SVD-based Identification Algorithm for Hammerstein-typed Nonlinear Systems

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***Abstract* - Hammerstein-typed nonlinear models can be used to represent nonlinear systems in the areas of chemical processes, biological processes, signal processing, etc. Firstly a novel multi-channel algorithm for the identification of Hammerstein-typed nonlinear systems is presented, in which the coefficient parameters of the dynamic linear block and memoryless nonlinear block are identified by least squares estimation (LSE) combined with singular value decomposition (SVD). This identification algorithm can eliminate any needs for the mechanism or prior knowledge of the nonlinear or linear block. Furthermore, in comparison with traditional single-channel identification algorithm, this multi-channel one can increase the approximate accuracy remarkably. In addition, under weak assumptions on the persistency of excitation (PE) of the inputs, the algorithm provides consistent estimates in the presence of white output noise, moreover, its convergence can also be theoretically proved. At last, the performances of the identification algorithm are illustrated through simulations on a benchmark problem, a pH neutralization process, which validate the feasibility and superiority of these proposed algorithms.**

Index Terms –SVD, Hammerstein-typed nonlinear system, LSE, PE

I. INTRODUCTION

One of the most frequently studied classes of nonlinear models is Hammerstein-typed model [3], which consists of the cascade connection of a static (memoryless) nonlinear block followed by a dynamic linear block. This nonlinear system model structure have been successfully used to chemical processes (pH neuralization[4], distillation[5], etc), biological processes[1,6], signal processing[6], and communications[1], etc. Therefore, in recent years, the identification of Hammerstein-typed nonlinear systems has become one of the most urgent and difficult tasks in the fields of process control engineering, signal processing, etc.

Several techniques have been proposed in the references [1,2,7,8,9] for the identification of Hammerstein-typed nonlinearity. Of them, one of the most efficient methods, which is based on LSE, was introduced by Bai [7]. However, it can only deal with SISO (Single Input/ Single Output) system with output white noise. Inspired by Bai's work, Gómez and Baeyens[1,2] proposed a non-iterative identification, which can be applied to MIMO (Multi-Input/ Multi-Output) system and can guarantee consistent estimation even in the present of coloured output noise. Nevertheless, both of works of Gómez and Bai[1,2] use just one channel to identify the system, therefore, take the intrinsic of SVD into consideration, the identification error of these algorithms inevitably can not be minimized (or adjustable). The reason is that the identification error is determined by the 2nd largest singular value (for SISO system) or the $(n+1)$ th largest singular value (for MIMO system with n inputs).

Take the SISO system for instance, if the 2nd largest singular value is not small enough in comparison with the largest singular value the identification accuracy would be not satisfying, or even degrade to unacceptable level.

On the other hand, the achievements on the control of Hammerstein-typed nonlinearity are very limited. Most of the former related algorithms[10,11] rely on the mechanism or prior knowledge of the memoryless nonlinear block more or less. The precision of the mechanism or prior knowledge determines the performance of these algorithms, which limits the applications of them greatly.

Aimed at these two problems, we propose a SVD-based multi-channel identification algorithm in this paper, which can preserve all of the advantages of Gómez's algorithm, and minimize the identification error. More important is that it can also eliminate the reliance of the mechanism or prior knowledge of the memoryless nonlinear block. In addition, Due to its similarity to Padé Approximate[12], Lagurerre Functional Series has some advantages such as excellent capability to approximate the variances of control plant's input time-delay, order and other structural parameters[12-15], which are very common in real industrial productions. Thus, we take Laguerre Functional Series Model for example to approximate the dynamic linear block of each channel of our proposed multi-channel model, and then give the convergence theorem for this multi-channel model. At last, we presented the detailed identification performances of our proposed multi-channel algorithm in contrast with the performances of Gómez's single-channel identification algorithm[1].

The rest of the paper is organized as follows. In Section Ⅱ, the Hammerstein-typed system model is introduced, and the identification problem is formulated. Then, the multi-channel identification and modeling algorithm is derived and theoretically analyzed. Case studies are presented in Section III, which illustrate the performances of the modeling algorithms on a benchmark problem. Finally, conclusion remarks are made in Section IV.

II. IDENTIFICATION ALGORITHM

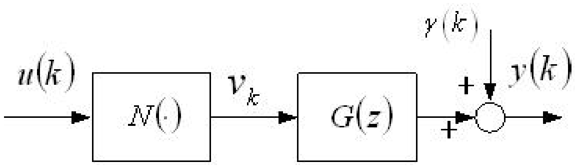

Fig. 1 Hammerstein-typed nonlinear system

A multivariable Hammerstein-typed nonlinear system is schematically represented in figure1. The model consists of memoryless nonlinear block $N(\cdot)$ in cascade with a dynamic linear block $G(z^{-1}) \in \mathbf{H}_2^{m\times n}(\mathbf{T})$ (Hardy space of $(m\times n)$ transfer matrices). The measured output $y(k)$ contains an unknown additive noise $\gamma(k)$.

The input/output relationship is then given by

$$y(k) = G(z^{-1})N(u(k)) + \gamma(k) \tag{1}$$

where $u(k) \in \Re^n$, $v(k) \in \Re^n$, $y(k) \in \Re^m$, $\gamma(k) \in \Re^m$ are the system input, mid output, system output and output noise vectors at time k, respectively. $\{\gamma(k)\}$ is a stochastic series defined in the probability space $(\boldsymbol{\Omega}, \mathbb{F}, \mathbf{P})$.

Then the output can be rewritten as

$$y(k) = \left(\sum_{l=1}^{N} c_l L_l(z^{-1})\right)\left(\sum_{i=1}^{r} a_i g_i(u(k))\right) + \gamma(k) = \sum_{l=1}^{N}\sum_{i=1}^{r} c_l a_i L_l(z^{-1})\left[g_i(u(k))\right] + \gamma(k) \tag{2}$$

where $g_i(\cdot): \Re^n \to \Re^n$, $(i = 1,\cdots,r)$ are known nonlinear bases, and $a_i \in \Re^{n\times n}$ are unknown matrix parameters. $c_l \in \Re^{m\times n}$ are unknown matrix parameters, and $\{L_l(\cdot)\}_{l=1}^{\infty}$ can be any rational orthonormal bases on the space $\mathbf{H}_2(\mathbf{T})$, N is the truncation length.

In order to make the parametrization unique, we normalize the parameter matrices a_i (or c_l), say

$$\|a_i\|_2 = 1 \ (i = 1,\cdots,r) \tag{3}$$

and define

$$\theta \stackrel{\Delta}{=} [c_1 a_1,\cdots,c_1 a_r,\cdots,c_N a_1,\cdots,c_N a_r]^T \tag{4}$$

$$\phi(k) \stackrel{\Delta}{=} [L_1(g_1^T(u(k))),\cdots,L_1(g_r^T(u(k))),\cdots, L_N(g_1^T(u(k))),\cdots,L_N(g_r^T(u(k)))]^T \tag{5}$$

Then (2) can be rewritten in linear regressor form [16] as

$$y(k) = \theta^T\phi(k) + \gamma(k). \tag{6}$$

Considering the S-point data set, we define

$$Y_S \stackrel{\Delta}{=} [y_1,\cdots,y_S]^T, \boldsymbol{\gamma}_S \stackrel{\Delta}{=} [\gamma_1,\cdots,\gamma_S]^T, \boldsymbol{\Phi}_S \stackrel{\Delta}{=} [\phi_1,\cdots,\phi_S]^T \tag{7}$$

then, we can obtain

$$Y_S = \boldsymbol{\Phi}_S^T\theta + \boldsymbol{\gamma}_S \tag{8}$$

Thus, provided the indicated inverse exists, it is well known that the estimate $\hat{\theta}$ of θ that minimize the prediction errors $\varepsilon_S = Y_S - \boldsymbol{\Phi}_S^T\theta$, which is the least square estimation [16] is given by

$$\hat{\theta} = \left(\boldsymbol{\Phi}_S\boldsymbol{\Phi}_S^T\right)^{-1}\boldsymbol{\Phi}_S Y_S \tag{9}$$

The problem is how to estimate the parameter matrices $a_i(i = 1,\cdots,r)$ and $c_l(l = 1,\cdots,N)$ from the estimate $\hat{\theta}$.

We define

$$\Theta_{ac} \stackrel{\Delta}{=} \begin{bmatrix} a_1^T c_1^T & a_1^T c_2^T & \cdots & a_1^T c_N^T \\ a_2^T c_1^T & a_2^T c_2^T & \cdots & a_2^T c_N^T \\ \vdots & \vdots & \ddots & \vdots \\ a_r^T c_1^T & a_r^T c_2^T & \cdots & a_r^T c_N^T \end{bmatrix} = ac^T \tag{10}$$

with the following definitions of matrices a, c,

$$a \stackrel{\Delta}{=} [a_1, a_2,\cdots,a_r]^T, c \stackrel{\Delta}{=} \left[c_1^T,\cdots,c_N^T\right]^T.$$

From the definition of θ, we can get that

$$\theta = blockvec(\in_{ac}) \tag{11}$$

where $blockvec(\in_{ac})$ is the block column matrix obtained by stacking the block columns of $\in_{ac}$ on the top of each other[1]. Then the estimate $\hat{\Theta}_{ac}$ of $\in_{ac}$ can be obtained from the estimate $\hat{\theta}$ in (11). The problem now is how to estimate the parameter matrices a and c from the estimate $\hat{\Theta}_{ac}$. It is clear that the closest, in the 2-norm sense [3], estimates $\hat{a}$ and $\hat{c}$ are the solutions of the optimization problem

$$(\hat{a},\hat{c}) = \arg\min_{a,c}\left\{\left\|\hat{\Theta}_{ac} - ac^T\right\|_2^2\right\} \tag{12}$$

Gómez [1] has given a technique which use SVD method to obtain the parameters matrices $a_i\ (i = 1,\cdots,r)$ and $c_l, (l = 1,\cdots,N)$. However, the approximation error of his algorithm is not small enough for most Hammerstein-typed nonlinear systems. So, an improved identification algorithm to minimize the approximation error must be proposed in this section.

Firstly, we will introduce a lemma proposed by Golub.

Lemma 1 [17]

Let $rank(\hat{\Theta}_{ac}) = q$, then the SVD of $\hat{\Theta}_{ac}$ is

$$\hat{\Theta}_{ac} = U_q\Sigma_q V_q^T = \sum_{j=1}^{q}\sigma_j\mu_j\varphi_j^T \tag{13}$$

where the singular matrix $\Sigma_q = diag\{\sigma_j\}$ fulfills that

$$\sigma_1 \geq \cdots \geq \sigma_q > 0 \qquad (14)$$

and

$$c_l = 0 \ (l > q) \qquad (15)$$

$\mu_j, \varphi_j, 1 = 1, 2, \cdots, q$, are pairwise orthogonal vectors, if $\left\| a^{(j)} \right\|_2 = 1$, then for $\forall N_1 : q$, we have

$$\left(\hat{a}^{(j)}, \hat{c}^{(j)}\right) = \underset{a^{(j)}, c^{(j)}}{\arg\min} \left[\left\| \hat{\Theta}_{ac} - \sum_{j=1}^{N_1} a^{(j)} \left(c^{(j)}\right)^T \right\|_2^2 \right] = \qquad (16)$$

$$\left(\mu_j, \sigma_j \varphi_j\right), \ j = 1, \cdots, N_1$$

and the approximation error is given by

$$\left\| \hat{\Theta}_{ac} - \sum_{j=1}^{N_1} a^{(j)} \left(c^{(j)}\right)^T \right\|_2^2 = \sum_{j=N_1+1}^{q} \sigma_j^2 \qquad (17)$$

where the superscript (j) indicate the j^{th} identification pair. ■

Based on **Lemma 1**, we can construct a multi-channel model, which is consisted by N_1 parallel channels (or submodels) and can remarkably increase the approximation accuracy, rather than a single-channel model [1] to identify the Hammerstein-typed system. Each channel is consists of the cascade connection of a static (memoryless) nonlinear block, which is represent by nonlinear bases, followed by a dynamic linear block.

In the following statement, without losing of generality, we choose $\{L_l(\cdot)\}_{l=1}^{\infty}$ (see (2)) as Laguerre Functional Series[17,18], which has excellent parameter robustness[14] and good capability to approximate the variance of the linear system's time-delay and orders[12], which are very common in modern industrial plants. In order to simplify the solution description of this identification problem, we will take SISO system, say, $u(k) \in \Re^1, y(k) \in \Re^1$, for instance to discuss. The results of MIMO system can be easily derived from the counterparts of SISO system. The details of Laguerre Functional Series Model can be seen in reference [13,15,19].

Now we will construct the multi-channel model, in which each channel's dynamic linear block is modeled by Laguerre Functional Series, to identify Hammerstein-typed systems.

Multi-channel Model:

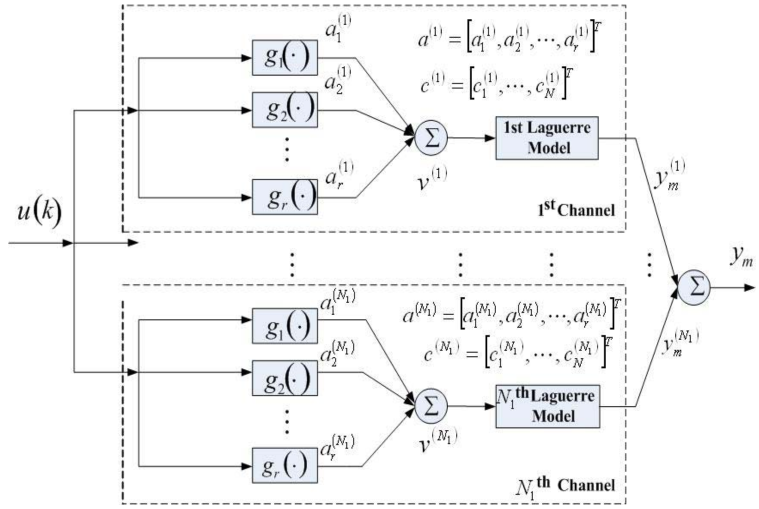

Fig. 2 Multi-channel nonlinear Laguerre model structure

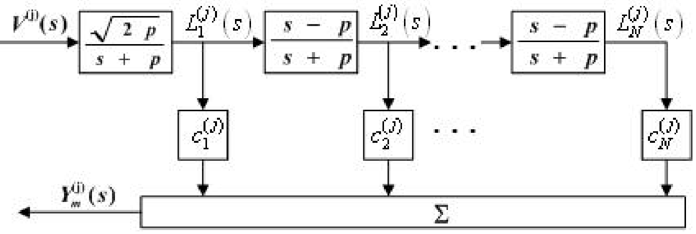

Fig. 3 Linear Laguerre model of the j^{th} channel, $(j = 1, \cdots, N_1)$

Define $a^{(j)} \overset{\Delta}{=} \left[a_1^{(j)}, \cdots, a_r^{(j)}\right]^T$,

$$c^{(j)} \overset{\Delta}{=} \left[c_1^{(j)}, \cdots, c_N^{(j)}\right]^T \ (j = 1, \cdots, N_1) \qquad (18)$$

where the superscript (j) indicate the j^{th} identification channel, which can be represented by a Laguerre Series Model as

$$L^{(j)}(k+1) = AL^{(j)}(k) + B\left[\sum_{i=1}^{r} a_i^{(j)} g_i(u(k))\right] (j = 1, \cdots, N_1) \qquad (19)$$

$$y_m^{(j)}(k) = \left(c^{(j)}\right)^T L^{(j)}(k) \qquad (20)$$

where

$y_m^{(j)}(k), L^{(j)}(k) = \left[L_1^{(j)}(k) \quad L_2^{(j)}(k) \quad \cdots \quad L_N^{(j)}(k)\right]^T$ are the output, and the state vector of the j^{th} identification channel in the k^{th} sampling period, respectively. $u(k)$ is the input of the multi-channel model in the k^{th} sampling period. The expressions of A, B can be seen in reference [13,15,19]. Thus, the out put of the multi-channel model is

$$y_m(k) = \sum_{j=1}^{N_1} y_m^{(j)}(k) \qquad (21)$$

The multi-channel model is shown in figure 2. In each channel, the linear block is represented by Laguerre Functional Series Model which is shown in figure 3. Thus, the multi-channel Laguerre Model for Hammerstein-typed nonlinear system is constituted by the equations (18-21), whose convergence analysis will be given in the following theorem.

Theorem 2:

For the Hammerstein-typed nonlinear system

$$y(k)=\sum_{l=1}^{N} c_l L_l\left(z^{-1}\right)\left[\sum_{i=1}^{r} a_i\left(g_i\left(u(k)\right)+\delta_i(k)\right)\right]+\gamma(k) \quad (22)$$

where $\|a_i\|_2=1$, $\delta_i(k)$ is the unmodeled part in the k^{th} sampling period; $L\left(z^{-1}\right)=\left[L_1\left(z^{-1}\right),L_2\left(z^{-1}\right),\cdots,L_N\left(z^{-1}\right)\right]$ is the Laguerre state vector in Z-domain. The unmodeled part $|\delta_i|$ and the nonlinear bases $g_i(\cdot)$ $(i=1,\cdots,r)$ are both bounded. If the input $u(k)$ is **PE** (persistence exiting), and is uncorrelated with the output white noise $\gamma(k)$, then for $\forall\,\varepsilon>0$, $\exists N_1\geq 1$ and $N_\varepsilon>0$ which make the output of the multi-channel Laguerre Model (18-21) satisfy that

$$\left[y_m(k)-y(k)\right]^2\leq\varepsilon,\ \left(\forall k>N_\varepsilon\right) \quad (23)$$

Proof: Firstly, apply (9) to identify the system (2) to get the LSE matrix $\hat{\theta}$ of θ, and then compute the matrix $\hat{\Theta}_{ac}$.

The linear block is stable, and $g_i\left(u(k)\right),(i=1,2,\cdots,r)$ is bounded, so the model output $y_m(k)$ is bounded. Take equations (18-21) and (5) into consideration, we have the 2-norm of $\phi(k)$ is bounded, in other words, $\exists R_L>0$ which makes $\phi(k)$ satisfy that $\|\phi(k)\|_2^2\leq R_L$. (24)

For $\forall\,\varepsilon>0$, $\exists\varepsilon_1,\varepsilon_2>0$, which fulfill that $\varepsilon=\varepsilon_1+\varepsilon_2$, let $\varepsilon_3=\varepsilon_1/(\max(r,N)R_L)$ and $\varepsilon_4=\varepsilon_2/R_L$. Because $u(k)$ is **PE** and unrelated with $\gamma(k)$, from reference [16], we have that for $\forall\,\varepsilon_4>0$, $\exists N_\varepsilon>1$, when the number of samples $S>N_\varepsilon$, the following inequality is satisfied

$$\left\|\hat{\theta}-\theta\right\|_2^2\leq\varepsilon_4 \quad (25)$$

Use **Lemma 1** to make SVD for the matrix $\hat{\Theta}_{ac}$, then the influence of the unmodelled dynamics $\delta_i(k)$ is contained in $\hat{\Theta}_{ac}$. Assume $rank\left(\hat{\Theta}_{ac}\right)=q$ where q is a finite integer, then from **Lemma 1**, we have that, for $\forall\,\varepsilon_3>0$, $\exists N_1\leq q$ which satisfies

$$\sum_{m=N_1+1}^{q}\sigma_m^2\leq\varepsilon_3 \quad (26)$$

in other words

$$\left\|\sum_{j=1}^{N_1}\sigma_j\mu_j\varphi_j-\hat{\Theta}_{ac}\right\|_2^2\leq\varepsilon_3 \quad (27)$$

Define

$$\hat{\theta}^{(j)}\overset{\Delta}{=}[c_1^{(j)}a_1^{(j)},c_1^{(j)}a_2^{(j)},\cdots,c_1^{(j)}a_r^{(j)},c_2^{(j)}a_1^{(j)},\cdots,c_2^{(j)}a_r^{(j)},\cdots,c_N^{(j)}a_r^{(j)}]^T \quad (28)$$

then from definition of matrix 2-norm, when $k>N_\varepsilon$, we can get

$$\left[y_m(k)-y(k)\right]^2=[\phi^T(k)(\sum_{j=1}^{N_1}\hat{\theta}^{(j)}-\hat{\theta}+\hat{\theta}-\theta)]^2\leq$$

$$\leq R_L\max(r,N)\left\|\sum_{j=1}^{N_1}\sigma_j\mu_j\varphi_j-\hat{\Theta}_{ac}\right\|_2^2+R_L\left\|\hat{\theta}-\theta\right\|_2^2=$$

$$R_L\max(r,N)\varepsilon_3+R_L\varepsilon_4=\varepsilon \quad \blacksquare$$

The theoretical foundation of our proposed multi-channel model is established by **Theorem 2**.

III. CASE STUDIES

Plant: simplified pH neutralization process model [20]

$$x(k)=f(u(k))=u(k)-1.207u^2(k)+1.15u^3(k)+\delta(k) \quad (29)$$

$$G(z)=\frac{y(k)}{x(k)}=\frac{0.0185z^{-2}+0.0173z^{-3}+0.00248z^{-4}}{1-1.558z^{-1}+0.597z^{-2}} \quad (30)$$

where, assume the input

$$u(k)=0.2\cos(0.015k)+1.3\sin(0.005k)+0.4\sin(0.01k) \quad (31)$$

the unmodelled nonlinear part

$$\delta(k)=0.2\sqrt{|u(k)|}+0.083u^6(k) \quad (32)$$

Besides, the output white noise $\gamma(k)$ fulfills that

$$\mathbf{E}\left\{\gamma^2(k)/\mathbb{F}_{t-1}\right\}\overset{a.s.}{=}0.083 \quad (33)$$

The identification performances of the multi-channel Laguere model (see figure 2 and 3) are shown in figure 4 and 5. In detail, in the left subfigures, the blue dashdot and the purple solid curves are the system's input $u(k)$ and output $y(k)$, respectively. The blue solid, black dot, and black solid curves represent the outputs $y_m(k)$ of the models with 1 channel ($N_1=1$), 2 channels ($N_1=2$), and 3 channels ($N_1=3$) respectively. The parameters and the identification error characteristics are shown in table 1, in which, $r\left(\hat{\Theta}_{ac}\right)$ is the rank of $\hat{\Theta}_{ac}$. $E\left\{\left|e(k)\right|\right\}$ and $max\left\{\left|e(k)\right|\right\}$ are the mean and the maximum of the identification error's absolute value series, respectively.

From the experiment results, we can see that Gomez's single channel identification algorithm [1] has larger error, which may even degrade to unacceptable level (see table1 for $N_1=1$). Our proposed multi-channel identification algorithm can greatly enhance the identification accuracy, especially in the case that $\left|\sigma_{i+1}/\sigma_i\right|$ $\left(i=1,\cdots,r\left(\hat{\Theta}_{ac}\right)-1\right)$ are not small enough. Thus, the feasibility and superiority of this proposed algorithm are validated.

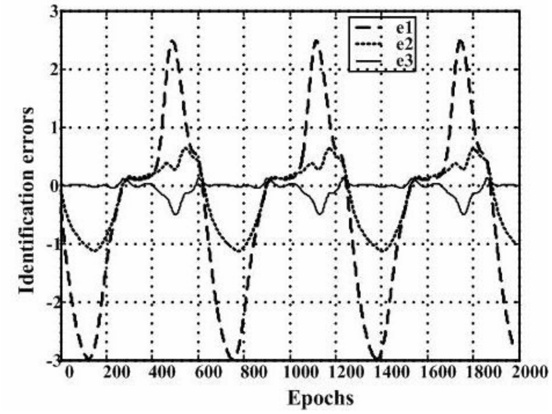

Fig. 4 Identification errors

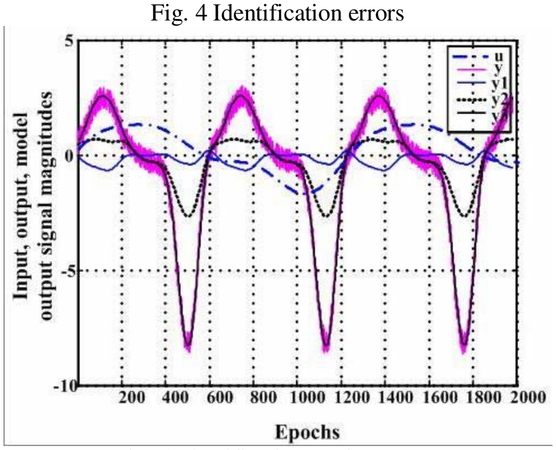

Fig. 5 Identification performances

TABLE I Modeling parameters and identification error characteristics

p	N	T	$r\left(\hat{\Theta}_{ac}\right)$	N_1	$E\left\{\left\|e(k)\right\|\right\}$	$max\left\{\left\|e(k)\right\|\right\}$
1.9	5	2	4	1	2.1810	7.9684
				2	0.6313	5.5323
				3	0.0727	0.2183

IV. CONCLUSIONS

In this paper, a SVD-based multi-channel algorithm for the identification of Hammerstein-typed nonlinear system has been proposed. The algorithm is numerically robust, since it relies only on LSE and SVD. Under the weak assumptions on the persistency of excitation of the input, the algorithm provides consistent estimates even in the presence of output noise. More important is that the algorithm can eliminate any needs for the mechanism or prior knowledge about the nonlinear block and can greatly reduce the identification error by using multi-channel model. The cost for the increase of accuracy is the moderate enhancement of the computational load. In addition, as the foundation of this multi-channel identification algorithm, we also give the convergence theorem. At last, a number of simulation experiment results on a benchmark problem, a pH neutralization process, validate the feasibility and superiority of the proposed multi-channel identification algorithm.

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