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Adaptive predictive control algorithm based on Laguerre Functional Model

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SUMMARY

Laguerre Functional Model has many advantages such as good approximation capability for the variances of system time-delay, order and other structural parameters, low computational complexity, and the facility of online parameter identification, etc., so this model is suitable for complex industrial process control. A series of successful applications have been gained in linear and non-linear predictive control fields by the control algorithm based on Laguerre Functional Model, however, former researchers have not systemically brought forward the theoretical analyses of the stability, robustness, and steady-state performance of this algorithm, which are the keys to guarantee the feasibility of the control algorithm fundamentally. Aimed at this problem, we introduce the principles of the Incremental Mode Linear Laguerre Predictive Control (IMLLPC) algorithm, and then systemically propose the theoretical analyses and proofs of the stability and robustness of the algorithm, in addition, we also put forward the steady-state performance analysis. At last, the control performances of this algorithm on two different physical industrial plants are presented in detail, and a number of experimental results validate the feasibility and superiority of IMLLPC algorithm. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS: incremental mode; Laguerre Functional Model; model predictive control (MPC); stability; robustness

1. INTRODUCTION

Due to its similarity to Padé approximate, Orthonormal Functional Series (OFS) Model has some advantages such as good approximation capability for the variances of control plant's input time-delay, order and other structural parameters [1]. Moreover, combined with Volterra

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Series, this model can be easily extended to the field of non-linear predictive control. Therefore, it is more applicable to process control than other traditional linear models such as Controlled Auto-Regressive Integrated Moving Average (CARIMA) model [2], and pure data-driven models including Artificial Neural Network (ANN), Supporting Vector Machine (SVM), etc.

Laguerre series is one of the most elegant techniques [3] of OFS. It can date back to Lee (1931 [4] and 1961 [5]) and Wiener (1956 [6]). They found that the Laplace transforms of the classical orthonormal Laguerre functions, introduced in 1879 by the French mathematician—Laguerre, are very useful for approximating linear dynamic systems. In 1965 [7], another elegant technique of OFS, Kautz series, was described by Horowitz, who explained how to apply it to approximation methods in feedback system design. Owing to the attractive advantages of OFS, recently, there has been considerable interest in using OFS to design effective adaptive controllers. In 1995, Heuberger [8] and Wahlberg [7] proposed summaries on approximating dynamic linear systems by OFS in Z-domain and S-domain, respectively, which established the modelling foundation of the adaptive OFS control.

In 1988 [9], Zervos and Dumont proposed a novel linear MPC algorithm based on Laguerre series in which the control horizon equals one. The original analyses of robust stability and steady-state performance were given in their paper as well. In 1990 [1], they applied this scheme to pH control in an industrial bleach plant extraction stage, which was the first successful industrial application of OFS based control algorithms. The results were well received by the mill's personnel, because they gained a better closed-loop performance than the traditional algorithms with substantial savings on the operational costs to the management. Furthermore, based on Samll Gain Theorem, they gave a robust stability theorem for this algorithm. From 2000 to 2004, Zhang presented a lot of successful industrial applications of Laguerre functional series based control algorithm on high temperature semiconductor diffusion furnace [10], double water tanks [11], distillation columns [12] and water recycling irrigation system [13], etc. Meanwhile, he has also made some theoretical progresses for this algorithm [10–13]. In 2000 [9], Olivera *et al.* extended the Laguerre functional series based control algorithm to a robust one for systems with hard input constraints. Processes including integral action and hard input signal can also be considered in this strategy. In 2004 [3], Wang extended the design methodology from a continuous-time frame to a discrete-time one, of which the closed-loop stability was proved by imposing terminal states constraints.

However, most of the former OFS based control algorithms are MPC ones, so it is hard to analyse their closed-loop stability and robustness [2]. In order to guarantee the stability, several strict conditions must be imposed first, which inevitably adds conservativeness of these promising algorithms. For instance, the assumptions of Dumont's robust theorem [1] based on Small Gain Theorem are so rigid that the theorem is hard to use; Wang [3] imposed strict terminal states constraints; Olivera [14] required a very long predictive horizon; Agamennoni [15] required a series of different frequency response, which are hard to obtain in large-scale industrial processes; Jordán [16] proposed strict preconditions to ensure the existence of Kharitonov hypercube in the space of controller coefficients. In addition, when these conditions are applied to constrained plants, an optimization problem with a large amount of constraints must be solved in each sampling period, which imposes heavy computational loads on these methods and greatly limits their applications.

In a word, these theoretical works about stability, robustness, and steady-state performance are the foundation to guarantee the feasibility and superiority of the algorithms based on Laguerre Model. However, till now, there is no systematical theory for them.

The contributions of this papers are: (1) propose an effective adaptive predictive control algorithm—IMLLPC; (2) systematically present the theory on the stability, robustness and steady-state performance of this IMLLPC algorithm, and relax the sufficient conditions of former theorems [1, 14, 16]; (3) validate the feasibility and superiority of our IMLLPC algorithm by using successful applications on two different industrial plants.

The paper is organized as follows: The principles of IMLLPC are introduced in the next section. In Section 3, the stability proofs are given, while the robustness analyses are demonstrated in Section 4. Then, the theoretical analysis and proof of steady-state performance and some issues for unstable plants are presented in Section 5. The results of this algorithm's applications in semiconductor diffusion furnace's temperature control system and double tanks' water level control system are shown in Section 6. Finally, conclusion remarks are made in Section 7.

2. IMLLPC ALGORITHM

Definition 1 (*Wahlberg and Mäkilä [7], Heuberger et al. [8]*)
Laguerre Function is defined as a functional series

$$\Phi_i(t) = \sqrt{2p}\frac{e^{pt}}{(i-1)!}\cdot\frac{d^{i-1}}{dt^{i-1}}[t^{i-1}\cdot e^{2pt}], \quad i = 1, 2, \ldots, \infty \tag{1}$$

where p is a constant called time scaling factor [17], and $t \in [0, \infty)$ is a time variable.

Theorem 1 (*Wahlberg and Mäkilä [7], Heuberger et al. [8]*)
Laguerre Function series constructs a group of complete orthonormal bases in the function space $L_2(R^+)$ (square integrable function space in $[0, \infty)$).

The Laplace transformation of Laguerre function is

$$\Phi_i(s) = L[\Phi_i(t)] = \sqrt{2p}\frac{(s-p)^{i-1}}{(s+p)^i}, \quad i = 1, 2, \ldots, \infty \tag{2}$$

From Theorem 1 any open-loop stable system can be approximated by N order Laguerre series as shown in Figure 1

$$Y_m(s) = \sum_{i=1}^{N} C_i\Phi_i(s)U(s) = \sum_{i=1}^{N} C_i l_i(s) \tag{3}$$

The state space expression of Incremental Mode Laguerre Functional Model after discretization is

$$\Delta L(k+1) = A\Delta L(k) + b\Delta u(k) \tag{4}$$

$$\Delta y_m(k) = C^{\mathrm{T}}\Delta L(k) \tag{5}$$

where $\Delta L(k) = L(k) - L(k-1) = [\Delta l_1(k)\ \Delta l_2(k)\ \cdots\ \Delta l_N(k)]^{\mathrm{T}}$ is the state vector of the Incremental Mode Laguerre Functional Model; $\Delta y_m(k) = y_m(k) - y_m(k-1)$, $\Delta u(k) = u(k) -$

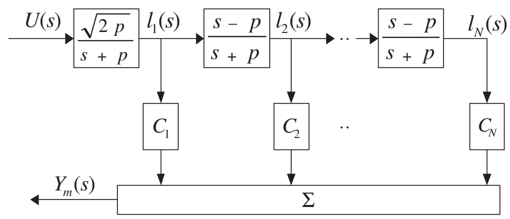

Figure 1. Laguerre series model structure.

$u(k-1)$ are the input and output of this model in the kth sampling period, respectively; $C^{\mathrm{T}} = [c_1 \;\; c_2 \;\; \cdots \;\; c_N]$ is the Laguerre coefficients vector.

$$A = \begin{bmatrix} \tau_1 & 0 & \cdots & 0 \\ \dfrac{-\tau_1\tau_2 - \tau_3}{T} & \tau_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \dfrac{(-1)^{N-1}\tau_2^{N-2}(\tau_1\tau_2+\tau_3)}{T} & \cdots & \dfrac{-(\tau_1\tau_2+\tau_3)}{T} & \tau_1 \end{bmatrix}$$

$$b^{\mathrm{T}} = [\tau_4, (-\tau_2/T)\tau_4, \ldots, (-\tau_2/T)^{N-1}\tau_4], \quad \tau_1 = \mathrm{e}^{-pT}, \quad \tau_2 = T + \frac{2}{p}(\mathrm{e}^{-pT}-1)$$

$$\tau_3 = -T\mathrm{e}^{-pT} - \frac{2}{p}(\mathrm{e}^{-pT}-1), \quad \tau_4 = \sqrt{2p}\,\frac{(1-\tau_1)}{p}, \quad T: \text{sampling period} \tag{6}$$

We calculate Δu instead of u in the controller, because this method can import integral mechanism, which can guarantee zero steady-state error in the closed-loop system [18, 19]. This fact will be proved in Section 5. Choosing p according to the theoretical method [17], combined with the known sampling period T, we can compute the above matrices A, b offline, which can reduce the online computational burden greatly.

Equations (4) and (5) yield

$$\begin{aligned} \Delta L(k+2) &= A^2\Delta L(k) + Ab\Delta u(k) + b\Delta u(k+1) \\ &\vdots \\ \Delta L(k+M) &= A^M\Delta L(k) + \sum_{i=0}^{M-1} A^{M-i}b\Delta u(k+i) \\ &\vdots \\ \Delta L(k+P) &= A^P\Delta L(k) + \sum_{i=0}^{M-1} A^{P-1-i}b\Delta u(k+i) \end{aligned} \tag{7}$$

and

$$
\begin{aligned}
\Delta y_m(k+1) &= C^{\mathrm{T}} A \Delta L(k) + C^{\mathrm{T}} b \Delta u(k) \\
&\vdots \\
\Delta y_m(k+M) &= C^{\mathrm{T}} A^{M} \Delta L(k) + \sum_{i=0}^{M-1} C^{\mathrm{T}} A^{M-i} b \Delta u(k+i) \\
&\vdots \\
\Delta y_m(k+P) &= C^{\mathrm{T}} A^{P} \Delta L(k) + \sum_{i=0}^{M-1} C^{\mathrm{T}} A^{P-1-i} b \Delta u(k+i)
\end{aligned} \tag{8}
$$

Consider that

$$
\begin{aligned}
y_m(k+1) &= y_m(k) + \Delta y_m(k+1) \\
y_m(k+2) &= y_m(k) + \Delta y_m(k+1) + \Delta y_m(k+2) \\
&\vdots \\
y_m(k+P) &= y_m(k) + \Delta y_m(k+1) + \cdots + \Delta y_m(k+P)
\end{aligned} \tag{9}
$$

Then, let $Y_m(k+1) = [y_m(k+1), \ldots, y_m(k+P)]^{\mathrm{T}}$ be the system output vector of future P steps, and let $\Delta U_M(k) = [\Delta u(k), \ldots, \Delta u(k+M-1)]^{\mathrm{T}}$ be the system input vector of future M steps, where P is prediction horizon, M is control horizon, and $P \geqslant M$. We have

$$
Y_m(k+1) = SH_l \Delta L(k) + SH_u \Delta U_M(k) + \Phi y_m(k) \tag{10}
$$

where

$$
S = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & & \\ \vdots & & \ddots & \\ 1 & 1 & \cdots & 1 \end{bmatrix}_{P\times P}, \quad \Phi = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}_{P\times 1}
$$

$$
H_l = \begin{bmatrix} C^{\mathrm{T}} A \\ C^{\mathrm{T}} A^2 \\ \vdots \\ C^{\mathrm{T}} A^{P} \end{bmatrix}_{P\times N}, \quad H_u = \begin{bmatrix} C^{\mathrm{T}} b & 0 & \cdots & 0 \\ C^{\mathrm{T}} A b & C^{\mathrm{T}} b & & \\ & & \ddots & \\ C^{\mathrm{T}} A^{M-1} b & \cdots & \cdots & C^{\mathrm{T}} b \\ \vdots & & & \vdots \\ C^{\mathrm{T}} A^{P-1} b & \cdots & \cdots & C^{\mathrm{T}} A^{P-M} b \end{bmatrix}_{P\times M} \tag{11}
$$

After feedback rectification [2] $K[y(k) - y_m(k)]$, the future P steps of output vector of Laguerre Model is

$$\hat{Y}_m(k+1) = SH_l\Delta L(k) + SH_u\Delta U_M(k) + \Phi y_m(k) + K[y(k) - y_m(k)] \tag{12}$$

Where, $y(k)$ is plant's output in the kth sampling period. K is rectification gain vector, in general, $K = [1, \ldots, 1]^{\mathrm{T}}_{P\times 1}$.

The quadratic control cost function is

$$J = \left\| Y_r(k+1) - \hat{Y}_m(k+1)\right\|_Q^2 + \left\|\Delta U_M(k)\right\|_R^2 \tag{13}$$

where $Y_r(k+1) = [y_r(k+1) \ \cdots \ y_r(k+P)]^{\mathrm{T}}$ is system's future P steps of output reference vector and

$$y_r(k+i) = \alpha^i y(k) + (1-\alpha^i)w, \quad i = 1, 2, \ldots, P \tag{14}$$

w is set point. Q, R are diagonal weighted matrices, $Q = \mathrm{diag}\{q_1, \ldots, q_P\}$, $R = r \cdot I_{M\times M}$ where $q_i\,(1 \leqslant i \leqslant P)$ and r are weighted factors, $I_{M\times M}$ is a unit matrix, $0 < \alpha < 1, \alpha$ is the soften factor.

Use $\partial J/\partial \Delta U_M(k) = 0$ to minimize the cost function (13), then the control law is

$$\Delta U_M(k) = (H_u^{\mathrm{T}} S^{\mathrm{T}} Q S H_u + R)^{-1} H_u^{\mathrm{T}} S^{\mathrm{T}} Q[Y_r(k+1) - \hat{Y}_p(k+1)] \tag{15}$$

where

$$\hat{Y}_p(k+1) = SH_l\Delta L(k) + \Phi y_m(k) + K[y(k) - y_m(k)] \tag{16}$$

the first element of the control vector is selected as current control signal

$$\Delta u(k) = D\Delta U_M(k) \tag{17}$$

where $D = [1, 0, \ldots, 0]_{1\times M}$.

C^{T} can be identified online by RLS (recursive least square) [20] algorithm with a forgetting factor

$$\hat{C}(k) = \hat{C}(k-1) + \frac{P(k-1)\Delta L(k)}{\lambda + \Delta L^{\mathrm{T}}(k)P(k-1)\Delta L(k)} \cdot [\Delta y(k) - \hat{C}^{\mathrm{T}}(k)\Delta L(k-1)] \tag{18}$$

$$P(k) = \frac{1}{\lambda}\left[P(k-1) - \frac{P(k-1)\Delta L(k)\Delta L^{\mathrm{T}}(k)P(k-1)}{\lambda + \Delta L^{\mathrm{T}}(k)P(k-1)\Delta L(k)}\right] \tag{19}$$

where $0 < \lambda < 1$, λ is the forgetting factor.

3. NOMINAL STABILITY ANALYSIS OF IMLLPC

3.1. IMAC (Incremental Model Algorithm Control) algorithm [18,19]

Let $\{h_i, 1 \leqslant i < \infty\}$ be the impulse response series of controlled plant, then the output of plant's model described by the impulse response is

$$y_m(k) = \sum_{i=1}^{\infty} h_i u(k-i) \tag{20}$$

When the controlled plant is asymptotically stable, we can conclude $\lim_{i\to\infty} h_i = 0$. So, let the truncation length of impulse response be N', in other words, $h_i = 0$ for $\forall i > N'$, then

$$y_m(k) = \sum_{i=1}^{N'} h_i u(k-i) \tag{21}$$

and

$$y_m(k-1) = \sum_{i=1}^{N'} h_i u(k-i-1) \tag{22}$$

Subtract (22) from (21), we have

$$\Delta y_m(k) = \sum_{i=1}^{N'} h_i \Delta u(k-i) \tag{23}$$

(23) yields that

$$\begin{aligned} \Delta y_m(k+j) &= \Delta y_{mf}(k+j) + \Delta y_{mp}(k+j) \\ &= \sum_{i=1}^{j} h_i \Delta u(k+j-i) + \sum_{i=j+1}^{N'} h_i \Delta u(k+j-i), \quad (j = 1, 2, \ldots, P) \end{aligned} \tag{24}$$

where Δy_{mp} and Δy_{mf} are the parts inspired by the system's past input and future input, respectively.

$$y_m(k+j) = y_m(k) + \Delta y_m(k+1) + \cdots + \Delta y_m(k+j) \tag{25}$$

After output feedback rectification, system's future output is

$$\hat{y}_m(k+j) = y_m(k) + \Delta y_m(k+1) + \cdots + \Delta y_m(k+j) + [y(k) - y_m(k)] \tag{26}$$

Let $e(k) = y(k) - y_m(k)$, then

$$\hat{Y}_m(k+1) = \Phi y_m(k) + S(H_{uf}\Delta U_M + H_{up}\Delta U_p) + Ke(k) \tag{27}$$

where

$$\hat{Y}_m(k+1) = [\hat{y}_m(k+1) \;\; \hat{y}_m(k+2) \;\; \cdots \;\; \hat{y}_m(k+P)]^{\mathrm{T}}, \quad \Phi = K = [1 \cdots 1]^{\mathrm{T}}_{P\times 1}$$

$$H_{uf} = \begin{bmatrix} h_1 & & & \\ h_2 & h_1 & & \\ & & \ddots & \\ h_M & h_{M-1} & \cdots & h_1 \\ & \cdots & \cdots & \\ h_P & h_{P-1} & \cdots & h_{P-M+1} \end{bmatrix}_{P\times M} \tag{28}$$

$$H_{up} = \begin{bmatrix} h_2 & h_3 & & & h_{N'} \\ h_3 & h_4 & & h_{N'} & \\ \vdots & & \iddots & & \\ h_{P+1} & \cdots & h_{N'} & & \end{bmatrix}_{P\times(N'-1)} \tag{29}$$

$$\Delta U_M = \begin{bmatrix} \Delta u(k) \\ \vdots \\ \Delta u(k+M-1) \end{bmatrix}, \quad \Delta U_p = \begin{bmatrix} \Delta u(k-1) \\ \vdots \\ \Delta u(k-N'+1) \end{bmatrix}$$

The quadratic control cost function is set as

$$J = \| Y_r(k+1) - \hat{Y}_m(k+1)\|_Q^2 + \|\Delta U_M(k)\|_R^2 \tag{30}$$

Let $\partial J/\partial \Delta U_M(k) = 0$, then

$$\Delta U_M(k) = (H_{uf}^{\mathrm{T}} S^{\mathrm{T}} Q S H_{uf} + R)^{-1} H_{uf}^{\mathrm{T}} S^{\mathrm{T}} Q [Y_r(k+1) - \hat{Y}_p(k+1)] \tag{31}$$

where

$$\hat{Y}_p(k+1) = S H_{up} \Delta U_p + \Phi y_m(k) + K e(k) \tag{32}$$

This is the IMAC algorithm based on plant's impulse response series.

3.2. *IMLLPC's equivalence*

Now, we analyse IMLLPC algorithm as follows.

From (4) and (5), we have

$$\Delta L(k) = (zI - A)^{-1} b \Delta u(k-1) = \sum_{i=1}^{\infty} A^{i-1} b \Delta u(k-i) \tag{33}$$

$$\Delta y_m(k) = C^{\mathrm{T}} (zI - A)^{-1} b u(k-1) = \sum_{i=1}^{\infty} C^{\mathrm{T}} A^{i-1} b \Delta u(k-i) \tag{34}$$

Because $C^{\mathrm{T}} A^{i-1} b$ $(i = 1, 2, \ldots)$ are Markov series of Incremental Mode Laguerre Functional Model, if the model is matching, these series are exactly the impulse response series of controlled plant, say,

$$h_i = C^{\mathrm{T}} A^{i-1} b \quad (i = 1, 2, \ldots) \tag{35}$$

For asymptotically stable system, let the truncation length of impulse response be N', then

$$\Delta L(k) = \sum_{i=1}^{N'} A^{i-1} b \Delta u(k-i) \tag{36}$$

and

$$\Delta y_m(k) = \sum_{i=1}^{N'} h_i \Delta u(k-i) \tag{37}$$

Then, combined with (36), (16) yields that

$$\begin{aligned}
\hat{Y}_P(k+1) &= SH_l \Delta L(k) + Ke(k) + \Phi y_m(k) \\
&= SH_l \sum_{i=1}^{N-1} A^{i-1} b \Delta u(k-i) + Ke(k) + \Phi y_m(k) \\
&= S \begin{bmatrix} C^{\mathrm{T}} A \\ \vdots \\ C^{\mathrm{T}} A^P \end{bmatrix} \sum_{i=1}^{N-1} A^{i-1} b \Delta u(k-i) + Ke(k) + \Phi y_m(k) \\
&= S \begin{bmatrix} C^{\mathrm{T}} Ab & C^{\mathrm{T}} A^2 b & \cdots & C^{\mathrm{T}} A^{N'-1} b \\ C^{\mathrm{T}} A^2 b & C^{\mathrm{T}} A^3 b & \cdots & C^{\mathrm{T}} A^{N'} b \\ \vdots & \vdots & \ddots & \vdots \\ C^{\mathrm{T}} A^P b & C^{\mathrm{T}} A^{P+1} b & \cdots & C^{\mathrm{T}} A^{P+N'-2} b \end{bmatrix} \cdot \begin{bmatrix} \Delta u(k-1) \\ \Delta u(k-2) \\ \vdots \\ \Delta u(k-N'+1) \end{bmatrix} \\
&\quad + Ke(k) + \Phi y_m(k) \\
&= S \begin{bmatrix} h_2 & h_3 & \cdots & h_{N'} \\ h_3 & h_4 & \cdots & h_{N'+1} \\ \vdots & \vdots & \vdots & \vdots \\ h_{P+1} & h_{P+2} & \cdots & h_{N'+P-1} \end{bmatrix} \begin{bmatrix} \Delta u(k-1) \\ \Delta u(k-2) \\ \vdots \\ \Delta u(k-N'+1) \end{bmatrix} \\
&\quad + Ke(k) + \Phi y_m(k)
\end{aligned} \tag{38}$$

Consider that, when $i > N'$, $h_i = 0$, so

$$\hat{Y}_P(k+1) = SH_p \Delta U_p(k) + Ke(k) + \Phi y_m(k) \tag{39}$$

where

$$H_p = \begin{bmatrix} h_2 & h_3 & & & h_{N'} \\ h_3 & h_4 & & h_{N'} & \\ \vdots & & \iddots & & \\ h_{P+1} & \cdots & h_{N'} & & \end{bmatrix}_{P\times(N'-1)}$$

$$\Delta U_p(k) = \begin{bmatrix} \Delta u(k-1) \\ \Delta u(k-2) \\ \vdots \\ \Delta u(k-N'+1) \end{bmatrix} \tag{40}$$

thus

$$\hat{Y}_m(k+1) = SH_p\Delta U_p(k) + SH_u\Delta U_m(k) + Ke(k) + \Phi y_m(k) \tag{41}$$

where

$$H_u = \begin{bmatrix} h_1 & & & \\ h_2 & h_1 & & \\ & & \ddots & \\ h_M & h_{M-1} & \cdots & h_1 \\ & \cdots & \cdots & \\ h_P & h_{P-1} & \cdots & h_{P-M+1} \end{bmatrix}_{P\times M} \tag{42}$$

Consequently, Equation (41) is equivalent to IMAC's Equation (27), and the control law

$$\Delta U_M(k) = (H_u^{\mathrm{T}} S^{\mathrm{T}} Q S H_u + R)^{-1} H_u^{\mathrm{T}} S^{\mathrm{T}} Q[Y_r(k+1) - \hat{Y}_p(k+1)] \tag{43}$$

is equivalent to IMAC's control law (31). Therefore, IMLLPC is equivalent to IMAC, and all the stability theorems of IMAC can be imported to IMLLPC algorithm.

3.3. The stability theorems of IMAC

Based on the theory on polynomial roots' location, Shu [18] systematically gave stability theorems for IMAC.

Lemma 1 (*Shu [18]*)
For the closed-loop system determined by IMAC control law (31), if the parameters are set as $Q = I_{P\times P}$ (a unit matrix), $r = 0$, $M = 1$, then there exists an long enough predictive time horizon P which can make system asymptotically stable.

Proof
Refer to Reference [18]. □

Lemma 2 (*Shu [18]*)
For the closed-loop system determined by IMAC control law (31), let the $r > 0$ in weighted matrix $R = rI_{M\times M}$, then the sufficient and necessary condition which guarantees closed-loop asymptotically stability only by increasing r is

$$\sum_{i=1}^{P}\left(\sum_{j=1}^{i} h_j\right) q_i \cdot \sum_{l=1}^{N'} h_l > 0 \tag{44}$$

where q_i, $i = 1, 2, \ldots, P$ are diagonal elements of weighted matrix Q.

Proof
Refer to Reference [18]. □

Use the equivalence of IMAC and IMLLPC algorithms, we can get the following two theorems.

Theorem 2
For the closed-loop system determined by IMLLPC control law (15) and (17), if the model determined by (4) and (5) is matching, and the parameters are set as $Q = I$, $r = 0$, $M = 1$, then there exists an enough long predictive time horizon P which can make system asymptotically stable.

Theorem 3
For the closed-loop system determined by IMLLPC control law (15) and (17), if the model determined by (4) and (5) is matching, let $r > 0$ in weighted matrix $R = rI$, then the sufficient and necessary condition which guarantees closed-loop asymptotically stability only by increasing r is

$$\sum_{i=1}^{P} \beta_i q_i \cdot C^{\mathrm{T}}(I - A)^{-1} b > 0 \tag{45}$$

where $\beta_i = C^{\mathrm{T}}(A^{i-1} + A^{i-2} + \cdots + I)b$ is the sum of front i elements of the Markov series of the Incremental Mode Laguerre Functional Model; q_i ($i = 1, 2, \ldots, P$) are the diagonal elements of the weighted matrix Q.

Proof
Because

$$\beta_i = \sum_{j=1}^{i} C^{\mathrm{T}} A^{j-1} b = \sum_{j=1}^{i} h_j \tag{46}$$

and each of the eigenvalues of A is inside the unit circle of Z-plane, say, $|\lambda_i(A)| < 1$, $i = 1, 2, \ldots, N$, then we can get

$$\lim_{N'\to\infty} \sum_{l=1}^{N'} h_l = \sum_{l=1}^{\infty} C^{\mathrm{T}} A^{l} b = \lim_{N'\to\infty} C^{\mathrm{T}}(I - A^{N'})(I - A)^{-1} b = C^{\mathrm{T}}(I - A)^{-1} b \tag{47}$$

Then (45) is equivalent to (44), therefore Theorem 3 can be deduced from Lemma 2. □

4. ROBUSTNESS ANALYSES OF IMLLPC

Assume the controlled plant is open-loop asymptotically stable and can be described as the following equations:

$$\Delta X(k+1) = A_0 \Delta X(k) + b_0 \Delta u(k) \tag{48}$$

$$\Delta y(k) = C_0^{\mathrm{T}} \Delta X(k) \tag{49}$$

Let

$$d^{\mathrm{T}} = D(H_u^{\mathrm{T}} S^{\mathrm{T}} Q S H_u + R)^{-1} H_u^{\mathrm{T}} S^{\mathrm{T}} Q \tag{50}$$

and ignore the output feedback rectification, the control law (15) and (17) can be rewritten as

$$\Delta u(k) = d^{\mathrm{T}}[Y_r(k+1) - SH_l \Delta L(k) - K y_m(k)] \tag{51}$$

Theorem 4

Consider an open-loop asymptotically stable system expressed by (48) and (49), if the last step's output $y(k-1) = \xi_k$ is identically bounded and the Incremental Mode Laguerre Functional Model determined by (4) and (5) is mismatching. Then, the closed-loop system determined by control law (51) is asymptotically stable if and only if each eigenvalue of $(A + \Psi_1 + \Psi_2)$ is inside the unit circle of Z-plane, where

$$\begin{aligned} \Psi_1 &= b d^{\mathrm{T}}(K_a - K) C^{\mathrm{T}}, \quad \Psi_2 = -b d^{\mathrm{T}} S H_l \\ K_\alpha &= [\alpha \ \cdots \ \alpha^P]^{\mathrm{T}} \end{aligned} \tag{52}$$

Proof

Let $K_{1-\alpha} = [(1-\alpha) \ \cdots \ (1-\alpha^P)]^{\mathrm{T}}$.

Then from Equations (4), (5), (48) and (49), we have

$$y_m(k) = y(k-1) + \Delta y_m(k) = \xi_k + C^{\mathrm{T}} \Delta L(k) \tag{53}$$

$$y(k) = y(k-1) + \Delta y(k) = \xi_k + C_0^{\mathrm{T}} \Delta X(k) \tag{54}$$

and

$$\begin{aligned} \Delta X(k+1) = A_0 \Delta X(k) + b_0 d^{\mathrm{T}}[&K_\alpha(\xi_k + C^{\mathrm{T}} \Delta L(k)) + K_{1-\alpha} w(k) \\ &- SH_l \Delta L(k) - K(\xi_k + C^{\mathrm{T}} \Delta L(k))] \end{aligned} \tag{55}$$

$$\begin{aligned} \Delta L(k+1) = A \Delta L(k) + b d^{\mathrm{T}}[&K_\alpha(\xi_k + C^{\mathrm{T}} \Delta L(k)) + K_{1-\alpha} w(k) \\ &- SH_l \Delta L(k) - K(\xi_k + C^{\mathrm{T}} \Delta L(k))] \end{aligned} \tag{56}$$

(55) and (56) can be rewritten as

$$\begin{bmatrix} \Delta X(k+1) \\ \Delta L(k+1) \end{bmatrix} = \begin{bmatrix} A_0 & b_0 d^{\mathrm{T}}[(K_a - K)C^{\mathrm{T}} - SH_l] \\ 0 & A + bd^{\mathrm{T}}[(K_a - K)C^{\mathrm{T}} - SH_l] \end{bmatrix} \cdot \begin{bmatrix} \Delta X(k) \\ \Delta L(k) \end{bmatrix} + \begin{bmatrix} b_0 \\ b \end{bmatrix} d^{\mathrm{T}}[(K_\alpha - K)\xi_k + K_{1-\alpha} w]$$
$$= \begin{bmatrix} A_0 & b_0 d^{\mathrm{T}}[(K_\alpha - K)C^{\mathrm{T}} - SH_l] \\ 0 & A + \Psi_1 + \Psi_2 \end{bmatrix} \cdot \begin{bmatrix} \Delta X(k) \\ \Delta L(k) \end{bmatrix} + \begin{bmatrix} b_0 \\ b \end{bmatrix} d^{\mathrm{T}}[(K_\alpha - K)\xi_k + K_{1-\alpha} w] \quad (57)$$

Then the eigenvalues of this system's closed-loop state matrix are composed of the eigenvalues of A_0 and $(A + \Psi_1 + \Psi_2)$. The controlled plant is open-loop asymptotically stable, each eigenvalue of A_0 is inside the unit circle, say, $|\lambda(A_0)| < 1$, consider that the output of the last step ξ_k is identically bounded, therefore, the necessary and sufficient condition is that each eigenvlaue of $(A + \Psi_1 + \Psi_2)$ is inside the unit circle. □

Remark 1
Because

$$H_u^{\mathrm{T}} S^{\mathrm{T}} S H_u = \sum_{j=0}^{P-1} \left(\sum_{i=0}^{j} C^{\mathrm{T}} A^i b \right)^2 \quad (58)$$

Let

$$\delta = \frac{1}{\sum_{j=0}^{P-1} \left(\sum_{i=0}^{j} C^{\mathrm{T}} A^i b \right)^2} = \frac{1}{\sum_{j=0}^{P-1} \left(\sum_{i=0}^{j} h_i \right)^2} \quad (59)$$

where $h_i = C^{\mathrm{T}} A^i b,\ (0 \leqslant i \leqslant P)$ are the Markov series of Incremental Mode Laguerre Model.

Consider

$$\Psi_1 = bd^{\mathrm{T}}(K_a - K)C^{\mathrm{T}} = bD(H_u^{\mathrm{T}} S^{\mathrm{T}} Q S H_u + R)^{-1} H_u^{\mathrm{T}} S^{\mathrm{T}} Q (K_a - K) C^{\mathrm{T}} \quad (60)$$

When $Q = I$, $r = 0$, $M = 1$, we have

$$\Psi_1 = b(H_u^{\mathrm{T}} S^{\mathrm{T}} S H_u)^{-1} H_u^{\mathrm{T}} S^{\mathrm{T}} (K_a - K) C^{\mathrm{T}} = \delta \sum_{j=0}^{P-1} \sum_{i=0}^{j} C^{\mathrm{T}} A^i b \cdot (\alpha^i - 1) b C^{\mathrm{T}}$$
$$= \delta \sum_{j=0}^{P-1} \sum_{i=0}^{j} [h_i(\alpha^i - 1)] \cdot bC^{\mathrm{T}} \quad (61)$$

and

$$\Psi_1 b = \delta \sum_{j=0}^{P-1} \sum_{i=0}^{j} [h_i(\alpha^i - 1)] \cdot bC^{\mathrm{T}} b = \delta \sum_{j=0}^{P-1} \sum_{i=0}^{j} [h_i(\alpha^i - 1) h_0] \cdot b \quad (62)$$

Ψ_1 can be expressed as the product of a column vector and a row vector, so Ψ_1 have only one non-zero eigenvalue, which is assumed to be $\lambda(\Psi_1)$. Then, from the changeability of column vector b, we conclude that

$$\lambda(\Psi_1) = \delta \sum_{j=0}^{P-1} \sum_{i=0}^{j} [h_i(\alpha^i - 1)h_0] \tag{63}$$

Because $h_0 \approx 0$, and $|\alpha^i - 1| < 1$, when P is big enough, we have

$$\lambda(\Psi_1) \to 0 \tag{64}$$

Consider

$$\Psi_2 = -bd^{\mathrm{T}}SH_l = -bD(H_u^{\mathrm{T}}S^{\mathrm{T}}QSH_u + R)^{-1}H_u^{\mathrm{T}}S^{\mathrm{T}}QSH_l \tag{65}$$

When $Q = I$, $r = 0$, $M = 1$, we have

$$\Psi_2 = -b(H_u^{\mathrm{T}}S^{\mathrm{T}}SH_u)^{-1}H_u^{\mathrm{T}}S^{\mathrm{T}}SH_l \tag{66}$$

$$= -\delta b\left[C^{\mathrm{T}}b, C^{\mathrm{T}}b + C^{\mathrm{T}}Ab, \ldots, \sum_{i=0}^{P-1} C^{\mathrm{T}}A^i b\right] \begin{bmatrix} C^{\mathrm{T}}A \\ \vdots \\ \sum_{i=1}^{P} C^{\mathrm{T}}A^i \end{bmatrix} \tag{67}$$

and

$$\begin{aligned} \Psi_2 b = &-\delta b\left[C^{\mathrm{T}}b, C^{\mathrm{T}}b + C^{\mathrm{T}}Ab, \ldots, \sum_{i=0}^{P-1} C^{\mathrm{T}}A^i b\right] \begin{bmatrix} C^{\mathrm{T}}Ab \\ \vdots \\ \sum_{i=1}^{P} C^{\mathrm{T}}A^i b \end{bmatrix} \\ = &-\delta b \sum_{i=1}^{P} \left(\sum_{j=0}^{i-1} C^{\mathrm{T}}A^j b \cdot \sum_{j=1}^{i} C^{\mathrm{T}}A^j b \right) \\ = &-\delta b \sum_{i=1}^{P} \left(\sum_{j=0}^{i-1} h_j \cdot \sum_{j=1}^{i} h_j \right) = -\delta \sum_{i=0}^{P-1} \left(\sum_{j=0}^{i} h_j \cdot \sum_{j=0}^{i} h_{j+1} \right) b \end{aligned} \tag{68}$$

Ψ_2 can be expressed as the product of a column vector and a row vector, so Ψ_2 have only one non-zero eigenvalue, which is assumed to be $\lambda(\Psi_2)$. Then, from the changeability of column vector b, we can conclude that

$$\lambda(\Psi_2) = -\delta \sum_{i=0}^{P-1} \left(\sum_{j=0}^{i} h_j \cdot \sum_{j=0}^{i} h_{j+1} \right) \tag{69}$$

Because $\{h_i\}$ are the Markov series of Incremental Mode Laguerre Model which is open-loop stable, when i is big enough, $h_i > h_{i+1}$ we have

$$-1 < \lambda(\Psi_2) < 0 \tag{70}$$

Therefore, the eigenvalues of A, Ψ_1, Ψ_2 can be thoroughly analysed, respectively, however, the eigenvalues of $(A + \Psi_1 + \Psi_2)$ can only be gained by matrix perturbation theory [21].

Lemma 3 (*Generalized Bauer–Fike Theorem, Kahan et al. [21]*)
Consider matrices A, B, E, where $B = A + E \in C^{n\times n}$, if $A = Q^{-1}JQ$, where J is the Jordan standard canonical of A, then for any eigenvalue λ of matrix B, there must exist an eigenvalue μ of matrix A, which satisfies the following inequality:

$$\frac{|\mu - \lambda|^m}{(1 + |\mu - \lambda|)^{m-1}} \leqslant \|Q^{-1}EQ\|_2 \tag{71}$$

where m is the order of the maximal Jordan block which belongs to eigenvalue μ in matrix J.

Proof
Refer to Reference [21]. □

Corollary 1
The conditions are the same as Theorem 4, besides, assume Laguerre series order $N \geqslant 2$, then, a sufficient condition of system closed-loop asymptotically stable is

$$\|Q^{-1}EQ\|_2 < \frac{(1 - \mathrm{e}^{-pT})^N}{(2 - \mathrm{e}^{-pT})^{N-1}} \tag{72}$$

where

$$A = Q^{-1}JQ, \quad E = \Psi_1 + \Psi_2$$

$$J = \begin{bmatrix} \mathrm{e}^{-pT} & 1 & \cdots & 0 \\ 0 & \mathrm{e}^{-pT} & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & \mathrm{e}^{-pT} \end{bmatrix} \tag{73}$$

Proof
From Theorem 4, we have, if $|\lambda(B)| < 1$, where $B = A + E$, then the closed-loop system expressed by (4), (5), (48), (49) and (51) is asymptotically stable. From (6), A is a lower triangular matrix, and the exclusive eigenvalue of A is e^{-pT}. Besides, we can find that the maximal Jordan block order which belongs to eigenvalue e^{-pT} in matrix J is N. In addition, because $pT > 0$, we have $0 < 1 - \mathrm{e}^{-pT} < 1$.

For an arbitrary eigenvalue λ of B, then, from Lemma 3 and inequality (72), we have

$$\frac{|\lambda - \mathrm{e}^{-pT}|^N}{(1 + |\lambda - \mathrm{e}^{-pT}|)^N} \leqslant \|Q^{-1}EQ\|_2 < \frac{(1 - \mathrm{e}^{-pT})^N}{(2 - \mathrm{e}^{-pT})^{N-1}} \tag{74}$$

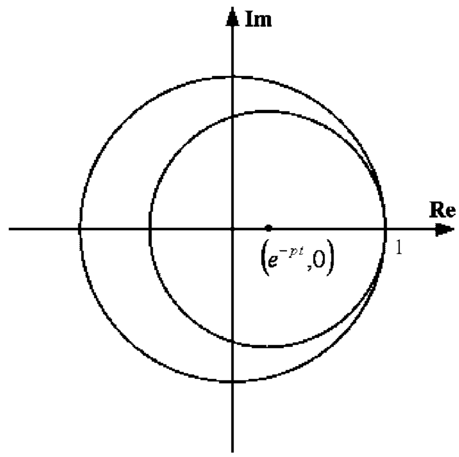

Figure 2. Illustration of Corollary 1 in Z-plane.

and so

$$\frac{|\lambda - \mathrm{e}^{-pT}|^N}{(1 + |\lambda - \mathrm{e}^{-pT}|)^{N-1}} < \frac{(1 - \mathrm{e}^{-pT})^N}{(1 + 1 - \mathrm{e}^{-pT})^{N-1}} \tag{75}$$

Consider that the function $f(x) = x^N/(1 + x)^{N-1}$ is monotonously increasing when $N \geqslant 2$, thus $|\lambda - \mathrm{e}^{-pT}| < 1 - \mathrm{e}^{-pT}$. As shown in Figure 2, each eigenvalue of closed-loop system matrix $(A + E)$ is inside the small circle, whose centre and radius are $(\mathrm{e}^{-pT}, 0)$ and $(1 - \mathrm{e}^{-pT})$, respectively. This small circle is inside the unit circle, so each eigenvalue of closed-loop system matrix $(A + E)$ is inside the unit circle, too. Therefore, the closed-loop system is asymptotically stable. □

Remark 2

Consider Corollary 1, the nearer the center $(\mathrm{e}^{-pT}, 0)$ is to the origin, the closer the small circle is to the unit circle, and thus, the stable region given by Corollary 1 becomes larger and more applicable. In industrial applications, suitable p, T are always selected to satisfy $\mathrm{e}^{-pT} \in [0.03, 0.06]$, which makes the stable region large enough.

Matrix perturbation theory, Theorem 4, Corollary 1, Remark 1 and Remark 2 can be used to research further in the robustness of IMLLPC, and then, to discover more applicable robustness theorems of IMLLPC.

5. STEADY-STATE PERFORMANCE ANALYSIS OF IMLLPC

Theorem 5

The control law determined by (15) and (17) can eliminate steady-state error under output disturbance.

Proof
Equations (4) and (5) yield that

$$\Delta L(k) = (zI - A)^{-1} b \Delta u(k-1) \tag{76}$$

$$y_m = C^{\mathrm{T}}(zI - A)^{-1} b z^{-1} u(k) \tag{77}$$

So the control law is

$$\Delta u(k) = d^{\mathrm{T}}[Y_r(k+1) - F(z^{-1})u(k) - \Phi e(k)] \tag{78}$$

where

$$F(z^{-1}) = [SH_l(1 - z^{-1}) + \Phi C^{\mathrm{T}}] \cdot (zI - A)^{-1} b z^{-1} \tag{79}$$

The internal model control structure of IMLLPC can be seen in Figure 3. After unifying

$$u(k) = \frac{1}{\tilde{F}(z^{-1})}[D_r(z^{-1}) y_r(k+P) - K_f e(k)] \tag{80}$$

where

$$\tilde{F}(z^{-1}) = \frac{1}{d_s}[1 - z^{-1} + d^{\mathrm{T}} F(z^{-1})], \quad d_s = \sum_{i=1}^{P} d_i, \quad K_f = \sum_{i=1}^{P} \frac{d_i}{d_s} = 1,$$

$$D_r = \frac{1}{d_s} \sum_{i=1}^{P} d_i z^{-(P-i)}, \quad d = [d_1, d_2, \ldots, d_P]^{\mathrm{T}}$$

Without loss of generality, we ignore the soften mechanism, say, $G_r = 1$.
Thus, system's steady-state error is

$$E(\infty) = \lim_{k \to \infty,\, z^{-1} \to 1} \left[\frac{\tilde{F}(z^{-1}) - G(z^{-1}) + G_f(z^{-1}) K_f (G(z^{-1}) - \hat{G}(z^{-1}))}{\tilde{F}(z^{-1}) + G_f(z^{-1}) K_f (G(z^{-1}) - \hat{G}(z^{-1}))} W \right.$$
$$\left. - \frac{K_f G_f(z^{-1}) \hat{G}(z^{-1}) - \tilde{F}(z^{-1})}{[\hat{G}(z^{-1}) - G(z^{-1})] K_f G_f(z^{-1}) - \tilde{F}(z^{-1})} \bar{d} \right] \tag{81}$$

As shown in Figure 3, $G(z^{-1})$ and $\hat{G}(z^{-1})$ in (81) represent the plant's and Laguerre mode's transfer functions, respectively, $G_f(z^{-1})$ is the feedback transfer function which satisfies $\lim_{z^{-1} \to 1} G_f(z^{-1}) = 1$, W is set point, and $\bar{d}(k)$ is outside perturbation.

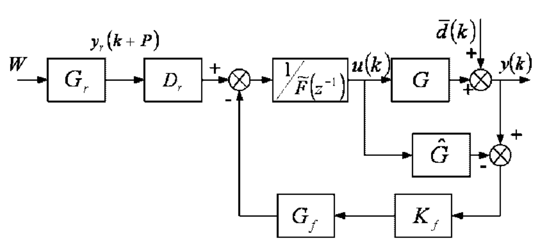

Figure 3. Internal model control structure of IMLLPC.

In steady-state, we have

$$\lim_{z^{-1}\to 1} \hat{G}(z^{-1}) = C^{\mathrm{T}}(I - A)^{-1}b = \lim_{z^{-1}\to 1} \tilde{F}(z^{-1}) \tag{82}$$

and

$$\lim_{z^{-1}\to 1} G(z^{-1}) = \lim_{z^{-1}\to 1} \hat{G}(z^{-1}) \tag{83}$$

Thus, substitute (82) and (83) into (81), we have $E(\infty) = 0$. □

Theorem 5 proves that IMLLPC can trace set point curves without error, which is an excellent capability.

Remark 3
IMAC is based on the impulse response of the plant, so the feasible condition for IMAC is that the controlled plant should be open-loop stable. Moreover, only open-loop stable plant which belongs to the space $L_2(R^+)$ can be modelled by Laguerre Series. Consequently, these above theoretical results for IMLLPC are suitable for open-loop stable plants. However, that does not mean that IMLLPC cannot control open-loop unstable plants. Indeed, after some improvements [13, 22], IMLLPC can be applied to some unstable plants including integral plants and plants with unstable poles or zeros. Furthermore, the theoretical results for these improved IMLLPC algorithms, such as closed-loop stability, robustness, etc. can also be obtained based on the above theoretical results for standard IMLLPC.

6. APPLICATIONS OF IMLLPC

6.1. Water level control of double tanks

Double tanks water level control system can be seen in Figure 4. As shown in the right subfigure this figure, control variables of this system are the control currents u_1, u_2 of electromagnetic valves R_1, R_2, and the outputs are the water levels of the double tanks h_1, h_2. Q_i $(i = 1, \ldots, 5)$ are water flows. The control signals u_1, u_2 are transferred to standard electronic signals between 4

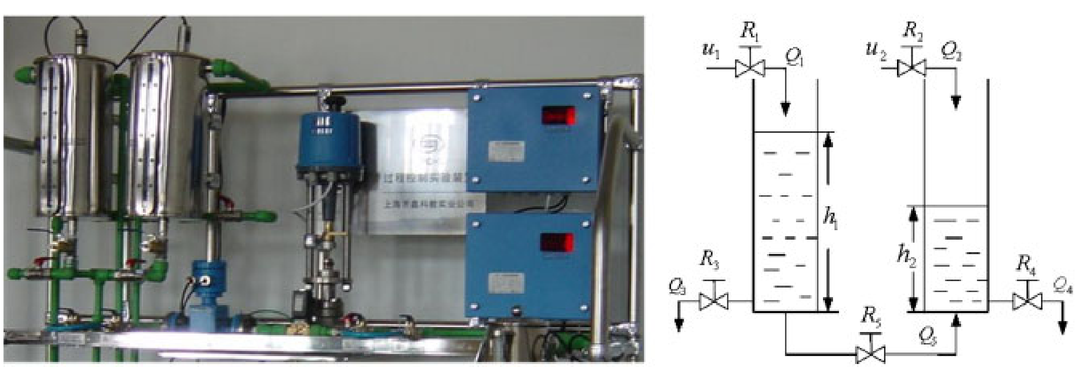

Figure 4. Water control system of double tank.

and 20 mA, while the output signals (0–300 mm water levels) h_1, h_2 are transferred to standard electronic signals between 1 and 5 V by HMPK2-0.00588-A-0.5-AGAB-typed pole-inputting water level sensors, whose accuracy is $\pm 0.25\%$. Hand valves R_3, R_4 are used to adjust double tanks' drain flows Q_3, Q_4, respectively, while hand valve R_5 is used to modulate the flow Q_5 between the two ranks.

6.1.1. SISO (single input/single output) water level control of cascaded tanks. In this experiment, R_5 is completely opened, R_2, R_3 are completely closed, and R_4 is partly opened. u_1 is used to control h_2. This cascaded water level SISO control system can be described by mechanism differential equations (84) [23], where $\varphi(u_1)$ is the input flow of left tank, and $g(h_1, h_2)$ is the flow passing R_5

$$\begin{aligned} A_1 \frac{\mathrm{d}h_1}{\mathrm{d}t} &= \varphi(u_1) - g(h_1, h_2) \\ A_2 \frac{\mathrm{d}h_2}{\mathrm{d}t} &= g(h_1, h_2) - K\sqrt{h_2} \end{aligned} \tag{84}$$

Because of the long pipe line between the electromagnetic valves and the water tanks' entries, the system has long time-delay. Moreover, the fluctuation of the water flow velocity makes the time-delay varying. Therefore, besides the non-linearity in Equations (84), varying long time-delay also makes this plant difficult to control.

Initial water levels of the double tanks are set to be 0 mm. Valve opening of R_5 is decreased by 20% at the 480th second (GPC) and the 420th (IMLLPC), which equals to importing a big disturbance to plant's characteristics. Control parameters are set as: In GPC predictive horizon $P = 8$, control horizon $M = 5$, soften factor $\alpha = 0.6$, control weighted factor $r = 0.2$, sampling period $T = 1$ s; In IMLLPC: $P = 7$, $M = 5$, $\alpha = 0.5$, $r = 0.2$, $T = 1$ s, time scaling factor $p = 1.5$, Laguerre series order $N = 7$, forgetting factor $\lambda = 0.8$.

Figure 5 shows the control performances of IMLLPC. The overshooting, the response time of the output and the oscillation amplitudes of the control and output variables are much smaller than the counterparts of GPC under perturbation of control plant. Moreover, the control precision in steady-state of IMLLPC ($\pm 0.5\%$) is much higher than that of GPC $\pm$ 0.8%.

6.1.2. Double tanks' 2 inputs and 2 outputs water level control. In this experiment, initial water levels of the double tanks are set to be 0. R_3, R_4 are completely opened, R_5 is partly opened. u_1 and u_2 are used together to control h_1, h_2. The valve opening R_5 determines the coupling intensity.

This 2 inputs and 2 outputs control system can be described by mechanism differential equations (85) [23], where $\varphi(u_1)$, $\varphi(u_2)$ are the input flows of left tank and right tank, respectively, and $g(h_1, h_2)$ is the flow passing R_5

$$\begin{aligned} A_1 \frac{\mathrm{d}h_1}{\mathrm{d}t} &= \varphi(u_1) - g(h_1, h_2) - K\sqrt{h_1} \\ A_2 \frac{\mathrm{d}h_2}{\mathrm{d}t} &= \varphi(u_2) + g(h_1, h_2) - K\sqrt{h_2} \end{aligned} \tag{85}$$

Parameters are set as: In GPC, $P = 8$, $M = 5$, soften factor vector $\alpha = [0.6\ \ 0.5]$, control weighted factor vector $r = [0.2\ \ 0.2]$; In IMLLPC: $\alpha = [0.7\ \ 0.5]$, forgetting factor $\lambda = [0.8\ \ 0.8]$, $N = 8$. The other parameters are the same as above IMLLPC for SISO system. It should be

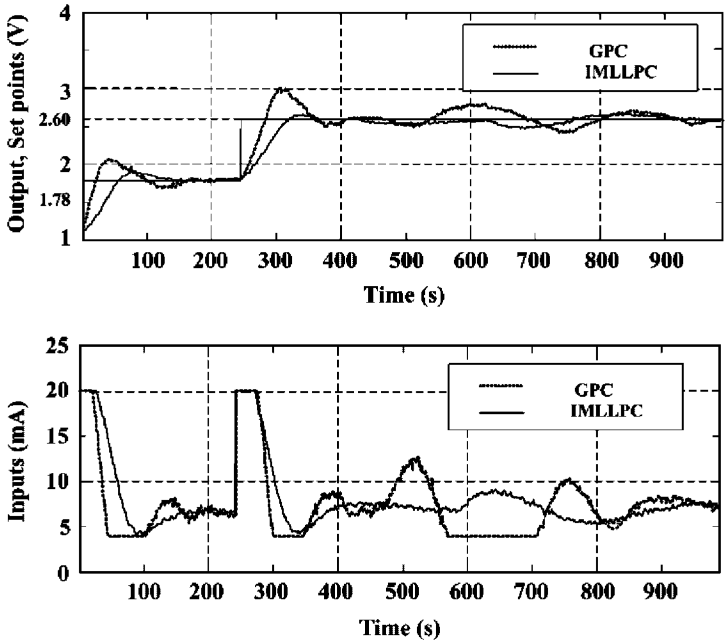

Figure 5. Control performances of SISO system.

noted that, in multi-input/multi-output (MIMO) system, the scalar factors α, λ, r of SISO system are extended into vector factors. Set points w_1, w_2 are step curves. Valve opening of R_5 is increased by 20% at the 210th second, which increases the coupling intensity of the two tanks. As shown in Figure 6, both GPC and IMLLPC can trace set curves. However, the overshootings, the response time of the output and the oscillation amplitudes of the control and output variables of IMLLPC are much smaller then the counterparts of GPC. Moreover, the control precision of IMLLPC ($\pm 0.8\%$) is much higher than that of GPC ($\pm 1.4\%$). Thus, the control algorithm's superiority is verified.

6.2. Temperature control of semiconductor diffusion furnace

Diffusion furnace is an equipment in semiconductor apparatus production which can be used to diffuse semiconductor particulates [10]. It is a pipe shaped resistance heater whose main heating section is enlaced by heating wires evenly. As shown in Figure 7, the sensor of the diffusion furnace is a thermal couple, whose voltage signal is transferred by the ICP 7018 module to temperature signal whose accuracy is $\pm 0.1^\circ$C. A controlled silicon component acts as the executer (controller). The ICP 7043 and ICP 7520 modules are the A/D transmitter and the transmitter between RS-232 bus signals and RS-485 bus signals, respectively.

As to the furnace chamber's temperature, in reality, the diffusion furnace is a two dimension distributed parameters dynamic system. However, consider the evenly heating structure, this system is generally simplified as a centralize parameters system whose transfer function is

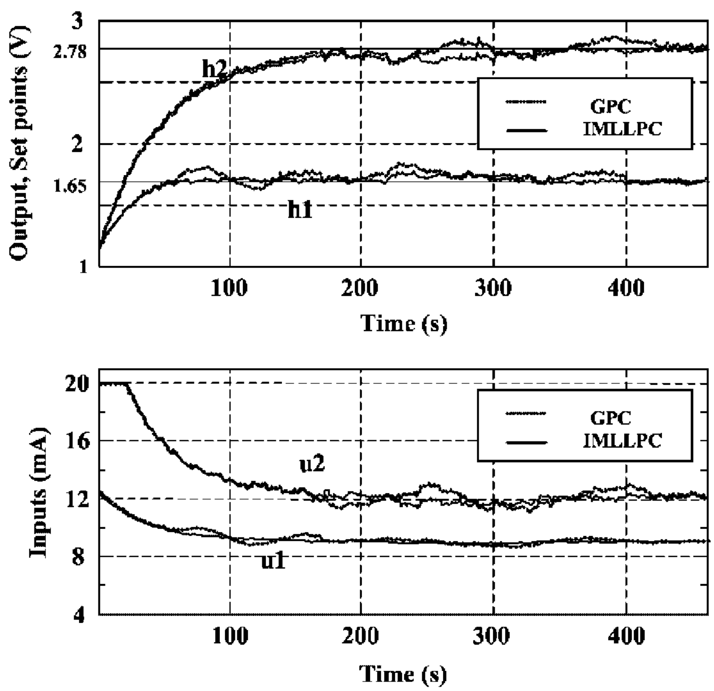

Figure 6. Control performance of 2 inputs and 2 outputs system.

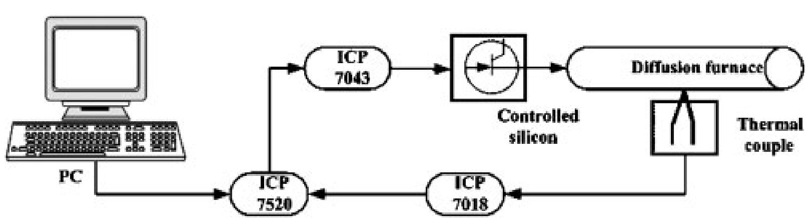

Figure 7. Structure of diffusion furnace control system.

$G(s) = K\mathrm{e}^{-\tau s}/(TS + 1)$. The input and output of this system are the heating power and the furnace chamber's temperature, respectively.

The requirements of steady-state performances in temperature control are very strict by semiconductor production technics. Generally, the steady-state error between the measured output value and the set value is required to be under $\pm 0.5^{\circ}\mathrm{C}$. However, the furnace is fairly hard to control, the reasons are stated as follows.

The heating signals can affect the temperature after a comparatively long time, moreover, the dynamic characteristics of this furnace can be easily influenced by the variance of surrounding temperature and device aging of itself, so this plant has a long varying time-delay. Besides, there is no cooling equipment in this furnace and the adiabatic material in the chamber makes the temperature difficult to fall, so the cooling process of the furnace chamber is much slower than the heating process, which causes a big dynamic characteristic difference between these two processes. Consequently, some traditional predictive controllers such as GPC [2, 24], and DMC [18, 25], etc. cannot give satisfying control performances so far.

IMLLPC is applied to diffusion furnace system. Control parameters in IMLLPC is set as: $T = 5$ s; $p = 1.5$; $N = 7$; $\lambda = 0.9$; $\alpha = 0.5$; $P = 7$; $M = 5$. The control variable value is required to be transferred to an integer between 5 and 20 mA by the ICP7520 module, in which 5 mA is the lower limit representing zero heating power and 20 mA is the upper limit indicating full heating power.

Control performances of IMLLPC are presented in Figure 8. There are 3 curves tracing 945, 895 and 845°C, respectively. These experiments show that the steady-state errors are smaller than ± 0.3°C, which can satisfy the requirements of the semiconductor production technics. In the course of experiment tracing 895°C, we purposely open the cover at one end of the furnace to quicken the cooling speed. In this way, output perturbation is imported, meanwhile, the furnace's characteristic suffers a big variance. However, it can be seen from the experiment results that this disturbance influences little on diffusion furnace's temperature control performance. Therefore, conclusion can be drawn that this furnace's IMLLPC system has shown great adaptation to the variances of the plant's time-delay and other structural characteristics. In addition, IMLLPC algorithm has shown good robustness to outside disturbance.

Remark 4

In these experiments, the strategy to choose p follows Reference [17]. Theorems 2–4 and Corollary 1 are used to choose suitable values for the other parameters, such as N, λ, α, P, M,

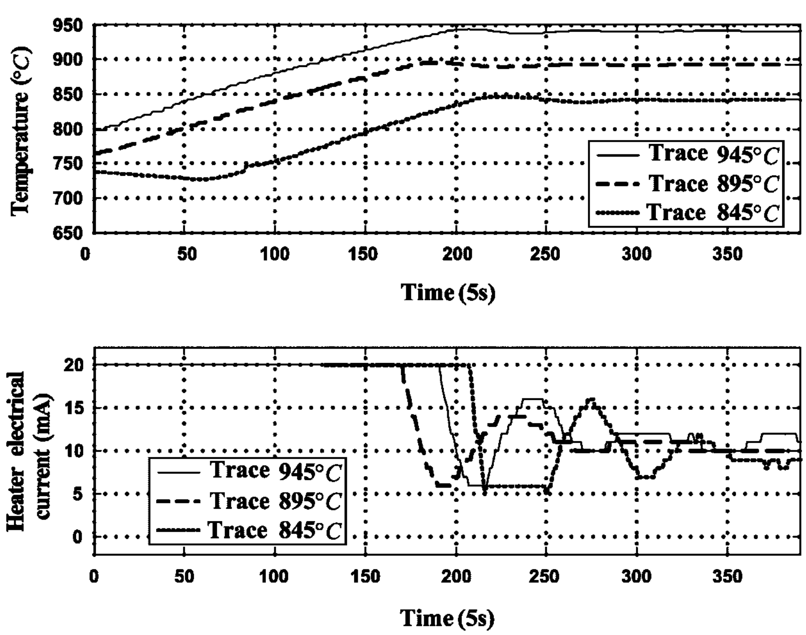

Figure 8. Temperature control performance of diffusion furnace.

etc. In addition, there are also some empirical or experimental methods to optimize these control parameters [2, 8, 11–14].

7. CONCLUSION

Laguerre functional model has advantages such as good approximation capability for the variances of control plant's time-delay, order and other structural parameters. Moreover, if combined with Volterra Series, Wiener Series, etc. this model can be easily extended to the domain of non-linear predictive control. Therefore, it is more applicable to process control than some traditional linear models such as CARIMA model [2, 23], etc. However, it is a pity that existing researches in this field have not systemically presented the theoretical analyses for control algorithms based on Laguerre Functional Model so far.

A new multi-step prediction and multi-step control adaptive predictive control algorithm, IMLLPC, is proposed in this paper. Then, the theoretical analyses of stability, robustness and steady-state performance are presented systemically. After that, the extensions to open-loop unstable plants are introduced briefly. At last, the applications of IMLLPC on two different real industrial plants are given in detail. A large amount of industrial application results have validated the feasibility and superiority of this control algorithm.

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