-linear systems; 2) to prove the stability of the closed-loop system both in time-domain and frequency-domain; 3) to prove zero steady-state error property of the closed-loop system determined by our algorithm; and 4) to successfully apply this algorithm to a physical Hammerstein-typed non-linear system with long time-delay.

The paper is organized as follows: typical OFS models are introduced in the next section. In section 3, the adaptive non-linear state feedback control algorithm with hard input constraints is presented in detail. In section 4, the robust stability and the steady-state performance analyses are given. Section 5 shows the simulations and applications of this proposed algorithm. Finally, conclusions are drawn in section 6.

2. OFS Model theory

For any function $f(x) \in \mathbf{L}^2(R^+)$ (square integrable space in positive real number set) can be approximate by a complete orthornormal function series (OFS) with arbitrary accuracy (Parker and Doyle, 1998). Pulse function, Laguerre function and Kautz function are three typical orthonormal functions of 0,1 and 2 OFS order, respectively.

The S-transforms of Laguerre function are (Wahlberg and Mäkilä, 1996)

$$\Phi_i(s,p) = \sqrt{2p}\frac{(s-p)^{i-1}}{(s+p)^i} \tag{1}$$

and the S-transforms of Kautz function are (Wahlberg and Mäkilä, 1996):

$$\Phi_{2i-1}(s,b,c) = \frac{\sqrt{2b}s}{s^2+bs+c}\left[\frac{s^2-bs+c}{s^2+bs+c}\right]^{i-1}, \Phi_{2i}(s,b,c) = \frac{\sqrt{2bc}}{s^2+bs+c}\left[\frac{s^2-bs+c}{s^2+bs+c}\right]^{i-1} \tag{2}$$

In (1) and (2), $i = 1, 2, \cdots, \infty, p > 0, b > 0, c > 0$

The advantages of the OFS model are

1) owing to its similarity to Páde approximants (Dumont *et al.*, 1990), OFS models have excellent approximation ability for the system's varying time-delay and order;
2) extensions to multivariable schemes do not require interactive matrices (Wang *et al*., 2004);
3) low sensitivity to the modelling parameters.

3. Algorithm

3.1 OFS model for dynamic linear system

Any stable linear system can be represented by an OFS model with sufficient accuracy (Wang, 2004). After discretization, the state and output equations are (Dumont *et al.*, 1990; Zervos and Dumont, 1998):

$$\Phi(k+1) = A\Phi(k) + Bu(k) \tag{3}$$

$$y_m(k) = C^T\Phi(k) \tag{4}$$

where $\Phi(k) = [\phi_1(k), \cdots, \phi_N(k)]^T$ is the state vector of OFS model and $C^T = [c_1, c_2, \cdots, c_N]$ is the coefficient vector of OFS model. The expressions of matrices A, B for Laguere model and Kautz model, which are computed offline, can be seen in Dumont *et al.*, (1990).

3.2 OFS model for dynamic non-linear system

Single-input–single-output (SISO) time-invariant causal non-linear discrete system $y(k)$ with input $u(t) \in \mathbf{L}^2(R^+)$ can be expressed by a Volterra functional series as:

$$y(k) = h_0 + \sum_{i=0}^{\infty} h_1(i)u(k-i) + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} h_2(i,j)u(k-i)u(k-j) + \cdots \tag{5}$$

$h_0, h_1(\tau), h_2(\tau_1, \tau_2)\cdots$ are called zero-order kernel, first-order kernel, second-order kernel, and so on. For a stable non-linear discrete system, the Volterra kernel of each order belongs to an $\mathbf{L}^2[R^+]$ space, so it can be reproached by an N order OFS model with sufficient accuracy, say

$$y_m(k) = h_0 + \sum_{n=1}^{N} c_n\phi_n(k) + \sum_{n=1}^{N}\sum_{m=1}^{N} c_{nm}\phi_n(k)\phi_m(k) + \cdots \tag{6}$$

where $c_n, c_{nm}, \cdots$ are coefficients of OFS, $\phi_n(k) = \Sigma_{i=0}^{\infty}\varphi_n(i)u(k-i), \quad (n = 1, 2, \cdots, N)$ are the states of OFS model, and $\varphi_n(i)$is the nth element of OFS. Based on the non-linear OFS model above, the system can be represented by state space as follows

$$\begin{cases}\Phi(k+1) = A\Phi(k) + Bu(k)\\ y_m(k) = c_0 + C^T\Phi(k) + \Phi^T(k)D\Phi(k) + \cdots\end{cases} \tag{7}$$

where$c_0 = h_0, C = [c_1, \cdots, c_N]^T$, and $D = [c_{ij}]_{1\le i\le N, 1\le j\le N}$. Because Volterra kernels can be transferred to symmetric kernels, D is a Hermite matrix, in other words $c_{ij=}c_{ij}$ $(i, j = 1, \cdots, N)$.

3.3 Adaptive non-linear state feedback control algorithm with hard input constraints

The algorithm is deduced as follows:

Assume

$$u(k) = K\Phi(k) + \delta(k) \tag{8}$$

where the state feedback vector K satisfies that 1)$\Psi = A + bK$ is Hurwitz, 2) $\Sigma_{k=0}^{\infty}\Phi(k)^T Q\Phi(k) + Ru(k)^2$is minimized$\delta(k)$is determined by the set point r substitute (8) into (7) we have

$$\Phi(k) = (zI - A - BK)^{-1}B\delta(k) \tag{9}$$

Substitute (9) into (8), and let $t \to \infty$, in other words $z \to 1$, then we have

$$\alpha_0 + \alpha_1\delta(k) + \alpha_2\delta(k)^2 + \alpha_3\delta(k)^3 + \cdots = r \tag{10}$$

where $\alpha_0 = C_0$, $\alpha_1 = C^T(I - A - BK)^{-1}B\ \alpha_2 = B^T[(I - A - BK)^{-1}]^T D(I - A - BK)^{-1}B, \ \cdots,$ r is the set point. δ (k) can be obtained by solving the polynomial equation (10); accordingly, the control law $u(k)$ can be obtained.

If the plant is not subject to hard input constraints, K can be computed offline. Otherwise, K must be obtained by solving a quadratic optimization problem online. In order to constitute the adaptive mechanism, we must update the non-linear OFS model's coefficients online. Assume

$$\theta = [c_0 \quad c_1 \quad c_2 \quad \cdots \quad c_N \quad c_{11} \quad c_{12} \quad \cdots \quad c_{21} \quad c_{22} \cdots c_{NN}] \tag{11}$$

$$\Psi(k) = [1 \quad \phi_1 \quad \phi_2 \quad \cdots \quad \phi_N \quad \phi_1^2 \quad \phi_1\phi_2 \quad \cdots \quad \phi_2\phi_1 \quad \phi_2^2 \quad \cdots \quad \phi_N^2 \ ] \tag{12}$$

thus

$$y_m(k) = \theta^T \Psi(k). \tag{13}$$

This is a linear regression form, so at each sampling period, the coefficients θ can be identified by RLSE (recursive least-square estimation) with a forgetting factor λ online (Ljung, 1999).

4. Theoretical analysis

4.1 Robust stability

Theorem 1. If the controlled plant satisfies the following three assumptions:

1) *it is a linear system, a Hammerstein-typed non-linear system, or a Wiener-type non-linear system;*

2) *the dynamic linear part of this plant is open-loop stable;*

3) *the static non-linear block of this plant is identically continuous; then, the closed-loop system determined by control law (8)and (10) is asymptotically stable.*

Proof. Let the dynamic linear block of this plant be represented by the state-space equations (Yu, 2002)

$$X(k+1) = (A_0 + \Delta A_0)X(k) + (B_0 + \Delta B_0)u(k) \tag{14}$$

ΔA_0 and Δb_0 are uncertain matrices, assume that they have the standard formulations

$$[\Delta A_0 \ \ \Delta B_0] = D_0 F_0 [E_A \ \ E_B] \tag{15}$$

where D_0, E_A, E_B are constant matrices, which represent the structure of system's uncertainty, F_0 is an alterable matrix which satisfies $F_0^T F_0 \leq I$.

The control law is shown in (8) and (10), in which $\delta(k)$ is independent of $L(K)$ and $X(K)$. By using this control law, we can obtain the following closed-loop state equations

$$\begin{bmatrix} X(k+1) \\ \Phi(k+1) \end{bmatrix} = \begin{bmatrix} A_0 + \Delta A_0 & (B_0 + \Delta b_0)K \\ 0 & A + BK \end{bmatrix} \cdot \begin{bmatrix} X(k) \\ \Phi(k) \end{bmatrix} + \begin{bmatrix} (B_0 + \Delta B_0)\delta(k) \\ B\delta(k) \end{bmatrix} \tag{16}$$

Because the linear part is open-loop stable, $(A_0 + \Delta A_0)$ is always Hurwitz. Furthermore, our control algorithm guarantees that $(A + BK)$ is Hurwitz at each sampling period. Consequently, the closed-loop state matrix $\begin{bmatrix} A_0 + \Delta A_0 & (B_0 + \Delta B_0)K \\ 0 & A + BK \end{bmatrix}$ is Hurwitz. So the linear dynamic block is closed-loop stable. In addition, from the assumptions 3) and the definitions of Hammerstein-type and Wiener-type systems, the non-linear block of the controlled plant is static and identically continuous, which cannot affect the closed-loop stability property, so the closed-loop system is asymptotically stable. □

In the frequency domain, Agamennoni *et al.* (1990) has also presented a methodology to characterize the uncertainty description of nominal stable, time-invariant linear control systems based on frequency response measurement by using a Laguerre functional model. Based on the robust theorem of Agamennoni *et al.*, (1990), and noting that the memoryless non-linear block of the block-oriented non-linear cannot affect the closed-loop stability, we can easily obtain the robust stability theorem of our proposed algorithm in the frequency domain.

4.2 Steady-state error analysis

Lemma 1 (Campello et al., 2004). Assume a non-linear time-invariant stable system can be represented by a Laguerre–Volterra model with mth-order Volterra kernel, and the truncation length of the Laguerre functional series is Nε. In addition, the time-scaling factor P of Laguerre series (1) is given by

$$p = \frac{2Q_{1,m} - 1 - Q_{2,m}}{2Q_{1,m} - 1 + \sqrt{4Q_{1,m}Q_{2,m} - Q_{2,m}^2 - 2Q_{2,m}}} \tag{17}$$

Then, the square norm of the error resulting from the truncation of the series expansion satisfies

$$\|e_m\|^2 \le \frac{J_m}{m(N_\varepsilon + 1)} \tag{18}$$

where $e_m = \lim_{t\to\infty}[y(t) - y_m(t)]$. *The definitions of* $Q_{1,m}, Q_{2,m}, J_m, m, N_\varepsilon$ *can be referred to Campello et al., (2004).*

Theorem 2. The conditions are the same as Lemma 1, and the control law is given by (8) and (10), then the upper bound of the square norm of the steady-state error satisfies:

$$\|e\|^2 \le \frac{J_m}{m(N_\varepsilon + 1)} \tag{19}$$

where $e = \lim_{t\to\infty}[y(t) - r(t)]$, *and* $r(t)$ *is the set point (output reference) signal.*

Proof: From Lemma 1, we have $\|e_m\|^2 = \lim_{t\to\infty}[y(t) - y_m(t)] \le \frac{J_m}{m(N_\varepsilon + 1)}$.

If the control law (8) and (10) is applied to system, then the output of Laguerre functional model can track the set point curve without error (Chen, 1999)

$$\lim_{t\to\infty}[y_m(t) - r(t)] = 0 \tag{20}$$

Therefore, the steady-state error

$$e = \lim_{t\to\infty}[y(t) - r(t)] = \lim_{t\to\infty}[y(t) - y_m(t) + y_m(t) - r(t)]$$
$$= \lim_{t\to\infty}[y(t) - y_m(t)] + \lim_{t\to\infty}[y_m(t) - r(t)] = e_m$$

thus

$$\|e\|^2 \le \frac{J_m}{m(N_\varepsilon + 1)}.$$

□

5. Simulations and applications

5.1 Wiener-type non-linear system with uncertainties

$$X(k+1) = A_0 X(k) + b_0 u(k) \tag{21}$$

$$\eta(k-d) = C_0 X(k), y(k) = f[\eta(k)] = \eta^4(k) + 3\eta^3(k) \tag{22}$$

where

$$A_0 = \begin{bmatrix} 0.0579 & 0.8132 \\ 0.3529 & 0.0099 \end{bmatrix},\ b_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C_0 = [10]$$

system time-delay $d = 3$, and in the 500th epoch, d is changed into 5.

The set point curve is a square wave curve with amplitudes 20 and 10. Figure 1 shows the comparison of the performances of the following three different control algorithms. The output and control variable curves are shown in the left and right figures respectively.

A typical NMPC algorithm based on an NAARX (Henson, 1997) model, which is a special class of NARMAX models. Parameters: prediction horizon $P = 7$, control horizon $M = 5$, scalar non-linearities are set as polynomials, and the input and out memories are set to be 5 and 3, respectively.

A traditional non-linear NMPC (Parker and Doyle, 1998) based on a Laguerre–Volterra model. Parameters: $P = 7$, $M = 1$, $N = 10$, $\lambda = 0.98$, $P = 3$, $T = 2$.

Our proposed algorithm determined by (8) and (10). Parameters: Volterra functional series order $r_v = 2$, the other parameters are the same as algorithm b.

From the control performances based on our algorithm and on traditional one, we can conclude that, when controlling a wiener-type system, algorithm c can eliminate steady-state error while algorithm b cannot. In addition, compared with algorithm a, algorithm c has much smaller overshoots and shorter response times. Moreover, when the plant's time-delay is varying, the vibration intensity and the modulating time of algorithm c are much smaller than the algorithm b, which validates algorithm c's excellent adaptability for system time-delay. The control performances on a Hammerstein-type system can also validate the feasibility and superiority of algorithm c.

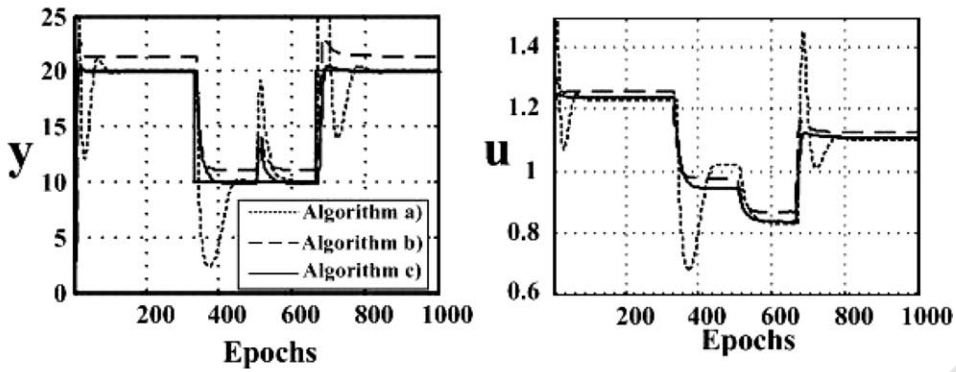

Figure 1 Control performances on Wiener-typed system

5.2 Temperature control of a simulating reactor with a cold water jacket

As Figure 2 shown, fluxes $Q_i, (i = 1, \cdots, 4)$, are adjusted by valves $R_i, (i = 1, \cdots, 4)$, respectively. The controlled variable is the temperature y of the inner tank, and the control variable is the current u of the electrically heated wire, which fulfils $4 \le u \le 20$ and $|\Delta u| \le 2$. The water levels of the inner tank and the jacket are h_1 and h_2, respectively. There are two PT thermal resistance sensors (WZP-270S-typed), whose accuracy is ± 0.1°C, to measure the temperatures of the tank's hot water and the jacket's cold water, respectively. Because of to its intrinsic mechanism, this system can be approximated by a Hammerstein-type non-linear system with long time-delay, hard input constraints and uncertainties.

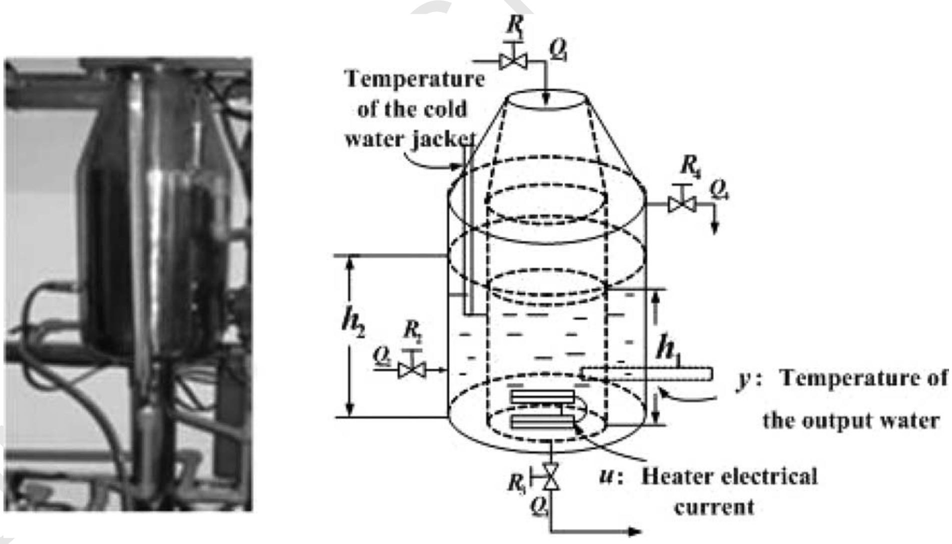

Figure 2 Control system of simulating reactor with cold water jacket

The controllable silicon's conduction angle is adjusted to heat up the heated wire. Q_2 and h_1 are initially set to be 60 L/h and 200 mm respectively. Control performances of our algorithm on this system are shown in Figure 3. In the upper figure, the solid curve is the system's output to trace the double-step of 60°C and 65°C, while the dashed curve is the system's output to trace the double-step of 32°C and 38°C. The corresponding control variable curves are shown in the lower figure. In the course of tracing 65°C, we purposely reduce the flow passing through the cold water jacket from 60 to 50 L/h in the 1300th second, which corresponds to a disturbance of the plant's characteristic. Control parameters are set as: , $N = 7$, $\lambda = 0.98$, $P = 2.5$, $T = 2$.

Experimental results show that the steady-state error is smaller than ± 0.3°C, the overshoots are smaller than ± 2°C, and the modulating times are lesson than 200 s as well. Therefore, for a physical Hammerstein-type non-linear system, with long time-delay, hard input constraints and uncertainties, our algorithm shows great adaptability and can effectively eliminate steady-state error.

6. Conclusions

The OFS model is very suitable for dealing with systems' varying time-delay, which widely exists in modern industrial engineering. Accordingly, combining it with a Volterra series, we constructed a non-linear OFS model as the internal model of our adaptive control algorithm to solve block-oriented non-linear systems with time-delay. Because the system matrix of the OFS model can be computed offline, a stable state feedback vector based on an OFS filter is used to design a stable control law. Added to an offset value, this state feedback control law can be used to track set point curves without error. In this way, an adaptive non-linear algorithm is designed for

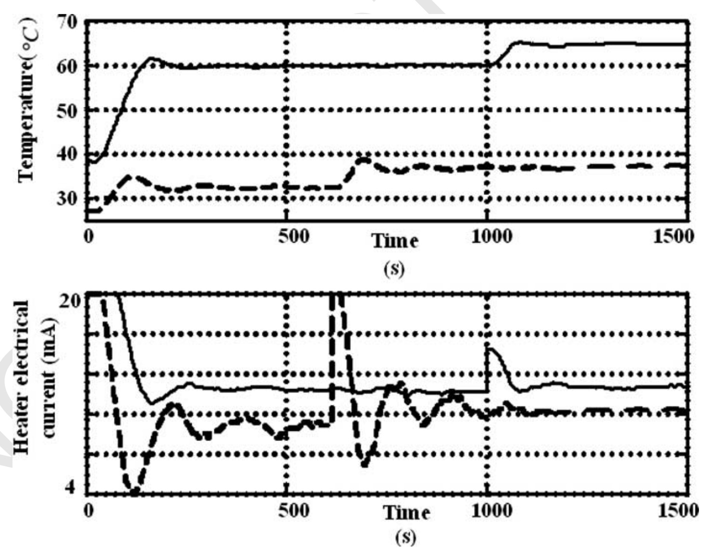

Figure 3 Temperature control performance of the proposed algorithm

block-oriented non-linear systems. To support this algorithm, we presented the theoretical analyses of stability, robustness and steady-state performance systemically.

Finally, simulations on a Wiener-type non-linear system, and experiments on a time-varying Hammerstein-type non-linear process (a chemical rector's temperature control system) are shown, which have long time-delay, hard input constraints and uncertainties. A large amount of control results validate the feasibility and superiority of our proposed algorithm.

Acknowledgements

This paper is funded by the '985' High Level University Construction Fund of China in 2003 (KY2701), and Key Scientific Research Fund of Hefei City of China in 2002 (He Ke 2002-45).

References

Agamennoni, O., **Paolini, E.** and **Desages, A.** 1990: On robust stability analysis of a control system using Laguerre series. *Automatica* 28, 815–18.

Campello, R.J.G.B., Favier, G. and **Wagner, C.D.A.** 2004: Optimal expansions of discrete-time Volterra models using Laguerre functions. *Automatica* 40, 815–22.

Chen, C.T. 1999: *Linear system theory and design.* Oxford University Press, 269–80.

Dumont, G.A., Zervos, C.C. and **Pageau, G.L**. 1990: Laguerre-based adaptive control of pH in an industrial bleach plant extraction stage. *Automatica* 26, 781–87.

Gómez, J.C. and Baeyens, E. 2004: Identification of block-oriented non-linear systems using orthonormal bases. *Journal of Process Control* 14, 685–97.

Henson, M.A. 1997: *Non-linear process control.* Prentice Hall PTR, 30–37.

Mäkilä, P.M. 1990: Approximation of stable systems by Laguerre filters. *Automatica* 26, 332–45.

Ljung, L. 1999: *System identification: theory for the user*, 2nd edition. Prentice-Hall.

Oliveira, G.H.C., Wagner, C.D.A, Favier, G., and **Dumont, G.A.** 2000: Constrained robust predictive controller for uncertain processes modelled by orthonormal series functions. *Automatica* 36, 563–71.

Parker, R.S. 2002: Non-linear model predictive control of a continuous bioreactor using approximate data-driven models, *Proceedings of American Control Conference*, Anchorage, 8–10 May 2885–90.

Parker, R.S. and **Doyle, F.J.** 1998: Non-linear model predictive control of a continuous bioreactor at near-optimum conditions, *Proceedings of American Control Conference*, Philadelphia, 24–26 June, 2549–53.

Wahlberg, B. and **Mäkilä, P.M.** 1996: On approximation of stable linear dynamical systems using Laguerre and Kautz Functions. *Automatica* 32, 693–708.

Wang L.P. 2004: Discrete model predictive controller design using Laguerre functions, *Journal of Process Control* 14, 131–42.

Yu L. 2002: *Robust control- LMI processing method*. Tsinghua University Press, 86–88.

Zervos, C.C. and **Dumont, G.Y.** 1988: Deterministic adaptive control based on Laguerre series representation, *International Journal of Control* 48, 2333–59.

Zhang, H.T., Chen, Z.H. and **Li, S.F**. 2004a: Multi-variable system's Laguerre predictive control algorithm and the improvement for this algorithm. *Journal of Control Engineering of China* 11, 55–58.

Zhang, H.T., Chen, Z.H. and **Qin T.** 2004b: An Laguerre function model adaptive predictive control based on a chaos neural network for heavy oil distillation column. *Journal of Information and Control of China* 33, 13–17.

Zhang, H.T., Xu, Z.F. and **Li, S.F.** 2002: An adaptive predictive control based on Laguerre function model for diffusion furnace. *Journal of System Engineering and Electronics of China* 24, 54–57.