Brief Papers

On Stability Margins of the Fiat Dedra Engine Model

Masaaki Kanno and Tai Cheng Yang

***Abstract*—In this paper, we evaluate the stability margin of a Fiat Dedra engine model to explore possible applications of theoretical achievements in the parametric approach to robust control problems. The study is based on two versions of the model with uncertainties. One is the original model, which has a multilinear characteristic polynomial, while the other is an affine-linearized model which is obtained by the affine linearization technique proposed in this paper. By applying some newly developed theoretical tools in the literature, the stability margins of the affine-linearized model in terms of the ℓ_∞ norm and the ℓ_2 norm and the stability margin of the original model in terms of the ℓ_∞ norm are obtained.**

***Index Terms*—Affine linear uncertainty, Fiat Dedra engine model, multilinear uncertainty, parametric approach, robust stability, stability margin.**

I. Introduction

RECENTLY, there has been an increased interest in the parametric approach to robust control problems. Many theoretical tools have been developed, while the application of these methods still lags behind. To stimulate application, a practical robust stability problem—the idle speed control of a spark ignition engine based on the Fiat Dedra engine model—has been used to demonstrate the application of some new results [1], [2]. Until now, the published work in this respect was limited to the robust stability analysis, i.e., to determine if the given system is robustly stable or not. Another important issue in the area is the stability margin, which indicates the size of allowable disturbance under which the system family is robustly stable. To follow the work in the direction of application, we apply some methods developed by Chapellat and Bhattacharyya [4], Tsypkin and Polyak [10], and Keel and Bhattacharyya [7], [8] to evaluate stability margins of the Fiat Dedra engine model. In addition, we propose a technique, affine linearization, to overcome some difficulties in evaluating stability margins.

This paper is organized as follows. Section II gives a brief introduction to the problem of stability margin determination for plants with uncertain parameters. The multilinear characteristic polynomial for the original Fiat Dedra engine model is presented in Section III. Then, a technique, affine linearization, is proposed and applied to obtain an affine linear polynomial corresponding to a simplified engine model, in terms that some uncertain parameters in the original model are treated as fixed in the simplified model. In Section IV, some newly developed methods are applied to the original and simplified models to evaluated stability margins. These are: 1) The ℓ_2 norm stability margin of the simplified model is evaluated by the method developed by Chapellat and Bhattacharyya [4]. 2) The ℓ_∞ norm stability margin of the simplified model is calculated using the Tsypkin–Polyak loci [10] and also the method proposed by Keel and Bhattacharyya [3], [7], [8]. 3) The stability margin of the original Fiat Dedra engine model in terms of the ℓ_∞ norm is estimated by the method proposed by Keel and Bhattacharyya [7], [8].

II. Stability Margin of Plants With Parametric Uncertainties

Given a polynomial with uncertain parameters $p(s, \boldsymbol{q})$ and a nominal parameter vector $\boldsymbol{q}_0$, if the nominal system $p(s, \boldsymbol{q}_0)$ is found stable, one then may want to know the maximum allowable parameter variations under which the system will remain robustly stable. To achieve this, the region of the uncertain parameter vector may be gradually expanded until the polynomial family contains at least one unstable polynomial. The size of the parameter region at this point is defined as stability margin. Mathematically, given a central, or nominal, parameter vector $\boldsymbol{q}_0$ where the polynomial $p(s, \boldsymbol{q}_0)$ is stable, the stability margin is defined as the largest ρ^* where $p(s, \boldsymbol{q})$ is stable for all $\boldsymbol{q}$ which satisfies $\|\boldsymbol{q} - \boldsymbol{q}_0\| < \rho^*$, i.e.,

$$\rho^*(\boldsymbol{q}_0) = \sup\{\rho \mid p(s, \boldsymbol{q}) \text{ is stable for all } \boldsymbol{q}, \|\boldsymbol{q} - \boldsymbol{q}_0\| < \rho\}. \quad (1)$$

The norm $\|\cdot\|$ can be any norms. Still, the main interests would be the ℓ_2 norm and the ℓ_∞ norm

$$\ell_2 \text{ norm}: \|\boldsymbol{q} - \boldsymbol{q}_0\|_2 = \sqrt{\sum_{i=1}^{l}(q_i - q_{i0})^2}$$
$$\ell_\infty \text{ norm}: \|\boldsymbol{q} - \boldsymbol{q}_0\|_\infty = \max_{1 \le i \le l} |q_i - q_{i0}|$$

where $\boldsymbol{q} = [q_1\ q_2\ \dots\ q_l]^T$ and $\boldsymbol{q}_0 = [q_{10}\ q_{20}\ \dots\ q_{l0}]^T$.

It is generally convenient to set $\boldsymbol{q}_0 = \boldsymbol{0}$ by shifting the origin of the parameter space. Then, the left-hand side of (1) can simply be expressed as ρ^*. Also, it may be useful to define frequency-dependent stability margin. From the zero exclusion theorem, it can be written as

$$\rho^*(\omega) = \sup\{\rho \mid p(j\omega, \boldsymbol{q}) \neq 0 \text{ for all } \boldsymbol{q},\ \|\boldsymbol{q}\| < \rho\} \quad (2)$$

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under the assumption that $p(s, \mathbf{0})$ is stable. Naturally

$$\rho^* = \inf_{\omega} \rho(\omega). \tag{3}$$

Stability margin determination encompasses a robust stability test, i.e., an examination of robust stability of the system in a given region. By properly shifting and scaling, uncertain parameters can be normalized as $q_i \in [-1;\ 1]$. The robust stability test is then equivalent to the task of determining whether the stability margin is greater than one. In fact, stability margin provides more information than just a "yes–no" answer to an ordinary robust stability test problem. When value sets are used in robust stability test, their graphical display shows the distance between the origin and the value sets. Nevertheless, it does not correspond to the distance to instability in the parameter space. On the other hand, the stability margin gives the direct indication of the distance to the unstable region from the nominal system in terms of uncertain parameter variations.

III. Characteristic Polynomials of the Fiat Dedra Engine Model

Using state-space modeling technique, the characteristic polynomial of the Fiat Dedra engine model is obtained as a fourth-order polynomial with seven uncertain parameters [9]

$$p(s, \boldsymbol{q}) = s^4 + a_3(\boldsymbol{q})s^3 + a_2(\boldsymbol{q})s^2 + a_1(\boldsymbol{q})s + a_0(\boldsymbol{q}) \tag{4}$$

where

$$\begin{aligned}
a_3(\boldsymbol{q}) =& k_{23}q_6q_7 + q_5q_7 + q_2 + k_{12}q_1 + k_{24} + 0.05\\
a_2(\boldsymbol{q}) =& k_{13}q_1q_4q_7 + k_{12}q_1q_5q_7 + (k_{12}k_{23} - k_{13}k_{22})\, q_1q_6q_7\\
&+ q_2q_5q_7 + k_{23}q_2q_6q_7 + q_3q_4q_7\\
&- k_{22}q_3q_6q_7 + (k_{24} + 0.05)q_5q_7 + k_{21}q_6q_7\\
&+ (k_{24} + 0.05)q_2 + ((k_{24} + 0.05)k_{12} - k_{22}k_{14})q_1\\
a_1(\boldsymbol{q}) =& (k_{11} - k_{14}k_{23} + k_{13}\,(k_{24} + 0.05))\, q_1q_4q_7\\
&+ (k_{12}\,(k_{24} + 0.05) - k_{14}k_{22})\, q_1q_5q_7\\
&+ (k_{12}k_{21} - k_{11}k_{22})\, q_1q_6q_7 + (k_{24} + 0.05)\, q_2q_5q_7\\
&+ k_{21}q_2q_6q_7 + (k_{24} + 0.05)\, q_3q_4q_7\\
a_0(\boldsymbol{q}) =& (k_{11}\,(k_{24} + 0.05) - k_{14}k_{21})\, q_1q_4q_7.
\end{aligned}$$

k_{ij} denotes the elements of the controller gain matrix [2]

$$K_C = \begin{bmatrix} 0.0081 & 0.1586 & 0.8072 & -0.1202\\ 0.0187 & 0.0848 & 0.1826 & -0.0224 \end{bmatrix}. \tag{5}$$

The coefficients of the characteristic polynomial depend multilinearly on the uncertain parameters. In [9], the operating domain is defined as a parameter box, or a Q-box

$$Q = \left\{ \boldsymbol{q} = \left[q_1\ q_2\ q_3\ q_4\ q_5\ q_6\ q_7 \right]^T \,\middle|\, q_i \in \left[q_i^-;\ q_i^+\right], i = 1, 2, \ldots, 7 \right\}. \tag{6}$$

The upper bounds and the lower bounds are given in Table I. We use the same operating domain and calculate from these values the nominal values and the weights of the uncertain parameters. That is, let q_{i0} and k_q_i denote the central value and the weight of q_i, respectively. Then

$$q_{i0} = \frac{q_i^- + q_i^+}{2},\ k\_q_i = \frac{q_i^+ - q_i^-}{2}. \tag{7}$$

TABLE I
THE RANGE OF PARAMETER VARIATIONS (REPRODUCED FROM [9])

Lower/Upper Bounds	q_1	q_2	q_3	q_4	q_5	q_6	q_7
q_i^-	2.1608	0.1027	0.0357	0.2539	0.0100	2.0247	0.1000
q_i^+	3.4329	0.1627	0.1139	0.5607	0.0208	4.4962	1.0000
q_{i0}	2.7969	0.1327	0.0748	0.4073	0.0154	3.2605	0.5500
k_q_i	0.6361	0.0300	0.0391	0.1534	0.0054	1.2357	0.4500

The uncertain parameters are normalized as

$$\tilde{q}_i = \frac{q_i - q_{i0}}{k\_q_1} \tag{8}$$

where $\tilde{q}_i$ denotes a normalized uncertain parameter and $\tilde{q}_i \in [-1;\ 1]$.

We consider

$$\begin{aligned}
q_1 =& q_{10} = 2.7969,\ q_2 = q_{20} = 0.1327\\
q_3 =& q_{30} = 0.0748,\ q_4 = q_{40} = 0.4073\\
q_5 =& q_{50} = 0.0154,\ q_6 = q_{60} = 3.2605\\
q_7 =& q_7^+ = 1.0000
\end{aligned}$$

as a set of the parameters corresponding to the nominal characteristic polynomial. The roots of this polynomial are

$$-0.5071 \pm 0.1709j, -0.1149, -0.0856$$

which show that it is Hurwitz. The stability margin for parameters in this given nominal polynomial will be studied in Section IV-B.

Remark: Instead of q_{70}, the value q_7^+ is used for q_7. The reason for this is discussed in Section IV-B. ◇

Characteristic Polynomial With Affine Linear Coefficient Uncertainty: More often than not, a model of a practical system is complicated and uncertainties appear in multilinear or polynomial form in the coefficients of its characteristic polynomial. Yet, by fixing some parameters in a polynomial with multilinear or polynomial uncertainty, it can be changed into an affine linear polynomial and we call such a process "affine linearization." Affine linearization will result in a subset of the original problem and its analysis will only lead to a necessary condition check of the whole polynomial family. However, there are effective methods for the robust stability analysis of affine linear polynomials and, thus, an affine-linearized polynomial is favorable in terms of computation cost. In addition, the information on an affine-linearized model can provide engineers with insight into the original problem they are facing. For instance, when bisection search is employed in the stability margin determination, the stability margin of the affine-linearized polynomial can be used as an initial upper bound.

Affine linearization of the Fiat Dedra engine model is conducted here. An investigation into the coefficients in the characteristic polynomial of (4) reveals that, if $q_4,\ q_5,\ q_6,$ and q_7 are fixed, the coefficients are dependent affine linearly on $q_1, q_2,$ and q_3. The fixed values

$$\begin{aligned}
q_4 =& q_4^- = 0.2539,\ q_5 = q_5^+ = 0.0208\\
q_6 =& q_6^- = 2.0247,\ q_7 = q_7^+ = 1.0000
\end{aligned}$$

are used here, as they correspond to the "nominal" condition [2].

Remark: In [2], the "nominal" operating point of the Fiat Dedra engine model is the most common operating point. It does not correspond to the set of the central values of the uncertain parameters. ◇

Furthermore, for simplicity of calculation and uniform dilation of the parameter box, the uncertain parameters are normalized. The uncertain polynomial then becomes the following simplified uncertain polynomial with coefficients that depend affine linearly on three scaled uncertain parameters:

$$\begin{aligned} p(s,\tilde{\boldsymbol{q}}) =& \underbrace{s^4 + 0.9944s^3 + 0.3341s^2 + 0.0424s + 0.0018}_{p_0(s)} \\ &+ \underbrace{(0.1009s^3 + 0.0532s^2 + 0.0084s + 0.0004)}_{p_1(s)}\tilde{q}_1 \\ &+ \underbrace{(0.0300s^3 + 0.0125s^2 + 0.0012s)}_{p_2(s)}\tilde{q}_2 \\ &+ \underbrace{(0.0032s^2 + 0.0003s)}_{p_3(s)}\tilde{q}_3. \end{aligned} \quad (9)$$

Remark: The same method of normalization of uncertain parameters can be applied to the multilinear coefficient polynomial given in (4). However, the resultant polynomial is lengthy and its coefficients are not directly used in the study of stability margin. Therefore, the process of normalization for the polynomial of (4) is not presented in this paper. ◇

Naturally, letting

$$\tilde{q}_1 = 0,\ \tilde{q}_2 = 0,\ \tilde{q}_3 = 0$$

the nominal characteristic polynomial is obtained. The roots of this polynomial are

$$-0.3871 \pm 0.0470j, -0.1344, -0.0859$$

which show that it is Hurwitz. The stability margin for parameters in this given nominal polynomial will be studied in Section IV-A.

IV. Some Stability Margins of the Fiat Dedra Engine Model

A. Stability Margins of the Affine Linear Polynomial

Largest Hypersphere: The stability margin in terms of the ℓ_2 norm, i.e., the largest hypersphere, is first investigated using the method proposed in [4].

The evaluation of $p(s,\tilde{\boldsymbol{q}}) = 0$, where $p(s,\tilde{\boldsymbol{q}})$ is given in (9), at $s = j\omega$ yields

$$\underbrace{\begin{bmatrix} \operatorname{Re} p_1(j\omega) & \operatorname{Re} p_2(j\omega) & \operatorname{Re} p_3(j\omega) \\ \operatorname{Im} p_1(j\omega) & \operatorname{Im} p_2(j\omega) & \operatorname{Im} p_3(j\omega) \end{bmatrix}}_{A(j\omega)} \tilde{\boldsymbol{q}} = \underbrace{\begin{bmatrix} -\operatorname{Re} p_0(j\omega) \\ -\operatorname{Im} p_0(j\omega) \end{bmatrix}}_{\boldsymbol{b}(j\omega)}. \quad (10)$$

For $\omega = 0$, the second row disappears and the size of the smallest $\tilde{\boldsymbol{q}}$ in terms of the ℓ_2 norm that satisfies (10), i.e., the stability margin at $\omega = 0$, is given by

$$\rho(0) = \frac{|p_0(0)|}{\|[p_1(0)\ \ p_2(0)\ \ p_3(0)]\|_2} = 4.40. \quad (11)$$

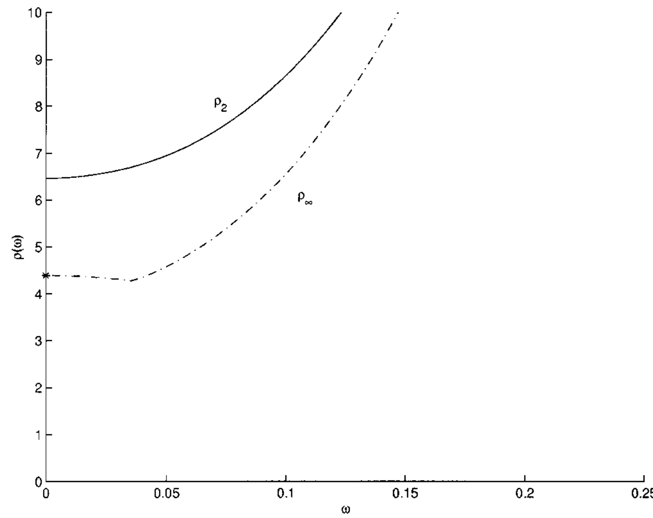

Fig. 1. ℓ_2 stability margin (solid) and ℓ_∞ stability margin (dash-dot) (affine linear case).

For any $\omega > 0$, rank $A(j\omega) = 2$ and the size of the smallest ℓ_2 norm solution of (10) is calculated by

$$\rho(\omega) = \left\| A^T(j\omega)\left[A(j\omega)A^T(j\omega)\right]^{-1}\boldsymbol{b}(j\omega)\right\|_2. \quad (12)$$

Since the characteristic polynomial is monic and its degree never drops irrespective of $\tilde{\boldsymbol{q}}$

$$\rho(\infty) = \infty. \quad (13)$$

Plotting $\rho(\omega)$ against ω, Fig. 1 is obtained (solid line and the asterisk at $\omega = 0$). From this graph, the stability margin is determined as

$$\rho^* = 4.40$$

and the critical frequency is $\omega = 0$.

Tsypkin–Polyak Loci: The Tsypkin–Polyak loci [10] suggest that the frequency-dependent ℓ_∞ norm stability margin of the polynomial (9) is given by

$$\rho(0) = \frac{|p_0(0)|}{\sum_{i=1}^{3}|p_i(0)|} = 4.40 \quad (14)$$

$$\rho(\omega) = \max_{1\le k\le 3} \frac{\left|\operatorname{Im}\left(\frac{p_0(j\omega)}{p_k(j\omega)}\right)\right|}{\sum_{i=1}^{3}\left|\operatorname{Im}\left(\frac{p_i(j\omega)}{p_k(j\omega)}\right)\right|}, 0 < \omega < \infty \quad (15)$$

$$\rho(\infty) = \frac{|p_0(j\infty)|}{\sum_{i=1}^{3}|p_i(j\infty)|} = \infty. \quad (16)$$

The plot of $\rho(\omega)$ is depicted in a dash-dot line in Fig. 1. The stability margin in terms of the ℓ_∞ norm is

$$\rho^* = 4.27$$

at $\omega = 0.036$ [rad/s].

Bounded Phase Theorem: Here, we use the bounded phase theorem [3], [7], [8] for the ℓ_∞ norm stability margin estimation which can, in fact, be applied to the analysis of a more general system, e.g, a family of quasipolynomials and a polytope of polynomials. The theorem says that whether a value set contains the origin can be determined by measuring the angle subtended

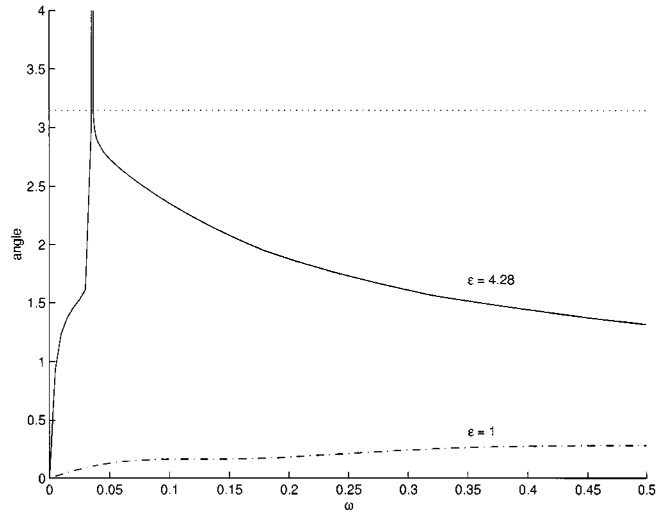

Fig. 2. Phase differences at $\epsilon = 1$ (dash-dot) and at $\epsilon = 4.28$ (solid) (affine linear case).

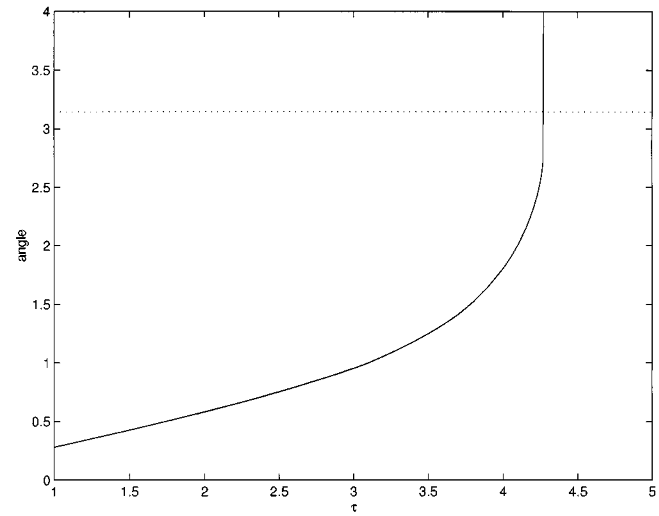

Fig. 3. Maximum phase differences against ϵ (affine linear case).

at the origin by the vertices of the value set and checking if the angle exceeds π. Let $v_i,\ i = 1, 2, \ldots$, be the vertices and

$$\phi^+ := \max_{v_i} \arg\left(\frac{v_i}{v_1}\right),\ 0 \le \phi^+ \le \pi \tag{17}$$

$$\phi^- := \min_{v_i} \arg\left(\frac{v_i}{v_1}\right), -\pi < \phi^- \le 0. \tag{18}$$

Then, the angle subtended at the origin is given by $\phi^+ - \phi^-$. The stability margin is evaluated by expanding the range of uncertain parameters $q_i \in [-\epsilon;\ \epsilon]$ and checking whether the maximum phase difference exceeds π.

The phase difference plot at $\epsilon = 1$ against frequency is plotted in a dash-dot line in Fig. 2. Increasing ϵ, the maximum phase differences corresponding to each ϵ can be obtained. The plot of the maximum phase differences against ϵ is shown in Fig. 3. It can be seen that the maximum phase differences exceed π at $\epsilon = 4.28$. The phase difference plot at $\epsilon = 4.28$ is shown in a solid line in Fig. 2. The stability margin is judged to be 4.27 and the critical frequency is $\omega = 0.036$ [rad/s]. The results agree with those obtained from the Tsypkin–Polyak loci.

The first two methods used in this section only require frequency grid, while ϵ has to be gridded as well in the last method. Hence, the last method is less attractive in terms of computational effort. Yet, the disadvantage can be offset by the fact that the method can deal with more general systems.

B. Stability Margin of the Multilinear Polynomial

It has been discovered [5], [6] that under special circumstances, the ℓ_∞ stability margin of a multilinear polynomial family can be obtained from subsets of an entire operating domain. In general, it is not easy to find the exact stability margin directly. Therefore, a method is proposed to evaluate the stability margin from a lower bound and an upper bound [3], [7], [8]. The ℓ_∞ stability margin can be obtained in the following way.

Let $P(j\omega, Q)$, $\overline{P}(j\omega, Q)$, and $P_V(j\omega, Q)$ denote the value set at the frequency ω, the convex hull of $P(j\omega, Q)$, and the images of the vertices of Q, respectively. Naturally, $P_V(j\omega, Q) \in P(j\omega, Q)$. The mapping theorem [11] suggests $P(j\omega, Q) \in \overline{P}(j\omega, Q)$. These imply that

$$\underline{\rho}(\omega) \le \rho(\omega) \le \overline{\rho}(\omega) \tag{19}$$

and that

$$\underline{\rho} \le \rho^* \le \overline{\rho} \tag{20}$$

where $\underline{\rho}$ and $\overline{\rho}$ are the stability margins approximated by $\overline{P}(j\omega, Q)$ and $P_V(j\omega, Q)$, respectively. Rather than calculating ρ^*, ρ^* is estimated from $\underline{\rho}$ and $\overline{\rho}$.

The upper bound $\overline{\rho}$ can be found by increasing ϵ and examining that all the polynomials corresponding to the vertices of the Q-box are stable. The estimation of the lower bound $\underline{\rho}$ can be made by means of the bounded phase theorem. That is, expand the Q-box and calculate the maximum phase difference of the images of the vertices of the Q-box in order to confirm the exclusion of the origin in the convex hull of $P(j\omega, Q)$ throughout the frequency.

The gap between the lower bound and the upper bound may be greater than acceptable and the actual stability margin may not be evaluated within a desired precision. In that case, the operating domain can be divided into several boxes for better approximation of the actual polynomial family and the stability margin may be evaluated within a desirable accuracy.

Now, q_7 of the Fiat Dedra engine model is fixed to the "nominal" value due to the following reason. It is known that the necessary condition of the robust stability of the uncertain polynomial

$$p(s, \boldsymbol{q}) = a_n(\boldsymbol{q})s^n + a_{n-1}(\boldsymbol{q})s^{n-1} + \cdots + a_0(\boldsymbol{q})a_n(\boldsymbol{q}) > 0 \tag{21}$$

is

$$a_i(\boldsymbol{q}) > 0,\ i = 0, 1, \ldots, n \ \text{ for all } \ \boldsymbol{q} \in Q. \tag{22}$$

As for the Fiat Dedra engine model

$$a_0(\boldsymbol{q}) = (k_{11}(k_{24} + 0.05) - k_{14}k_{21})q_1q_4q_7 = 0.0025q_1q_4q_7 \tag{23}$$

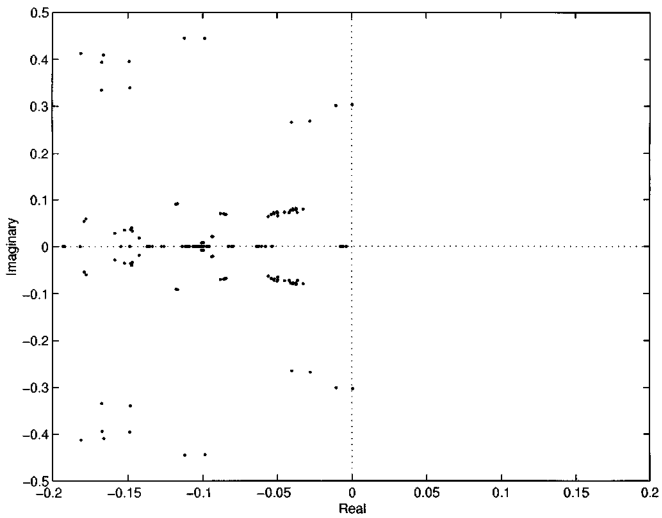

Fig. 4. Root set of the vertex polynomials at $\epsilon = 2.34$ (multilinear case).

where $q_1,\ q_4$, and q_7 are unscaled. Since the nominal values of q_1, q_4, and q_7 are all positive, it is necessary for the stability of the polynomial that all $q_1,\ q_4$, and q_7 are positive. Nevertheless, looking at the range of the unscaled q_7, [0.1 ; 1.0], it is easily found that there is only a small margin for $q_7 > 0$. Calculating the margin for $q_7 > 0$

$$\left| \frac{0 - \frac{0.1+1.0}{2}}{\frac{1.0-0.1}{2}} \right| = 1.22$$

is obtained. Consequently, the stability margin cannot be greater than 1.22. In fact, when the method described above is applied to this seven uncertain parameter problem, the resultant stability margin is 1.22. Therefore, the problem turns out to be trivial in spite of its appearance.

Recall that $q_7 = 1/J$ [9]. Therefore, $q_7 \rightarrow 0$ corresponds to the situation where the inertia moment J gets tremendously large. This will not be a real case. To exclude the unlikely situation and to make the task worth exploring, q_7 is fixed to the "nominal" value, i.e., $q_7^+ = 1.0$ [2] and the characteristic polynomial with six uncertain parameters $q_1,\ q_2, \ldots, q_6$ is analyzed.

In order to find the upper bound and the lower bound of the stability margin, the Q-box is dilated from $\epsilon = 1$. The root set of the vertex polynomials at $\epsilon = 2.34$ is shown in Fig. 4 and it is seen that one of the polynomials has roots with positive real part and becomes unstable. Therefore, the upper bound is tentatively given as $\overline{\rho} = 2.33$. It is also found that the angle subtended at the origin by the images of the vertices exceeds π at $\epsilon = 2.34$ (Figs. 5 and 6). Therefore, the lower bound is $\underline{\rho} = 2.33$, which coincides with the upper bound $\overline{\rho}$.

Consequently, there is no need to divide the Q-box for narrowing the gap between the lower bound and the upper bound and the stability margin is immediately determined as $\rho^* = 2.33$. The coincidence suggests that the unstable polynomial happens at one of the vertices of the Q-box.

For the stability margin determination of a multilinear polynomial, both ω and ϵ are in general to be gridded. Also, the Q-box may be divided. These imply the complexity of computation incurred by multilinear uncertainty.

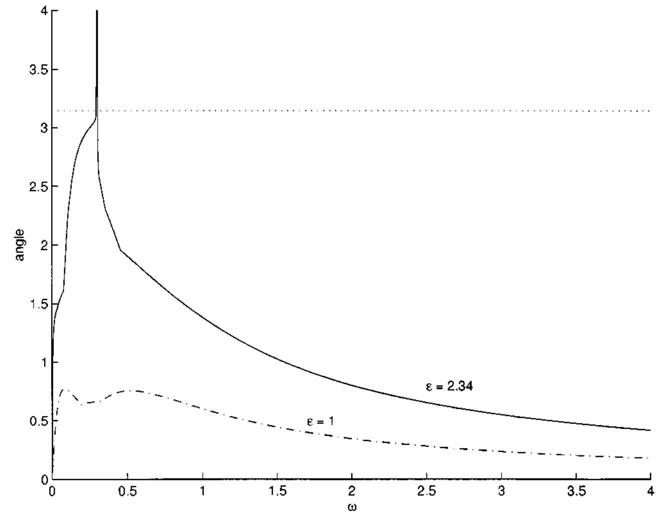

Fig. 5. Phase differences at $\epsilon = 1$ (dash-dot) and at $\epsilon = 2.34$ (solid) (multilinear case).

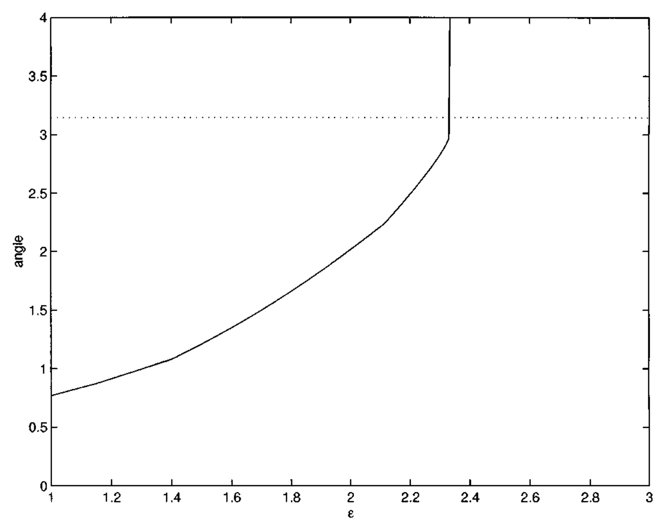

Fig. 6. Maximum phase differences against ϵ (multilinear case).

V. Conclusion

In this paper, some newly developed methods of obtaining stability margin are successfully applied to a practical model of the Fiat Dedra engine. Affine linearization is also proposed to overcome some difficulties in computation burdens. For the affine-linearized engine model, the stability margins in terms of the ℓ_2 norm and the ℓ_∞ norm are evaluated. For the original engine model with multilinear uncertainty, the ℓ_∞ norm stability margin is estimated by "sandwiching" it with an upper bound and a lower bound.

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