# TWO STABILISING CONTROL STRATEGIES FOR THE BRUSHLESS DOUBLY-FED MACHINE (BDFM)

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## ABSTRACT

The BDFM shows economic promise as a variable speed drive. One practical obstacle to commercial exploitation is the presence of operating speeds at which BDFM synchronous action cannot be maintained under open loop conditions. Two control strategies are proposed: *feedback linearisation* and *control winding phase angle control* which will enable BDFM synchronous action to be attained over the necessary operating speeds. This paper explains the theory of the two strategies, then suggests controllers for a specific BDFM. Finally the proposed control systems are tested by simulation and experiment. Both controllers successfully stabilised the BDFM. Whilst the feedback linearisation strategy gave superior performance in simulation, the difficulties of implementing it could limit its practical application.

## INTRODUCTION

Research into modern applications for the BDFM has accelerated over the last decade or so. As a motor the machine has attracted this attention due its potential to rival the induction machine as a variable speed drive (VSD), and as a generator where the prime mover speed can be variable. These advantages are realised through a reduced capital installation cost, due to a fractional inverter power requirement (as compared to that for an induction machine VSD (or generator)).

It is anticipated that to maximise the benefit of a BDFM VSD, an application with a low starting torque is required. This is because the inverter power requirements are greatest at zero speed, and so if the load at starting can be minimised so can the inverter size. Therefore a pump or fan application is being considered.

During investigations into the use of the BDFM Spée et al [1] noted that there are unstable speeds of operation ie speeds for which the BDFM would not maintain a synchronous speed under open-loop operation. Such unstable regions cannot be tolerated in a commercial implementation of the BDFM as a VSD. One approach is to establish the cause of the instability and then attempt to eliminate it by modifying the BDFM design in some way. However, an alternative approach is to design a controller to stabilise the system. This may well be easier, and advantageous because it does not necessarily constrain the BDFM design in terms of its steady-state performance (for example maximising efficiency, output torque, power factor etc., whilst minimising build costs). Furthermore a controller is required in a VSD to control the operating speed, so no further hardware is being introduced.

Several papers have already been published on the control of the BDFM. Li et al. published a significant paper in which they applied Lyapunov stability tests to the BDFM [1]. Various control strategies have been applied with the aim of improving the dynamic response (ie response to changes in load torque, speed) of the machine. However, none of the control schemes explicitly deal with stabilising the machine. In this context a *stable* speed is a shaft speed for which the BDFM can maintain synchronous action.

Previous work in this department by Healey [3], used the model described in the next section to investigate the stability of the BDFM. A specific machine configuration was chosen (identical to that used to produce the simulated results later in this paper), and an exhaustive search was performed to determine the effect of operating speed and load torque on stability. It was found that there is an unstable region around 650rpm, which is independent of load torque.

Therefore there is scope for considering stabilisation as a separate issue, such that once stabilisation has been achieved, performance optimising control schemes can be applied.

Two control stabilising strategies are presented in this paper, the first using a non-linear control technique called *feedback linearisation*. This technique allows the inherently non-linear, time-varying BDFM state-space system to be controlled in such a way that it will behave identically to a very simple linear time-invariant system. Standard linear control theory can then be applied to the resulting linear system. This has the advantage that an *optimal* controller can be designed much more easily than would be possible for a non-linear system.

The second strategy involves directly controlling the phase offset of the control winding supply to the power winding supply. This has a direct effect on the BDFM synchronous load angle, which determines the torque produced, thus enabling control of the machine.

## PRINCIPAL SYMBOLS

**i**: instantaneous coupled circuit current vector
**V**: instantaneous coupled circuit voltage vector
**M**: instantaneous coupled circuit inductance matrix
**R**: coupled circuit resistance matrix
$V_p$,$V_c$: rms power & control winding voltages
$\bar{x}(s)$: Laplace transform of $x(t)$
$\phi_c(t)$: control winding voltage phase offset
$p_p$,$p_c$: power and control winding pole pairs
$w_s$: steady state BDFM synchronous speed
$s$: complex variable of Laplace transform
$L$: steady state BDFM inductance parameter
$\theta_r'$: shaft position referred to the rotating reference frame
$\omega_p$, $\omega_c$: power & control winding angular supply frequencies
$V_i$: $i_{th}$ element of **V**
$\mathbf{i}^\mathbf{T}$: transpose of **i**
$\omega_r$: rotor shaft speed
$\theta_r$: rotor shaft position
$J$: system moment of inertia
$T_l$: load torque
$T_e$: electrical torque
$\dot{x}$: time derivative of $x(t)$
$\delta$: electrical load angle

## THEORY

### Dynamic BDFM modelling

The general coupled circuit model for the BDFM suitable for dynamic simulation, was devised from that described by Wallace et al [6]. The model assumes a magnetic circuit with laminations of infinite permeability, point rotor and stator conductors, and ignores slotting effects and conductor skin effect. These assumptions have been shown to lead to good accuracy providing the machine is not saturated.

The model equations can be written in state space form, where the elements of the vector quantities refer to individual coupled circuits in the model. For every circuit in the machine (a circuit being defined as any set of series connected conductors):

$$V_i = R_i i_i + \frac{d\Phi_i}{dt}, \text{and} \Phi_\text{i} = \sum_{\text{j}=1}^{\text{N}} \text{M}_\text{ij}\text{i}_\text{j} \tag{1}$$

Combining the above using matrix-vector notation:

$$\mathbf{V} = \mathbf{Ri} + \frac{d\,[\mathbf{Mi}]}{dt} = \mathbf{Ri} + \mathbf{M}\frac{d\mathbf{i}}{dt} + \omega_r\frac{d\mathbf{M}}{d\theta_r}\mathbf{i} \tag{2}$$

Rearranging and linking the equations by $T = J\ddot{\theta}_r$ and the electrical torque equation, $T_e = \frac{1}{2}\mathbf{i}^\mathbf{T}\frac{d\mathbf{M}}{d\theta_r}\mathbf{i}$ gives the equations in standard state-space form:

$$\begin{bmatrix} \dot{\mathbf{i}} \\ \theta_r \\ \omega_r \end{bmatrix} = \begin{bmatrix} -\mathbf{M}^{-1}\mathbf{Ri} \;-\; \omega_r\mathbf{M}^{-1}\frac{d\mathbf{M}}{d\theta_r}\mathbf{i} \\ \omega_r \\ \frac{1}{2J}\mathbf{i}^T\frac{d\mathbf{M}}{d\theta_r}\mathbf{i} \end{bmatrix} + \begin{bmatrix} \mathbf{M}^{-1}\mathbf{V} \\ 0 \\ -\frac{T_l}{J} \end{bmatrix} \tag{3}$$

(note that **M** is a function of rotor position)

### Stabilisation by Feedback Linearisation

Feedback linearisation is a control technique which can linearise a wide range of non-linear systems by applying the inverse model dynamics to the input to the system.

Following the criteria derived by Isidori [4], it can be shown that the coupled circuit BDFM model of (3) is output feedback linearisable with the shaft speed as the single output. To derive the linearising controller the system must be written in terms of the output and differentiated until fully defined, ie until all the system inputs appear on the RHS of the equation. Starting from the bottom row of (3) and differentiating:

$$\ddot{\omega}_r = \frac{1}{2J}\left[\frac{d\mathbf{i}}{dt}^T\frac{d\mathbf{M}}{d\theta_r}\mathbf{i} + \mathbf{i}^T\frac{d^2\mathbf{M}}{d\theta_r{}^2}\omega_r\mathbf{i} + \mathbf{i}^T\frac{d\mathbf{M}}{d\theta_r}\frac{d\mathbf{i}}{dt}\right] - \frac{1}{J}\frac{dT_l}{dt} \tag{4}$$

Noting that **M** is symmetric, therefore $\mathbf{M}^\mathbf{T} = \mathbf{M}$:

$$\Rightarrow \ddot{\omega}_r = \frac{1}{2J}\left[2\mathbf{i}^T\frac{d\mathbf{M}}{d\theta_r}\frac{d\mathbf{i}}{dt} + \mathbf{i}^T\frac{d^2\mathbf{M}}{d\theta_r{}^2}\omega_r\mathbf{i} - \frac{dT_l}{dt}\right] \tag{5}$$

then substituting for $\dot{\mathbf{i}}$ from the top row of (3):

$$\ddot{\omega}_r = \frac{1}{J}\left\{\mathbf{i}^\mathbf{T}\frac{d\mathbf{M}}{d\theta_r}\mathbf{M}^{-\mathbf{1}}\mathbf{V} - \mathbf{i}^\mathbf{T}\frac{d\mathbf{M}}{d\theta_r}\mathbf{M}^{-\mathbf{1}}\left[\mathbf{R} + \omega_r\frac{d\mathbf{M}}{d\theta_r}\right]\mathbf{i} + \frac{1}{2}\omega_r\mathbf{i}^\mathbf{T}\frac{d^2\mathbf{M}}{d\theta_r{}^2}\mathbf{i} - \frac{dT_l}{dt}\right\}$$

The system is now fully defined. Therefore we can now define a new control input, $u$:

$$u = \frac{d^2\omega_r}{dt^2} \tag{6}$$

$$\Rightarrow \mathbf{i}^\mathbf{T}\frac{d\mathbf{M}}{d\theta_r}\mathbf{M}^{-\mathbf{1}}\mathbf{V} = \mathbf{i}^\mathbf{T}\frac{d\mathbf{M}}{d\theta_r}\mathbf{M}^{-\mathbf{1}}\left[\mathbf{R} + \omega_r\frac{d\mathbf{M}}{d\theta_r}\right]\mathbf{i} - \frac{1}{2}\omega_r\mathbf{i}^\mathbf{T}\frac{d^2\mathbf{M}}{d\theta_r{}^2}\mathbf{i} + \frac{dT_l}{dt} + Ju \tag{7}$$

Hence if $u$ is set to be a desired speed, and then **V** is chosen so that it satisfies (7), the system is linearised, and the transfer function from $u$ to $\omega_r$ is:

$$\bar{\omega_r}(s) = \frac{1}{s^2}\bar{u}(s) \tag{8}$$

Notice that equation (7) is in fact a scalar equation. Thus for a BDFM, where there are two stator supplies, ie 6 controllable voltage supply inputs (two 3 phase windings), there is a whole family of solutions.

In the BDFM it is desired to only control one of the stator windings, the other being connected directly to the mains.

Therefore it is proposed that the least squares solution of (7) be used, ie the solution which minimises $\sum V_i{}^2$ where $i$ covers the three control winding voltages. This will ensure that the minimum input power solution is found. Notice that, from (7) alone, there is no restriction that the control winding be supplied with a balanced 3 phase sinusoidal voltage. Entirely arbitrary waveforms could be produced. In practice, it was found that, choosing the least squares solution of (7) restricts the solution to 3 phased balanced voltages, although not necessarily purely sinusoidal.

### Control Winding Phase Angle Control

It can be shown that the BDFM, under synchronous operating conditions has a *load angle* associated with it. Ferreira [5] derives equations (9) and (10) from which the concept of phase angle control can be understood.

$$\delta = \phi_c(t) + \beta - (p_p + p_c)\theta_r' \tag{9}$$

$$T_e = \frac{3V_c V_p}{\omega_p \omega_c L} \sin\delta \tag{10}$$

where $\theta_r'$ is the rotor shaft position referred to the rotor reference frame, as defined below, and $\beta$ is a machine dependent constant.

From consideration of (9), $\delta$ can be varied by varying the free variable $\phi_c(t)$. For small changes in $\delta$ at some $\omega_s$ then, (10) shows that $T_e \propto \phi_c$, since $\Delta\sin(\delta) \approx \Delta\delta$. Therefore by controlling the control winding phase parameter $\phi_c(t)$, it is possible to directly control the torque, and hence stabilise the machine. The following mathematics expresses the above ideas formally by linearising the equations about a suitable equilibrium point. The electrical ($\delta$) and physical ($\theta_r'$) load angles are related via the motor dynamics:

$$\ddot{\theta_r} = \frac{T}{J} = \frac{T_e - T_l}{J} \tag{11}$$

$$\theta_r' = \theta_r - \omega_s t \tag{12}$$

Differentiating (12) and substituting (10) and (11):

$$\ddot{\theta}_r' = \frac{T}{J} = \frac{T_e - T_l}{J} \tag{13}$$

$$= \frac{3V_c V_p}{J\omega_c\omega_p L} \sin\left(\phi_c(t) + \beta - (p_p + p_c)\theta_r'\right) - \frac{T_l}{J} \tag{14}$$

At equilibrium $\ddot{\theta}_r' = \dot{\theta}_r' = 0$, by definition, and $\phi_c = \phi_{c_e}$, $\theta_r' = \theta_{r_e}'$. From (14):

$$\theta_{r_e}' = \frac{-\arcsin\left(\frac{T_l\omega_c\omega_p L}{3V_c V_p}\right) + \beta + \phi_{c_e}}{p_p + p_c} \tag{15}$$

Hence, noting that: $\cos(-\arcsin x) = \sqrt{1 - x^2}$

$$\left.\frac{\partial\ddot{\theta}_r'}{\partial\phi_c}\right|_{\substack{\phi_c = \phi_{ce} \\ \theta_r' = \theta_{re}'}} = \frac{3V_c V_p}{J\omega_c\omega_p L}\sqrt{1 - \left(\frac{T_l\omega_c\omega_p L}{3V_c V_p}\right)^2}$$

$$= \Gamma \tag{16}$$

Hence the linearised transfer function from $\phi_c(t)$ to $\theta_r'$ for small changes in $\phi_c(t)$ is:

$$\bar{\theta}_r'(s) = \frac{\Gamma}{s^2}\bar{\phi}_c(s) \tag{17}$$

Notice that there is an immediate further restriction attached to transfer function: $\left|\frac{T_l\omega_c\omega_p L}{3V_c V_p}\right| < 1$. A controller has been designed and tested (both in simulation and on a real machine) using only position feedback.

## CONTROLLER IMPLEMENTATION

### Feedback Linearisation

Feedback linearisation requires complete state feedback at a rate much faster than the fastest dynamics present in the machine. This would mean, in practice, a sample rate of the order of kilohertz, which is certainly possible. The problem is obtaining complete state feedback with sufficient accuracy. In a commercial installation, certainly measurement of the rotor bar currents would not be desirable. Therefore it will be necessary to design an *observer* to determine the rotor currents. Observer design is hampered by the time varying nature of the system. However an initial study, by the author, has shown that an observer based on a d-q transformed version of the model is feasible [7].

A controller was designed for the linearised system, a phase lead compensator was chosen, with a closed loop bandwidth of 1000rad/s = 167Hz. Hence the a typical rise time of about 5ms can be expected:

$$\bar{K}(s) = 10^6 \frac{\sqrt{10}s + 1000}{s + 1000\sqrt{10}} \tag{18}$$

The simulated results of the feedback linearisation scheme presented in the *Results* section are based on exact knowledge of the state vector, and are implemented using the previously described coupled circuit model.

### Control Winding Phase Angle Control

Implementation of phase angle control is relatively straightforward. The only practical difficulty is the measurement of $\theta_r'$. A direct approach would be to apply equation (12). However in practice this requires knowledge of $\omega_s$ to a high precision. The approach adopted in this study was to high-pass filter the rotor position, with a ramp-rejecting filter:

$$\bar{H}(s) = \frac{s^2}{(s + 0.7)^2} \tag{19}$$

The choice of the cross-over frequency is important. If the cross-over is too high, then the filter will reject the unstable dynamics of the BDFM, thus preventing the controller from stabilising the machine. However, the lower the cross-over frequency the slower the system speed of response. $0.7\text{rads}^{-1}$ was chosen as a compromise between these competing constraints.

A stabilising controller for the control winding phase angle control scheme was initially designed to stabilise the system of equation (17). However, in practice it was found that additional phase lead compensation was required. This is most likely due to additional phase lag introduced by the input filter, $\bar{H}(s)$. The design was refined by using system identification techniques to determine the plant transfer function more accurately. The final design is given in equation (20).

$$\bar{K}(s) = \alpha \frac{\left(\frac{\sqrt{10}}{6}s+1\right)^2}{\left(\frac{1}{6\sqrt{10}}s+1\right)^2} \quad (20)$$

Presented in the *results* section are simulated and actual experimental results. The simulated results use the coupled circuit model previously described. The parameters of the coupled circuit model are not exactly tuned such that it accurately models at actual BDFM tested. However the model is based on the same machine configuration, and is close enough to the actual machine to illustrate the merits of the control strategy.

## RESULTS

In this section simulated results are presented for feedback linearisation, and simulated and experimental results are presented for the phase angle control scheme.

In the machine simulated the **V** vector comprises of 24 elements: 3 power winding supply voltages, 3 control winding supply voltages and 18 elements for the cage rotor loops (which are all zero). Both the simulated and experimental results are based on a D180 frame size BDFM with an 8 pole power winding and a, 4 pole control winding in a 48 slot stator. The rotor has 36 slots, and 6 nests with a common end-ring, each nest comprising of 3 loops. The *natural* speed (that is, the speed of operation with DC applied to the control winding) is 500rpm. If the control winding can vary in frequency from -50Hz to +50Hz the drive has a speed range of 0 to 1000rpm, as $\omega_s = \frac{\omega_c+\omega_p}{p_p+p+c}$.

Figures 1, 2 and 3 serve to illustrate the dynamic problems experienced with the BDFM. In figures 1 and 2 the initial and final synchronous speeds are stable, however the change in speed was sufficient to pull the machine out

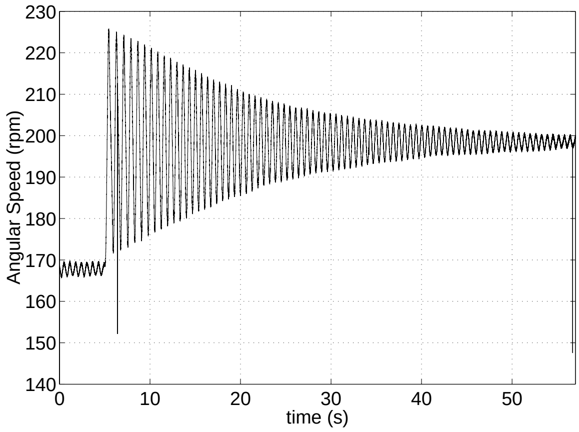


Figure 1: An illustration of the lightly damped behaviour of the test BDFM in response to a demanded step change in speed

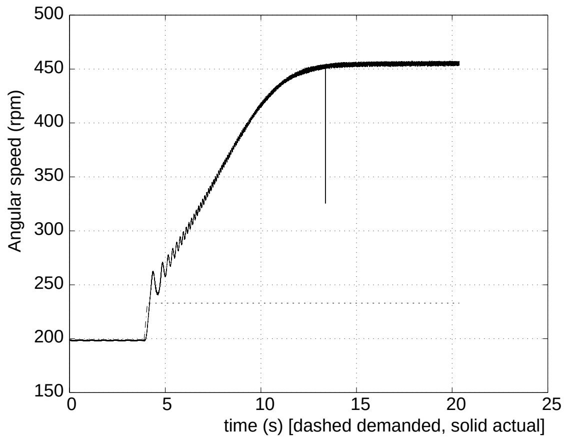


Figure 2: An illustration of an unstable demanded step change in speed

of synchronism in figure 2 and cause large oscillations in figure 1.

Figure 3 shows the BDFM tracking a ramp demanded speed input. Notice however, that the machine becomes unstable and looses synchronism at around 630rpm. This particular machine exhibited unstable regions from about 630rpm to 750rpm, where it was not possible to get the machine to lock into synchronism while driving a load.

### Feedback Linearisation (simulated)

Figures 4 and 5 illustrate control under feedback linearisation. The machine has been stabilised from 0 to 1000rpm (representing -50Hz to +50Hz frequency deviation on the control winding), and further the dynamic response is critically damped with a rise-time of about 5ms. The small change in phase angle after the load

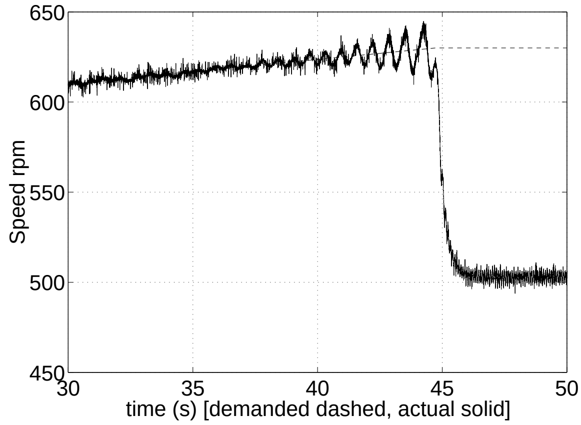


Figure 3: Open loop with a gentle demanded acceleration showing unstable region

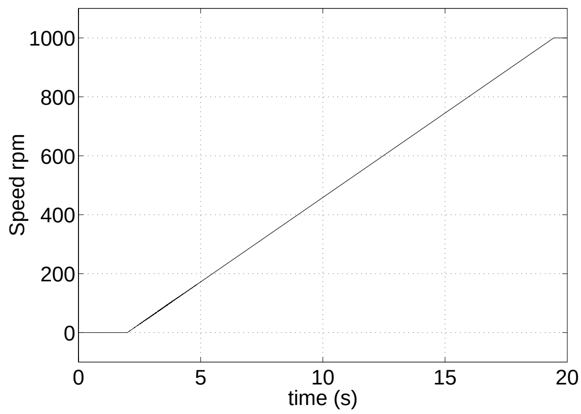


Figure 4: Demanded speed ramp tracking under feedback linearisation (simulated)

change is due to small imperfections in the linearisation due to computational issues.

### Control Winding Phase Angle Control

Comparing figure 6 with figure 3 it is evident that stabilisation of the BDFM synchronous operation has been achieved. The controller given by equation 20 stabilised this BDFM from 0 to 730rpm. The increase in output noise is due to position sensor noise which is not rejected due to the method of phase angle measurement.

Figure 7 shows the simulated BDFM following a demanded ramp under phase angle control. Notice that the machine remains in synchronism for the whole speed range from 0 to 1000rpm.

Figures 8 and 9 show the improved loading disturbance rejection of the phase control method.

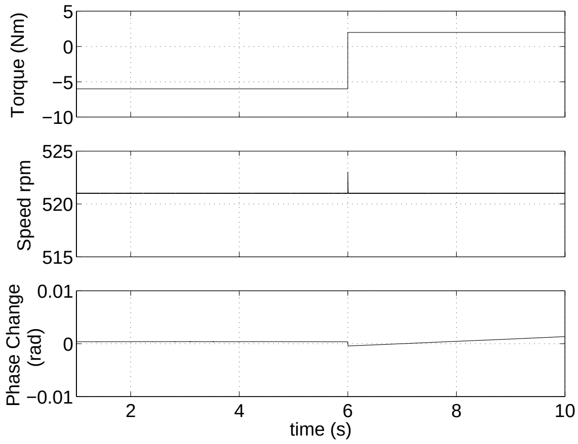


Figure 5: Demanded step change in load torque under feedback linearisation (simulated)

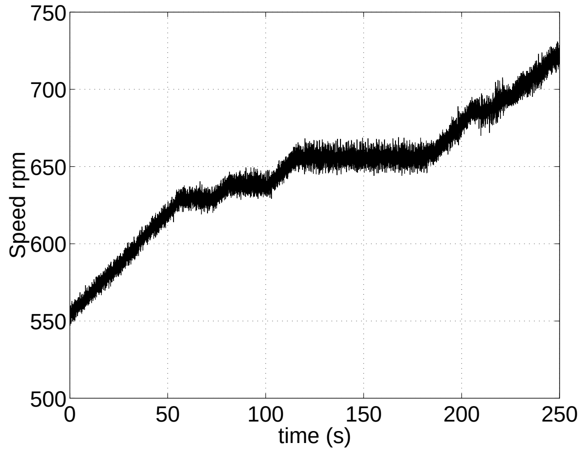


Figure 6: Closed loop phase control following a demanded speed ramp through unstable region (measured)

## CONCLUSIONS

The problem of stability in the synchronous mode of operation of a particular BDFM has been shown in both simulation and in practice. From the literature, it is clear that, similar problems have been experienced with other BDFM configurations. The approach of this paper has been to attempt to solve these stability problems by means of an external controller rather than a machine redesign. Two control strategies have been presented which can stabilise the BDFM. Although the results have been illustrated for a specific BDFM, the theory section shows that the same approach can be applied to any BDFM machine.

Feedback linearisation is a far more powerful control strategy than phase control. In addition to stabilising the BDFM it allows optimum control systems to be designed relatively simply. The only limitation on

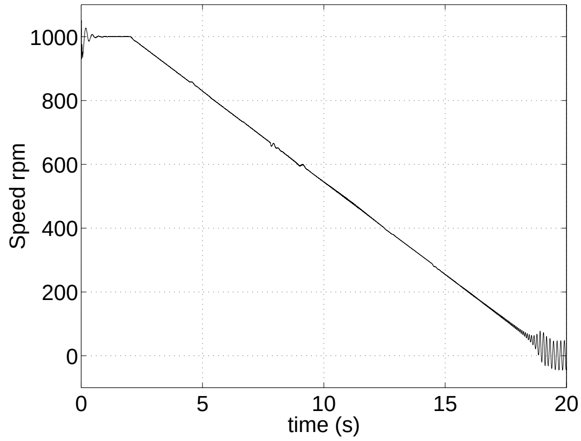


Figure 7: Simulation of demanded speed tracking under phase control

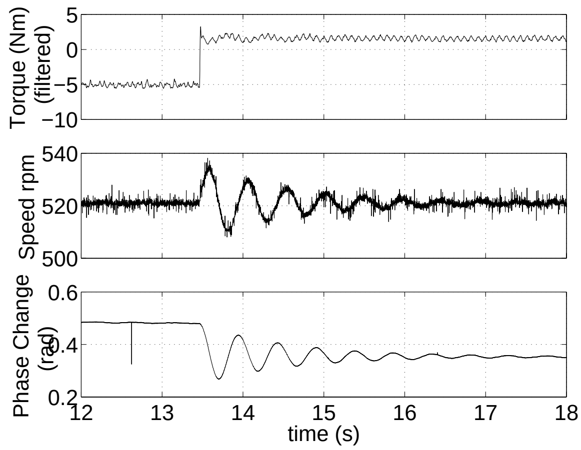


Figure 8: Uncontrolled step change in load

the performance of the control scheme is the physical limitations of the power supply. Phase control is based on a linearised steady state model. Therefore there will be performance limitations derived from the limited accuracy of the linearised model. Further, the present method of measurement of the referred rotor angle, $\theta_r'$ will only work when the system response is slow.

Feedback linearisation has major drawbacks in implementation. Certainly a state observer must be designed to 'measure' the rotor loop currents. Phase control is simple to implement, and once a stabilising strategy has been implemented system identification techniques can be used to optimise the controller design. Further work will be carried out to assess FBL performance and especially its robustness against inaccurate knowledge of the states predicted by the observer.

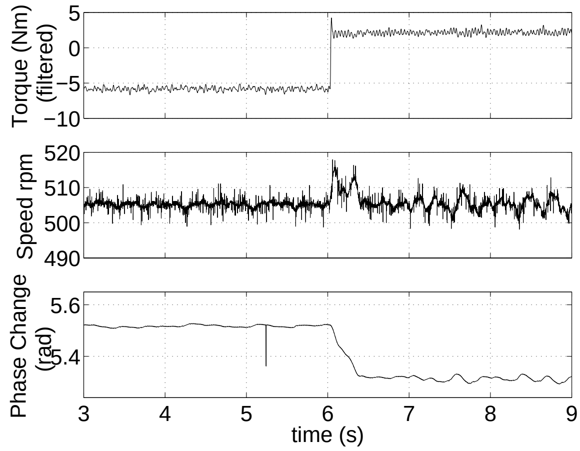


Figure 9: Step change in load under phase control

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