Discrete–event abstraction of quantised systems with asynchronous input and state events

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Abstract

The paper deals with the discrete–event modelling of continuous–variable dynamical systems whose input and state variables can only be measured through quantisers. It concerns the general case that input and state events occur asynchronously. The paper proposes an appropriate discrete–event model of the quantised system and presents an abstraction algorithm for determining the transition relation of this model. The abstraction is based on the reachability analysis. The abstracted models are proved to be complete.

Keywords: Asynchronous events, complete model, discrete–event abstraction, quantised system, reachability analysis

1 INTRODUCTION

Scope of the paper. In hybrid systems, continuous–variable and discrete–event subsystems are interconnected. One way to deal with such systems is to abstract a purely discrete–event description of the whole hybrid system. Then, analysis, control, supervision, verification, or diagnosis tasks can be solved by methods elaborated in discrete–event systems theory. This approach has been presented, for example, in [4, 7, 11, 13].

The main difficulty of this approach concerns the abstraction of a purely discrete–event description of the continuous subsystem including the input and state quantisers, which are the interfaces to the discrete subsystems. This setup is depicted in Figure 1. It is called a *quantised system* [5, 6, 9]. Only the qualitative values $[u]$ and $[x]$ of the input u and state x can be observed. Each change of the qualitative value of the state or the input is called an event. The problem is to find a discrete–event model that describes which qualitative input and state sequences can be generated by the quantised system (Figure 1).

Aim of the paper. The paper addresses two problems: First, a reasonable form of the discrete–event model has to be found for the quantised system (representation problem). Second, an abstraction procedure has to be found for determining the dynamical properties of this model for the given quantised system (abstraction problem).

Methods for abstracting discrete–event representations have either concerned discrete–time systems [5, 10] or continuous–time systems with synchronous input events [4, 7, 14]. In both situations, changes of the input $u(t)$ can only occur at the sampling instances or at the occurrence time of the output events. This simplifies the modelling task, because the input events take place at predefined time instances. This paper deals with the more general situation in which input events may occur at any time. This situation is typical in diagnostic applications or in distributed systems. It will be shown that the abstraction can be performed by a reachability analysis and, hence, has close connection to the results obtained, for example, in [1, 2, 12].

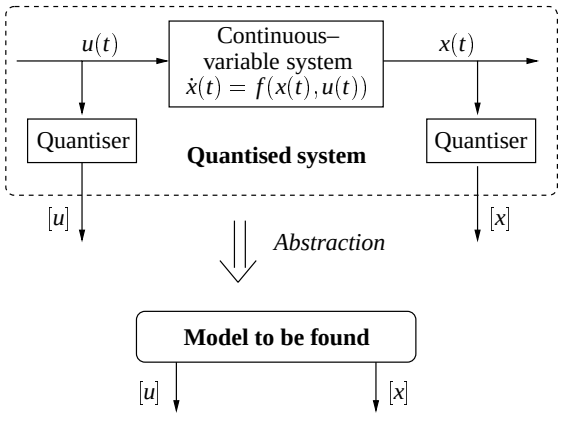

Figure 1: Quantised system and abstraction task

Structure of the paper. In Section 2, the quantised system is defined and it is explained that the quantised system has a nondeterministic behaviour. Section 3 states the modelling aim and proposes to use a non-

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