A simple rotor current observer with an arbitrary rate of convergence for the Brushless Doubly-Fed (Induction) Machine (BDFM)

P C Roberts (pcr20@cam.ac.uk), J M Maciejowski, R A McMahon, T J Flack
Engineering Dept., University of Cambridge,
Trumpington St., Cambridge CB2 1PZ, United Kingdom

I. Abstract

The BDFM shows economic promise as a variable speed drive or generator. One practical obstacle to commercial exploitation is the presence of operating speeds at which BDFM synchronous action cannot be maintained under open loop conditions. In [7] the authors proposed a control strategy using feedback linearization, requiring full state feedback. In this paper we present a simple rotor current observer with an arbitrary rate of convergence. Simulation results are given to validate the performance.

II. Introduction

Research into modern applications for the BDFM has accelerated over the last decade or so. As a motor the machine has attracted this attention due its potential to rival the induction machine as a variable speed drive (VSD), and as a generator where the prime mover speed can be variable, such as in a wind turbine. These advantages are realised through a reduced capital installation cost, due to a fractional inverter power requirement (as compared to that for an induction machine VSD (or generator)), and further, the advantages of brushless operation are maintained. This is of particular interest for off-shore wind turbine applications where maintenance costs are high.

Although the BDFM is similar in construction to an induction machine its operation is very different [9]. The machine has more in common with a synchronous machine, therefore induction motor control strategies are not directly applicable.

During investigations into the use of the BDFM as a VSD Spée et al [4] noted that there are regions of shaft speeds for which the machine shaft speed is very lightly damped or even unstable. The authors have noted similar stability problems in both simulation and experiment with a prototype machine [7].

High performance control of the machine requires knowledge of the rotor currents. However measurement of rotor currents would be prohibitively expensive in all but the largest of applications, and further would increase the maintenance requirements for the machine.

Control strategies proposed in [2], [10], [11] offer methods of estimating the rotor currents, however the rate of convergence cannot be controlled in these cases, and is dependent on the dynamics of the particular machine.

Extending the work of Martin and Rouchon [5], we present a rotor current observer for the BDFM. The proposed rotor current observer error can be made to converge to zero at an arbitrarily fast rate, while maintaining exponential stability of observer system. The observer requires measurement of the machine's stator currents, and shaft speed. In practice the rate of convergence will be limited by model accuracy and the level of sensor noise.

The observer exploits the form of the BDFM equations allowing derivation of an explicit solution for the observer error, where the rate of convergence of the error can be set by the designer. The advantage of this approach over a Kalman filter is one of simplicity, a Kalman filter for a time-varying system requires the state covariance estimate to be computed in real time.

III. Principal Symbols

X_{s1}, X_{s2}, X_r:	indicating a stator 1, 2, or rotor quantity X
X_d, X_q:	indicating *direct*, *quadrature* X
$X^{-1}, X^{\dagger}, X^{*}$:	indicating inverse, pseudo-inverse, complex-conjugate transpose of matrix X
X_{xy}:	element of matrix X at x^{th} row, and y^{th} column
$\hat{X}$:	estimated value of X
$\Re\{X\}, \Im\{X\}$:	denotes real, imaginary part of X
$\mathbf{X}$:	vector quantity $\mathbf{X}$
R, M, L:	resistance, mutual, self inductance
V, I:	instantaneous voltage, current
ϕ_{s1}, ϕ_{s2}:	stator winding 1,2 phase offset with rotor
P_{s1},P_{s2}:	stator winding pole pairs
ω_r:	BDFM rotational shaft speed (time-varying)
θ_r:	BDFM rotational shaft position (time-varying)

IV. Dynamic BDFM model

A reasonable dynamic model of the BDFM is given by Boger et al [1]. This model uses a standard *direct-*

quadrature (*d-q*) [3] transformation to represent the machine in the rotor reference frame. To apply the transformation, the full coupled circuit model (see e.g. [8]) is reduced in order by truncation of a Fourier series, and then transformed.

The electrical equations of the Boger model are given as follows. The state vector is the current vector, $\mathbf{I_{dq}}$:

$$\mathbf{V_{dq}} = M_{dq}\frac{d\mathbf{I_{dq}}}{dt} + (R_{dq} + \omega_r Z_{dq})\mathbf{I_{dq}}$$

where Z_{dq} is a spatial derivative of the mutual inductance matrix transformed into d-q axis coordinates.

$$\begin{bmatrix} V_{d1} \\ V_{q1} \\ V_{d2} \\ V_{q2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} L_{s1} & 0 & 0 & 0 & M_1\cos\phi_{s1} & M_1\sin\phi_{s1} \\ 0 & L_{s1} & 0 & 0 & -M_1\sin\phi_{s1} & M_1\cos\phi_{s1} \\ 0 & 0 & L_{s2} & 0 & M_2\cos\phi_{s2} & -M_2\sin\phi_{s2} \\ 0 & 0 & 0 & L_{s2} & -M_2\sin\phi_{s2} & -M_2\cos\phi_{s2} \\ M_1\cos\phi_{s1} & -M_1\sin\phi_{s1} & M_2\cos\phi_{s2} & -M_2\sin\phi_{s2} & L_r & 0 \\ M_1\sin\phi_{s1} & M_1\cos\phi_{s1} & -M_2\sin\phi_{s2} & -M_2\cos\phi_{s2} & 0 & L_r \end{bmatrix} \frac{d}{dt}\begin{bmatrix} I_{d1} \\ I_{q1} \\ I_{d2} \\ I_{q2} \\ I_{dr} \\ I_{qr} \end{bmatrix} +$$
$$\omega_r \begin{bmatrix} R_{s1}/\omega_r & P_{s1}L_{s1} & 0 & 0 & -P_{s1}M_1\sin\phi_{s1} & P_{s1}M_1\cos\phi_{s1} \\ -P_{s1}L_{s1} & R_{s1} & 0 & 0 & -P_{s1}M_1\cos\phi_{s1} & -P_{s1}M_1\sin\phi_{s1} \\ 0 & 0 & R_{s2}/\omega_r & P_{s2}L_{s2} & -P_{s2}M_2\sin\phi_{s2} & -P_{s2}M_2\cos\phi_{s2} \\ 0 & 0 & -P_{s2}L_{s2} & R_{s2}/\omega_r & -P_{s2}M_2\cos\phi_{s2} & P_{s2}M_2\sin\phi_{s2} \\ 0 & 0 & 0 & 0 & R_r/\omega_r & 0 \\ 0 & 0 & 0 & 0 & 0 & R_r/\omega_r \end{bmatrix} \begin{bmatrix} I_{d1} \\ I_{q1} \\ I_{d2} \\ I_{q2} \\ I_{dr} \\ I_{qr} \end{bmatrix} \quad (1)$$

Due to the structure of the model, the voltages and currents can be represented using complex phasors, for example:

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \sqrt{2}\begin{bmatrix} \Re V_c \\ \Im V_c \end{bmatrix}$$

Using the following transformation matrix the model can be converted into complex form:

$$\mathbf{T_{complex}} \triangleq \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & i \end{bmatrix}$$

$$i \triangleq \sqrt{-1}$$

Note that $\mathbf{T}^*_{complex} = \mathbf{T}^\dagger_{complex}$.

$$M_c \triangleq \mathbf{T_{complex}} M_{dq} \mathbf{T^*_{complex}}$$
$$R_c \triangleq \mathbf{T_{complex}} R_{dq} \mathbf{T^*_{complex}}$$
$$Z_c \triangleq \mathbf{T_{complex}} Z_{dq} \mathbf{T^*_{complex}}$$

We can now put the model in state-space form as follows:

$$\mathbf{V_c} = M_c\frac{d\mathbf{I_c}}{dt} + (R_c + \omega_r Z_c)\mathbf{I_c}$$

$$\begin{bmatrix} V_{s1} \\ V_{s2} \\ 0 \end{bmatrix} = \begin{bmatrix} L_{s1} & 0 & M_1 e^{-i\phi_{s1}} \\ 0 & L_{s2} & M_2 e^{i\phi_{s2}} \\ M_1 e^{i\phi_{s1}} & M_2 e^{-i\phi_{s2}} & L_r \end{bmatrix} \frac{d}{dt}\begin{bmatrix} I_{s1} \\ I_{s2} \\ I_r \end{bmatrix} +$$
$$\begin{bmatrix} R_{s1} - i\omega_r P_{s1} L_{s1} & 0 & -\omega_r P_{s1} M_1 i e^{-i\phi_{s1}} \\ 0 & R_{s2} + i\omega_r P_{s2} L_{s2} & \omega_r P_{s2} M_2 i e^{i\phi_{s2}} \\ 0 & 0 & R_r \end{bmatrix} \begin{bmatrix} I_{s1} \\ I_{s2} \\ I_r \end{bmatrix} \quad (2)$$

Defining:

$$A \triangleq -M_c^{-1}\left(R_c + \omega_r Z_c\right)$$

gives:

$$A = KA' = K\begin{bmatrix} A'_{11} & A'_{12} & M_1 e^{-i\phi_{s1}}(-iM_2^2\omega_r P_{s1} - iM_2^2\omega_r P_{s2} + iL_{s2}L_r\omega_r P_{s1} + R_r L_{s2}) \\ A'_{21} & A'_{22} & M_2 e^{i\phi_{s2}}(-iL_{s1}L_r\omega_r P_{s2} + L_{s1}R_r + iM_1^2\omega_r P_c + iM_1^2\omega_r P_{s1}) \\ A'_{31} & A'_{32} & (-R_r L_{s1} L_{s2} + iM_2^2 L_{s1}\omega_r P_{s2} - iM_1^2 L_{s2}\omega_r P_{s1}) \end{bmatrix} \quad (3)$$

where:

$$K = \frac{1}{L_{s1}L_{s2}L_r - L_{s1}M_2^2 - L_{s2}M_1^2}.$$

and A'_{ij} are of similar form to the terms in the 3rd column, but the details are not significant here.

Then:

$$\frac{d}{dt}\begin{bmatrix} I_{s1} \\ I_{s2} \\ I_r \end{bmatrix} = A\begin{bmatrix} I_{s1} \\ I_{s2} \\ I_r \end{bmatrix} + M_c^{-1}\begin{bmatrix} V_{s1} \\ V_{s2} \\ 0 \end{bmatrix} \quad (4)$$

in which both the variables and coefficients are complex.

V. An Observer

The following equation is an observer for the rotor currents in the BDFM. The poles of the error evolution can be set arbitrarily, so the observer can be made to converge at an arbitrarily fast rate.

$$\frac{d}{dt}\begin{bmatrix} \hat{I_{s1}} \\ \hat{I_{s2}} \\ \hat{I_r} \end{bmatrix} = A\begin{bmatrix} \hat{I_{s1}} \\ \hat{I_{s2}} \\ \hat{I_r} \end{bmatrix} +$$
$$\begin{bmatrix} \frac{Q}{R_r}\zeta_1 - A_{11} & -A_{12} \\ -A_{21} & \frac{Q}{R_r}\zeta_1 - A_{22} \\ \frac{Q}{R_r}\zeta_2\frac{P_{s2}}{P_{s1}+P_{s2}}\gamma - A_{31} & \frac{Q}{R_r}\zeta_2\frac{P_{s1}}{P_{s1}+P_{s2}} - A_{31} \end{bmatrix}\begin{bmatrix} \hat{I_{s1}} - I_{s1} \\ \hat{I_{s2}} - I_{s2} \end{bmatrix}$$
$$+ M_c{}^{-1}\begin{bmatrix} V_{s1} \\ V_{s2} \\ 0 \end{bmatrix} \quad (5)$$

where:

$$\gamma \triangleq \frac{L_{s1}M_2 e^{i(\phi_{s2}+\phi_{s1})}}{L_{s2}M_1}$$

and:

$$Q \triangleq -\frac{1}{R_r} + i\omega_r\frac{\beta_r}{R_r} \quad (6)$$

and:

$$\beta_r \triangleq \frac{M_2^2}{R_r L_{s2}}P_{s2} - \frac{M_1^2}{R_r L_{s1}}P_{s1} \quad (7)$$

and ζ_1, ζ_2 are gains chosen by the designer.

Which can also be written as:

$$\frac{d}{dt}\begin{bmatrix}\hat{I_{s1}}\\ \hat{I_{s2}}\\ \hat{I}_r\end{bmatrix} = A\begin{bmatrix}I_{s1}\\ I_{s2}\\ \hat{I}_r\end{bmatrix} + \begin{bmatrix}\frac{Q}{R_r}\zeta_1 & 0\\ 0 & \frac{Q}{R_r}\zeta_1\\ \frac{Q}{R_r}\zeta_2\frac{P_{s2}}{P_{s1}+P_{s2}}\gamma & \frac{Q}{R_r}\zeta_2\frac{P_{s1}}{P_{s1}+P_{s2}}\end{bmatrix}\begin{bmatrix}\hat{I_{s1}}-I_{s1}\\ \hat{I_{s2}}-I_{s2}\end{bmatrix} + M_c{}^{-1}\begin{bmatrix}V_{s1}\\ V_{s2}\\ 0\end{bmatrix} \quad (8)$$

in which the measured stator currents appear. Notice that the observed system, and the observer feedback itself are time-varying, and as such special treatment is required.

A. Proof

Let the error between the actual and estimated states be, $\Delta = I - \hat{I}$. Then from (8) and (4):

$$\frac{d}{dt}\begin{bmatrix}\Delta_{s1}\\ \Delta_{s2}\\ \Delta_r\end{bmatrix} = \begin{bmatrix}\frac{Q}{R_r}\zeta_1 & 0 & A_{13}\\ 0 & \frac{Q}{R_r}\zeta_1 & A_{23}\\ \frac{Q}{R_r}\zeta_2\frac{P_{s2}}{P_{s1}+P_{s2}}\gamma & \frac{Q}{R_r}\zeta_2\frac{P_{s1}}{P_{s1}+P_{s2}} & A_{33}\end{bmatrix}\begin{bmatrix}\Delta_{s1}\\ \Delta_{s2}\\ \Delta_r\end{bmatrix} \quad (9)$$

Notice that:

$$\frac{d}{dt}\begin{bmatrix}\frac{P_{s2}}{P_{s1}+P_{s2}}\gamma\Delta_{s1}\\ \frac{P_{s1}}{P_{s1}+P_{s2}}\Delta_{s2}\\ \Delta_r\end{bmatrix} =$$
$$\begin{bmatrix}\frac{Q}{R_r}\zeta_1\frac{P_{s2}}{P_{s1}+P_{s2}}\gamma & 0 & \frac{P_{s2}}{P_{s1}+P_{s2}}\gamma A_{13}\\ 0 & \frac{Q}{R_r}\zeta_1\frac{P_{s1}}{P_{s1}+P_{s2}} & \frac{P_{s1}}{P_{s1}+P_{s2}}A_{23}\\ \frac{Q}{R_r}\zeta_2\frac{P_{s2}}{P_{s1}+P_{s2}}\gamma & \frac{Q}{R_r}\zeta_2\frac{P_{s1}}{P_{s1}+P_{s2}} & A_{33}\end{bmatrix}\begin{bmatrix}\Delta_{s1}\\ \Delta_{s2}\\ \Delta_r\end{bmatrix} \quad (10)$$

Let:

$$\Delta_{sz} \triangleq \frac{P_{s2}}{P_{s1}+P_{s2}}\gamma\Delta_{s1} + \frac{P_{s1}}{P_{s1}+P_{s2}}\Delta_{s2}$$
$$\Rightarrow \frac{d\Delta_{sz}}{dt} = \frac{P_{s2}}{P_{s1}+P_{s2}}\gamma\frac{d\Delta_{s1}}{dt} + \frac{P_{s1}}{P_{s1}+P_{s2}}\frac{d\Delta_{s2}}{dt}$$

Adding the first row of (10) to the second gives:

$$\Rightarrow \frac{d}{dt}\begin{bmatrix}\Delta_{sz}\\ \Delta_r\end{bmatrix} = \begin{bmatrix}\frac{P_{s2}}{P_{s1}+P_{s2}}\gamma\frac{d\Delta_{s1}}{dt} + \frac{P_{s1}}{P_{s1}+P_{s2}}\frac{d\Delta_{s2}}{dt}\\ \frac{d\Delta_r}{dt}\end{bmatrix} =$$
$$\begin{bmatrix}\frac{Q}{R_r}\zeta_1\frac{P_{s2}}{P_{s1}+P_{s2}}\gamma & \frac{Q}{R_r}\zeta_1\frac{P_{s1}}{P_{s1}+P_{s2}} & A_{13}\frac{P_{s2}}{P_{s1}+P_{s2}}\gamma + A_{23}\frac{P_{s1}}{P_{s1}+P_{s2}}\\ \frac{Q}{R_r}\zeta_2\frac{P_{s2}}{P_{s1}+P_{s2}}\gamma & \frac{Q}{R_r}\zeta_2\frac{P_{s1}}{P_{s1}+P_{s2}} & A_{33}\end{bmatrix}\begin{bmatrix}\Delta_{s1}\\ \Delta_{s2}\\ \Delta_r\end{bmatrix}$$
$$= \begin{bmatrix}\frac{Q}{R_r}\zeta_1 & A_{13}\frac{P_{s2}}{P_{s1}+P_{s2}}\gamma + A_{23}\frac{P_{s1}}{P_{s1}+P_{s2}}\\ \frac{Q}{R_r}\zeta_2 & A_{33}\end{bmatrix}\begin{bmatrix}\Delta_{sz}\\ \Delta_r\end{bmatrix} \quad (11)$$

Defining:

$$\mathbf{\Delta} \triangleq \begin{bmatrix}\Delta_{sz}\\ \Delta_r\end{bmatrix}$$

From appendix IX-A, and equation (3), with some manipulation:

$$\frac{d\mathbf{\Delta}}{dt} = \frac{Q}{R_r}\begin{bmatrix}\zeta_1 & -KL_{s1}R_rM_2e^{i\phi_{s2}}\\ \zeta_2 & KL_{s1}L_{s2}R_r\end{bmatrix}\mathbf{\Delta} \quad (12)$$

And notice that, by choice, all the time-varying elements are now factorized into the scalar $\frac{Q}{R_r}$ term.

Following the method of Martin and Rouchon [5], we define a complex change of time, τ, given by:

$$\tau \triangleq -t + i\int_0^t \omega_r(\eta)\beta_r d\eta = -t + i\theta_r(t)\beta_r$$

since we can choose $\theta_r(0) = 0$.

$$\Rightarrow \frac{d\tau}{dt} = -1 + i\omega_r\beta_r = \frac{Q}{R_r}$$

We can then define a new state vector $\tilde{\mathbf{\Delta}}(\tau)$:

$$\tilde{\mathbf{\Delta}}(\tau) \triangleq \mathbf{\Delta}(t), \forall t$$

Substituting this back into equation (12) gives:

$$\frac{d\tilde{\mathbf{\Delta}}}{dt} = \frac{Q}{R_r}\begin{bmatrix}\zeta_1 & -KL_{s1}R_rM_2e^{i\phi_{s2}}\\ \zeta_2 & KL_{s1}L_{s2}R_r\end{bmatrix}\tilde{\mathbf{\Delta}}$$
$$\Rightarrow \frac{d\tilde{\mathbf{\Delta}}}{d\tau}\frac{d\tau}{dt} = \frac{Q}{R_r}\begin{bmatrix}\zeta_1 & -KL_{s1}R_rM_2e^{i\phi_{s2}}\\ \zeta_2 & KL_{s1}L_{s2}R_r\end{bmatrix}\tilde{\mathbf{\Delta}}$$
$$\Rightarrow \frac{d\tilde{\mathbf{\Delta}}}{d\tau}\frac{Q}{R_r} = \frac{Q}{R_r}\begin{bmatrix}\zeta_1 & -KL_{s1}R_rM_2e^{i\phi_{s2}}\\ \zeta_2 & KL_{s1}L_{s2}R_r\end{bmatrix}\tilde{\mathbf{\Delta}}$$
$$\Rightarrow \frac{d\tilde{\mathbf{\Delta}}}{d\tau} = \begin{bmatrix}\zeta_1 & -KL_{s1}R_rM_2e^{i\phi_{s2}}\\ \zeta_2 & KL_{s1}L_{s2}R_r\end{bmatrix}\tilde{\mathbf{\Delta}} \quad (13)$$

Notice that equation (13) is LTI, and can therefore be solved explicitly, as long as ζ_1 and ζ_2 are constant. ζ_1 and ζ_2 can be chosen to adjust the eigenvalues of the system arbitrarily. In order that we may later ensure exponential stability of the system, we assume that:

$$\forall t > 0 \ \mathrm{sign}(\omega_r(t)) = C,\ C \text{ constant.} \quad (14)$$

This condition on $\omega_r(t)$ may be relaxed for diagonalisable systems, however the derivation, and corresponding conditions for stability are not included in this paper.

Noting that $\tilde{\mathbf{\Delta}}(0) = \mathbf{\Delta}(0)$:

$$\tilde{\mathbf{\Delta}}(\tau) = \exp\left(\begin{bmatrix}\zeta_1 & -KL_{s1}R_rM_2e^{i\phi_{s2}}\\ \zeta_2 & KL_{s1}L_{s2}R_r\end{bmatrix}\tau\right)\tilde{\mathbf{\Delta}}(0)$$
$$\Rightarrow \mathbf{\Delta}(t) = \exp\left(\begin{bmatrix}\zeta_1 & -KL_{s1}R_rM_2e^{i\phi_{s2}}\\ \zeta_2 & KL_{s1}L_{s2}R_r\end{bmatrix}(-t + i\theta_r(t)\beta_r)\right)\mathbf{\Delta}(0) \quad (15)$$

Equation (15) shows the evolution of the error with time. In general the eigenvalues of equation (13) can be expressed as shown in equation (16) below (note they are not necessarily a conjugate pair as the matrix in equation (13) will, in general, have complex elements). Defining:

$$\lambda_1 \triangleq \lambda_{1a} + i\lambda_{1b}$$
$$\lambda_2 \triangleq \lambda_{2a} + i\lambda_{2b} \quad (16)$$

and:

$$k_1 \triangleq -KL_{s1}R_rM_2e^{i\phi_{s2}} \quad (17)$$
$$k_2 \triangleq KL_{s1}L_{s2}R_r. \quad (18)$$

From equation (13) the eigenvalues can be explicitly computed:

$$(\zeta_1-\lambda)(k_2-\lambda)-k_1\zeta_2=0$$
$$\Rightarrow \lambda^2-(\zeta_1+k_2)\lambda+(\zeta_1k_2-\zeta_2k_1)=0$$
$$\Rightarrow \lambda=\frac{(\zeta_1+k_2)\pm\sqrt{(\zeta_1+k_2)^2-4(\zeta_1k_2-\zeta_2k_1)}}{2} \quad (19)$$

therefore ζ_1 and ζ_2 can be computed, for a desired λ_1,λ_2:

$$\zeta_1 = \lambda_1+\lambda_2-k_2$$
$$\zeta_2 = \frac{(\lambda_2-k_2)(k_2-\lambda_1)}{k_1}. \quad (20)$$

So the solution must be bounded by linear combinations of:

$$e^{(\lambda_{1a}+i\lambda_{1b})(-t+i\beta_r\theta_r(t))} = e^{-(\lambda_{1a}t+\beta_r\lambda_{1b}\theta_r(t))}e^{(i\theta_r(t)\lambda_{1a}-i\lambda_{1b}t)} \quad (21)$$
$$e^{(\lambda_{2a}+i\lambda_{2b})(-t+i\beta_r\theta_r(t))} = e^{-(\lambda_{2a}t+\beta_r\lambda_{2b}\theta_r(t))}e^{(i\theta_r(t)\lambda_{2a}-i\lambda_{2b}t)} \quad (22)$$

So the solutions can be bounded by $e^{(-\lambda_{1a}t-\beta_r\lambda_{1b}\theta_r(t))}$ and $e^{(-\lambda_{2a}t-\beta_r\lambda_{2b}\theta_r(t))}$. Therefore ζ_1 and ζ_2 must be chosen such that these terms decay away. This can be satisfied by ensuring that:

$$\lambda_{1a},\lambda_{2a},\lambda_{1b}\text{sign}(\omega_r)\text{sign}(\beta_r),\lambda_{2b}\text{sign}(\beta_r)\text{sign}(\omega_r)>0, \quad (23)$$

but may be satisfied in other ways. Notice that if the eigenvalues were made entirely real (so $\lambda_{1b},\lambda_{2b}=0$), then the potentially advantageous convergence of these terms at significant rotor speeds is lost. For, an imaginary eigenvalue of a set magnitude will converge faster than a real one at higher motor speeds.

Further, let us now assume, that we have chosen our eigenvalues such that Δ_r decays exponentially. Then as long as ω_r is bounded then $A_{13}\Delta_r$ and $A_{23}\Delta_r$ also decay exponentially. From equation (9) we can write:

$$\frac{d}{dt}\begin{bmatrix}\Delta_{s1}\\ \Delta_{s2}\end{bmatrix}=\begin{bmatrix}\frac{Q}{R_r}\zeta_1 & 0\\ 0 & \frac{Q}{R_r}\zeta_1\end{bmatrix}\begin{bmatrix}\Delta_{s1}\\ \Delta_{s2}\end{bmatrix}+\begin{bmatrix}A_{13}\Delta_r\\ A_{23}\Delta_r\end{bmatrix} \quad (24)$$

Notice that equation (24) is really two similar scalar differential equations of the form:

$$\frac{d\Delta_{s_1}}{dt}=\frac{Q}{R_r}\zeta_1\Delta_{s_1}+f(t) \quad (25)$$

Equation (25) can be solved using an *integrating factor*, to give:

$$\Delta_{s_1}=\int_0^t e^{\left(\int_0^t \frac{Q}{R_r}\zeta_1 dt-\int_0^\eta \frac{Q}{R_r}\zeta_1 d\eta\right)}f(\eta)d\eta+ \Delta_{s_1}(0)e^{\left(\int_0^t \frac{Q}{R_r}\zeta_1 dt\right)} \quad \forall\eta:0\le\eta<t \quad (26)$$

Defining:

$$\alpha(t)\triangleq\Re\left\{\int_0^t \frac{Q}{R_r}\zeta_1 dt\right\}=-\Re\{\zeta_1\}t-\beta_r\Im\{\zeta_1\}\theta_r(t)$$
$$=-(\lambda_{1a}+\lambda_{2a}-k_2)t-\beta_r(\lambda_{1b}+\lambda_{2b})\theta_r(t) \quad (27)$$

From consideration of the initial condition response, a *necessary* condition for exponential decay of Δ_{s_1} is that:

$$\forall t\ge 0\,,\exists K_1>0:$$
$$|\Delta_{s_1}(0)|\left|e^{\left(\int_0^t \frac{Q}{R_r}\zeta_1 dt\right)}\right|=|\Delta_{s_1}(0)|\,e^{\alpha(t)}\le|\Delta_{s_1}(0)|\,e^{-K_1t} \quad (28)$$

From (27) this is satisfied if:

$$\lambda_{1a}+\lambda_{2a}>k_2,\text{ and}$$
$$(\lambda_{1b}+\lambda_{2b})\text{sign}(\beta_r)\text{sign}(\omega_r)>0. \quad (29)$$

From (14) $\theta_r(t)$ is monotonic. If we now assume the conditions expressed in (29) and (23) are satisfied, then from equation (27):

$$-\alpha(t)\ge-\alpha(\eta)\quad\forall\eta:0\le\eta\le t. \quad (30)$$

Therefore:

$$\forall t\ge 0,\left|e^{\left(\int_0^t \frac{Q}{R_r}\zeta_1 dt-\int_0^\eta \frac{Q}{R_r}\zeta_1 d\eta\right)}\right| = e^{(\alpha(t)-\alpha(\eta))}\le 1. \quad (31)$$

Recall that $|f(t)|$ is exponentially decaying, therefore we can say:

$$\forall t>0\,,\exists K_2,K_3>0:|f(t)|\le K_2e^{-K_3t}. \quad (32)$$

From (26), (29), (31), (32), and recalling $\left|\int f(z)dt\right|\le\int|f(z)|dt, z\in\mathbb{C}$:

$$|\Delta_{s_1}|\le\int_0^t K_2e^{-K_3\eta}d\eta+|\Delta_{s_1}(0)|e^{\alpha(t)} \quad (33)$$

which is guaranteed to converge.

From (25) it can be shown that:

$$\frac{d|\Delta_{s_1}|}{dt}=\Re\{\frac{Q}{R_r}\zeta_1\}|\Delta_{s_1}|+ \frac{\Re\{\Delta_{s_1}\}\Re\{f(t)\}+\Im\{\Delta_{s_1}\}\Im\{f(t)\}}{|\Delta_{s_1}|} \quad (34)$$

By choice we made $\Re\{\frac{Q}{R_r}\zeta_1\}<K,\forall t>0$, therefore, from (33):

$$\Re\{\frac{Q}{R_r}\zeta_1\}|\Delta_{s_1}|<0,\forall t>0 \quad (35)$$

$|f(t)|$ decays exponentially, the real and imaginary parts of $f(t)$ decay exponentially; and note that $\frac{|\Re\{z\}|}{|z|}\le 1,\forall z\in$

$\mathbb{C}$ (and similarly for the imaginary part). Therefore for any $\epsilon > 0$:

$$\exists t_0 > 0 : \\ -\epsilon < \frac{\Re\{\Delta_{s_1}\}\Re\{f(t)\} + \Im\{\Delta_{s_1}\}\Im\{f(t)\}}{|\Delta_{s_1}|} < \epsilon, \\ \forall t > t_0 \quad (36)$$

and hence:

$$\frac{d|\Delta_{s_1}|}{dt} < \Re\{\frac{Q}{R_r}\zeta_1\}|\Delta_{s_1}| + \epsilon, \forall t > t_0 \quad (37)$$

thus, $|\Delta_{s_1}|$ is guaranteed to decrease until $\Re\{\frac{Q}{R_r}\zeta_1\}|\Delta_{s_1}| = \epsilon$, as ϵ can be made arbitrarily small $|\Delta_{s_1}|$ must converge to zero.

Therefore the condition given in (29) for exponential convergence of Δ_{s_1}, is sufficient. It follows from (24), using a similar argument, that the same condition guarantees that Δ_{s_2} decay exponentially.

So we can ensure $\Delta_{s1}, \Delta_{s2} \,\&\, \Delta_r$ all to converge to zero, if $\lambda_{1a}, \lambda_{2a}, \lambda_{1b}\text{sign}(\omega_r)\text{sign}(\beta_r), \lambda_{2b}\text{sign}(\beta_r)\text{sign}(\omega_r)$ are strictly positive and $\lambda_{1a} + \lambda_{2a} > k_2$. As before, note that this is not a necessary condition.

VI. Example Observer Design

To illustrate this observer design technique, it will applied to a model of our BDFM. Physical machine data is available in the appendix.

From the machine data we calculate, $\beta_r = 0.02149$. We wish to design an observer for rotational speeds above 50 rad s^{-1}. Our objective is to ensure that the slowest mode of the error system has a time constant of no greater than $\frac{1}{40}$ s.

We want to ensure that the stator current errors also converge to zero, therefore, $\Re\lambda_1 + \Re\lambda_2 > k_2$. From the machine data $k_2 = 40.51$, therefore $\Re\lambda_1 + \Re\lambda_2 > 40.51$. If we assume $\Re\lambda_1 \geq 20$, to give some convergence at slower speeds, then $\Re\lambda_2 > 20.51$.

At 50 rad s^{-1}, $\omega_r\beta_r = 1.07$, therefore $\Im\lambda_{1,2} \geq (\frac{40}{1.07} - 20)$, from (21), (22).

These conditions can be satisfied by choosing $\lambda_1 = 20 + 20i, \lambda_2 = 25 + 20i$.

Figure 1 shows the response of the direct-axis rotor current to a step disturbance on the state of the observer at $t = 2\,$s of:

$$\begin{bmatrix} 28 \\ 35 \\ 707 \end{bmatrix} \text{amps.}$$

These values correspond to roughly 30% of the current amplitudes at this point in the simulation.

Figure 2 shows the envelope bounding the error decay computed from equation (19), and the evolving rotor current errors, calculated from the simulation.

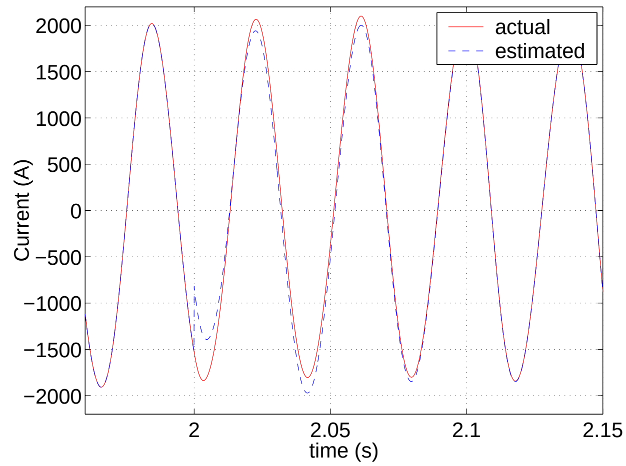

Fig. 1. d-axis estimated and actual rotor current step response, at around 75rads^{-1}

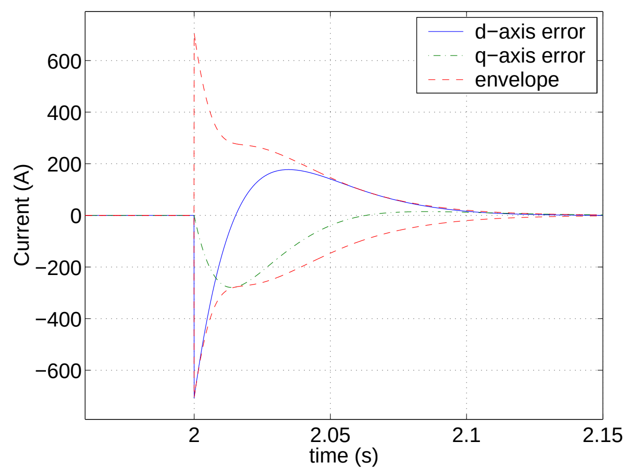

Fig. 2. d and q-axis observer error evolution from step response, at around 75rads^{-1}

VII. Conclusions and Further Work

We have presented an observer for the rotor current states of the BDFM. The observer has been shown to have a rate of convergence which can be set by the designer arbitrarily, if certain mild conditions hold.

Work is currently in progress implementing the observer on our prototype BDFM machine, and investigating the robustness of the observer to noise and modelling errors. The observer output is compared with the actual rotor bar currents measured using Bluetooth technology [6].

This work is being done in the context of devising robust controllers for the BDFM.

VIII. Acknowledgements

The UK Engineering and Physical Sciences Research Council (EPSRC) is thanked for its financial support through a research studentship. FKI Energy Technology (Laurence, Scott & Electromotors Ltd.) is thanked for assistance in supplying and fabricating the BDFM. Semikron

UK Ltd. is thanked for their provision of a high power inverter, and Cambridge Silicon Radio for supplying Bluetooth modules and software.

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IX. Appendix

TABLE I

TEST MACHINE DATA

L_{s1}	0.1022	L_{s2}	0.0931	L_r	7.3563×10^{-5}
P_{s1}	2	ϕ_{s1}	2.0508	M_1	2.1723×10^{-3}
P_{s2}	4	ϕ_{s2}	3.0544	M_2	1.4906×10^{-3}
R_{s1}	0.3190	R_{s2}	0.7000	R_r	1.4268×10^{-4}

A. $\frac{Q}{R_r}$ Factor

To show that a factor of $\frac{Q}{R_r}$ is present in $\frac{P_{s2}}{P_{s1}+P_{s2}}\frac{L_{s1}M_2e^{i\phi_{s2}}}{L_{s2}M_1e^{-i\phi_{s1}}}A_{13} + \frac{P_{s1}}{P_{s1}+P_{s2}}A_{23}$.

From (3):

$$
\begin{aligned}
&\frac{P_{s2}}{P_{s1}+P_{s2}}\frac{L_{s1}M_2e^{i\phi_{s2}}}{L_{s2}M_1e^{-i\phi_{s1}}}A_{13}+\frac{P_{s1}}{P_{s1}+P_{s2}}A_{23}\\
&=\frac{P_{s2}}{P_{s1}+P_{s2}}\frac{KL_{s1}M_2e^{i\phi_{s2}}}{L_{s2}M_1e^{-i\phi_{s1}}}M_1e^{-i\phi_{s1}}(R_rL_{s2}-iM_2^2\omega_rP_{s1}-iM_2^2\omega_rP_{s2}+\\
&\qquad iL_{s2}L_r\omega_rP_{s1})\\
&\quad+\frac{P_{s1}}{P_{s1}+P_{s2}}KM_2e^{i\phi_{s2}}(-iL_{s1}L_r\omega_rP_{s2}+L_{s1}R_r+iM_1^2\omega_rP_{s2}+\\
&\qquad iM_1^2\omega_rP_{s1})\\
&=\frac{P_{s2}}{P_{s1}+P_{s2}}KM_2e^{i\phi_{s2}}(-iM_2^2\frac{L_{s1}}{L_{s2}}\omega_rP_{s1}-iM_2^2\frac{L_{s1}}{L_{s2}}\omega_rP_{s2}+\\
&\qquad iL_{s1}L_r\omega_rP_{s1}+R_rL_{s1})\\
&\quad+\frac{P_{s1}}{P_{s1}+P_{s2}}KM_2e^{i\phi_{s2}}(-iL_{s1}L_r\omega_rP_{s2}+L_{s1}R_r+iM_1^2\omega_rP_{s2}+\\
&\qquad iM_1^2\omega_rP_{s1})\\
&=\frac{P_{s2}}{P_{s1}+P_{s2}}KM_2e^{i\phi_{s2}}(-iM_2^2\frac{L_{s1}}{L_{s2}}\omega_r(P_{s1}+P_{s2})+iL_{s1}L_r\omega_rP_{s1}+R_rL_{s1})\\
&\quad+\frac{P_{s1}}{P_{s2}+P_{s1}}KM_2e^{i\phi_{s2}}(-iL_{s1}L_r\omega_rP_{s2}+R_rL_{s1}+iM_1^2\omega_r(P_{s1}+P_{s2}))\\
&=KM_2e^{i\phi_{s2}}(-iM_2^2\frac{L_{s1}}{L_{s2}}\omega_rP_{s2}+\frac{P_{s2}P_{s1}}{P_{s1}+P_{s2}}iL_{s1}L_r\omega_r+\frac{P_{s2}}{P_{s1}+P_{s2}}R_rL_{s1})\\
&\quad+KM_2e^{i\phi_{s2}}(-\frac{P_{s1}P_{s2}}{P_{s2}+P_{s1}}iL_{s1}L_r\omega_r+\frac{P_{s1}}{P_{s1}+P_{s1}}R_rL_{s1}+iM_1^2\omega_rP_{s1})\\
&=KM_2e^{i\phi_{s2}}(-iM_2^2\frac{L_{s1}}{L_{s2}}\omega_rP_{s2}+\frac{P_{s2}P_{s1}}{P_{s1}+P_{s2}}iL_{s1}L_r\omega_r+\frac{P_{s1}+P_{s2}}{P_{s1}+P_{s2}}R_rL_{s1}\\
&\quad-\frac{P_{s1}P_{s2}}{P_{s1}+P_{s2}}iL_{s1}L_r\omega_r+iM_1^2\omega_rP_{s1})\\
&=KM_2e^{(i\phi_{s2})}(-iM_2^2\frac{L_{s1}}{L_{s2}}\omega_rP_{s2}+\frac{P_{s1}+P_{s2}}{P_{s1}+P_{s2}}R_rL_{s1}+iM_1^2\omega_rP_{s1})\\
&=\frac{M_2e^{(i\phi_{s2})}}{K}(-iM_2^2\frac{L_{s1}}{L_{s2}}\omega_rP_{s2}+R_rL_{s1}+iM_1^2\omega_rP_{s1})\\
&=\frac{KM_2}{L_{s2}}e^{i\phi_{s2}}(R_rL_{s1}L_{s2}-iM_2^2L_{s1}\omega_rP_{s2}+iM_1^2L_{s2}\omega_rP_{s1})\\
&=\frac{M_2}{L_{s2}}e^{i\phi_{s2}}(-A_{33})=-\frac{KM_2}{L_{s2}}e^{i\phi_{s2}}QL_{s1}L_{s2}
\end{aligned}
$$