## A discrete-event model of asynchronous quantised systems

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## Abstract

The paper deals with the discrete-event modelling of continuous-variable dynamical systems whose input and state signals can only be measured through quantisers. A change of the quantised input or state value is called an input or a state event. The paper concerns the general case in which these events may occur asynchronously to one another. A nondeterministic automaton is used as the discrete-event model of such quantised systems. The paper presents an abstraction algorithm for determining the state transition relation of the automaton from the state-space description of the continuous-time continuous-variable system and the signal space partitions introduced by the quantisers. The abstraction is based on the reachability analysis of quantised systems. A hierarchy of models with respect to the set of spurious behaviours is obtained by varying an abstraction parameter. The discrete-event models are proved to be complete models of the quantised system with the best possible transition relation.

Key words: Asynchronous events, discrete-event abstraction, quantised systems, reachability analysis, hybrid system

## 1 Introduction

**Scope of the paper.** The paper deals with the modelling of continuous-variable dynamical systems whose input and state signals can only be measured through quantisers (Figure 1). Such systems are called *quantised systems*. Changes of the quantised input or state values are called input event or state event, respectively. This paper concerns quantised systems where input and state events may occur asynchronously. The model to be found should refer directly to these events and is, therefore, called a *discrete-event abstraction* or a *qualitative model* of the continuous-variable system. An untimed purely discrete-event model is proposed, which describes the order in which the input and state events occur.

**Related work.** Results about quantised systems have been obtained within the research on discrete–event systems and hybrid systems [5,6,8–10], where the discrete–event model of the continuous–variable subsystem is combined with the model of the discrete–event subsystem to obtain a purely discrete–event representa-



Fig. 1. Quantised system and abstraction task

tion of the hybrid system, which simplifies the analysis compared to the hybrid model. Similarly, discrete–event abstractions are used for the design of discrete controllers for continuous or hybrid systems [10,12], for the verification of discrete control algorithms for continuous plants [13] or for diagnosing faults that occur in hybrid systems [4].

Most of the work known in literature assumes that input and state events occur at the same time instances. However, if the input is not chosen with respect to the system output, input events  $e_u$  may occur asynchronously

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