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Remarks on "Robustness Analysis of Nonlinear Feedback Systems: An Input–Output Approach"

Tryphon T. Georgiou and Malcolm C. Smith

Our previous paper [1, Sec. VII] contains two examples of adaptive controllers (Examples 8 and 9) for which the nonlinear gap robustness margin is zero and which are destabilized by arbitrarily small perturbations in the gap metric. The proof that the robustness margin is zero makes use of the fact that the parameter estimate $\theta(t)$ tends to infinity as $t \to \infty$ when there is a constant input disturbance of size $\epsilon > 0$. The purpose of this present note is: a) to complete the argument given in [1] to prove the unboundedness of $\theta(t)$ and b) to give an alternative proof that the convergence of the state $x(t) \to 0$ is a consequence of the unboundedness of $\theta(t)$ in Example 9.

We begin with a). In [1, p. 1214], (32) and (33) and the following five lines argue by contradiction. Assuming $\theta(t)$ is bounded, it is claimed that "(33) implies that $x(t) \to 0$" whence a contradiction is drawn via (32). In fact, (33) alone is not sufficient for the convergence of $x(t)$ to zero; (33) needs to be used in combination with the first-order equation (32) to assert this convergence. In particular, assuming $\theta(t)$ is bounded, and using the fact that $\theta(t)$ is monotonic, it can be shown with (32) that $x(t)$ is eventually monotonic, and hence from (33) $x(t) \to 0$. Rather than fill out the somewhat lengthy details of this reasoning we prefer to give an alternative direct proof of the unboundedness of $\theta(t)$. To make the result self-contained, we will restate it in the form of a lemma.

Lemma 1: Consider the equations

$$\dot{x}(t) = \epsilon + a(\theta(t))x(t), \tag{1}$$
$$\dot{\theta}(t) = x(t)^2 \tag{2}$$

where $\epsilon > 0$, $x(0) = \theta(0) = 0$, and $a(\theta)$ is any continuous function. Then, $\theta(t) \to \infty$ as $t \to \infty$.

Proof: Assume to the contrary that there exists $M_1 > 0$ such that $\theta(t) \le M_1$ for all t. Then, from (2), we see that $\int_0^t x^2(\tau)d\tau \le M_1$ for all t, and so $x(t) \in \mathcal{L}_2[0, \infty)$. Since $a(\theta)$ is continuous, there exists $M_2 > 0$ such that $a(\theta(t)) \le M_2$ for all t. It therefore follows from (1) that

$$\int_0^t (\dot{x}(\tau) - \epsilon)^2 \, d\tau \le M_2^2 M_1$$

for all t. Hence

$$\begin{aligned} M_2^2 M_1 &\ge \int_0^t \dot{x}(\tau)^2 \, d\tau - 2\epsilon \int_0^t \dot{x}(\tau) \, d\tau + \epsilon^2 t \\ &\ge -2\epsilon(x(t) - x(0)) + \epsilon^2 t \end{aligned}$$

for all t. Thus

$$x(t) \ge -\frac{M_2^2 M_1}{2\epsilon} + x(0) + \frac{\epsilon t}{2}$$

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T. T. Georgiou is with the Department of Electrical and Computer Engineering, University of Minnesota, Minneapolis, MN 55455, U.S.A.
M. C. Smith is with the Department of Engineering, University of Cambridge, Cambridge, CB2 1PZ, U.K.

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for all t. Hence, $x(t)$ grows without bound as $t \to \infty$ and cannot belong to $\mathcal{L}_2[0, \infty)$. This is a contradiction which establishes the lemma. ■

The lemma proves the relevant boundedness property of $\theta(t)$ for [1, Examples 8 and 9] with $a(\theta) = \theta^2 \cos\theta$ and $a(\theta) = -\theta$ respectively. We are most grateful to R. Ortega [2] for pointing out the need to clarify this step of our paper.

We now turn to b). The relevant fact is proved in the seven lines of parenthetic comments in [1, p. 1216, lines 32–38]. We now restate the claim in the form of a lemma and give an alternative proof.

Lemma 2: Let

$$\dot{x}(t) = \epsilon - \theta(t)x(t) \tag{3}$$

where $\epsilon > 0$ and $x(0) = \theta(0) = 0$. Assume $\theta(t)$ is continuous, monotonically nondecreasing and $\theta(t) \to \infty$ as $t \to \infty$. Then, $x(t) \to 0$ as $t \to \infty$.

Proof: Define

$$t' = \int_0^t \theta(\tau)\, d\tau =: f(t).$$

For some $t_0 > 0$, we have $\theta(t_0) > 0$ which means that $f(t)$ is monotonically (strictly) increasing on $[t_0, \infty)$. Thus, we can change variables in (3) from t to t' on this interval. Writing $\hat{x}(t') := x(f^{-1}(t'))$, and noting that $dt' = \theta(t)\, dt$, (3) becomes

$$\frac{d}{dt'}\hat{x}(t') = \frac{\epsilon}{\theta(f^{-1}(t'))} - \hat{x}(t') := u(t') - \hat{x}(t'). \tag{4}$$

Note that $u(t') \to 0$ monotonically as $t' \to \infty$. Let $t'_0 = f(t_0)$ and define $\delta_1 := |\hat{x}(t'_0)|$ and $\delta_2 := |u(t'_0)|$. Then, for $t' \geq t'_0$

$$\begin{aligned} |\hat{x}(t')| &= \left| e^{-t'+t'_0}\hat{x}(t'_0) + \int_{t'_0}^{t'} e^{-t'+\tau}u(\tau)\, d\tau \right| \\ &\leq e^{-t'+t'_0}\delta_1 + \delta_2. \end{aligned}$$

We now claim that $\hat{x}(t') \to 0$ as $t' \to \infty$, i.e., given any $\epsilon > 0$ there exists t'_1 such that $|\hat{x}(t')| \leq \epsilon$ for all $t' \geq t'_1$. To see this, choose t'_0 so that $\delta_2 \leq \epsilon/2$ and choose $t'_1 = t'_0 + \ln(2\delta_1/\epsilon)$ if $2\delta_1/\epsilon > 1$, otherwise, choose $t'_1 = t'_0$. This ensures that for any $t' \geq t'_1$

$$\begin{aligned} |\hat{x}(t')| &\leq e^{-t'+t'_0}\delta_1 + \delta_2 \\ &\leq e^{-t'_1+t'_0}\delta_1 + \delta_2 \leq \epsilon. \end{aligned}$$

Hence, for all $t \geq f^{-1}(t'_1)$, $|x(t)| \leq \epsilon$ which establishes the claim. ■

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Correction to "Passivity-Based Control of a Class of Blondel–Park Transformable Electric Machines"

L. U. Gökdere, W. Brice, P. J. Nicklasson, R. Ortega, and G. Espinosa-Pérez

In the above cited paper,[1], the third and fourth equations in the left column of p. 634 must be corrected to

$$\begin{aligned} \mathcal{L}_e(\dot{q}_e, q_m) &= \frac{1}{2}\dot{q}_e^T D_e \dot{q}_e + \mu^T \dot{q}_e - V(q_m) \\ \mathcal{L}_m(q_m, \dot{q}_m) &= \frac{1}{2} D_m \dot{q}_m^2. \end{aligned}$$

Thus, $\mathcal{L}_e$ contains the energy contributions of magnetic origin.

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The authors are with the CNRS-ESE, SUPELEC-LSS, Plateau de Moulon, 91192 Gif-sur-Yvette, France (e-mail: rortega@lss.supelec.fr).

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